Mechanics Of Solids
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Lecture – 42
Stress Components

Welcome back to the course Mechanics of Solids. So, in the last lecture we developed this strain components right.

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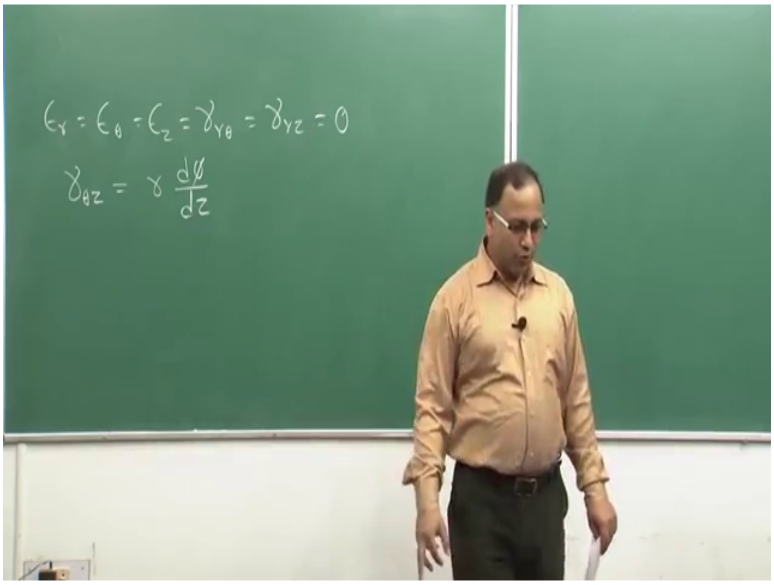

So, all the strain components are 0 except gamma theta z and that was derived as r into d phi d z and based on this strain components we can calculate the stress components using the Hooke's law whatever we have discussed earlier right. So, using Hooke's law we can simply write sigma r sigma theta sigma z equal to tau r theta equal to tau r z all are 0 and only remaining stress that is nothing, but tau theta z that is equal to G that is shear modulus multiplied by gamma theta z which is nothing, but G r d phi d z.

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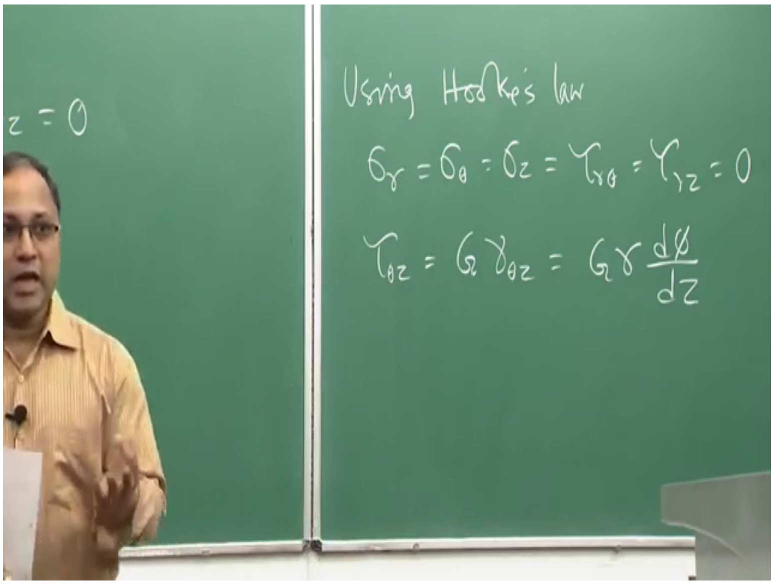

So, what we have got at this point that if you consider a circular shaft under constant twisting moment M t and if you consider the small deformation situation then you will be ending up with only one stress components that is tau theta z that is a shear stress happening on the on the cross section that is the shear stress happening on the cross section and based on this equation you can say shear stress is minimum that is 0 at the center of the circular shaft and that is becoming maximum at the periphery fine.

So, now let us come back to this figure. Now we will be talking about the equilibrium requirements, we will be talking about equilibrium requirements.

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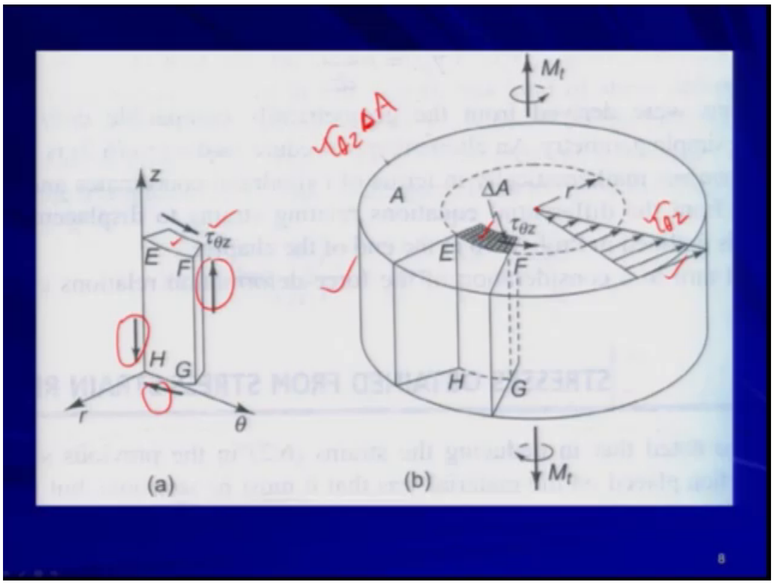

So, what is your equilibrium requirements because you are applying the twisting moment and that twisting moment is causing the development of this shear stress right.

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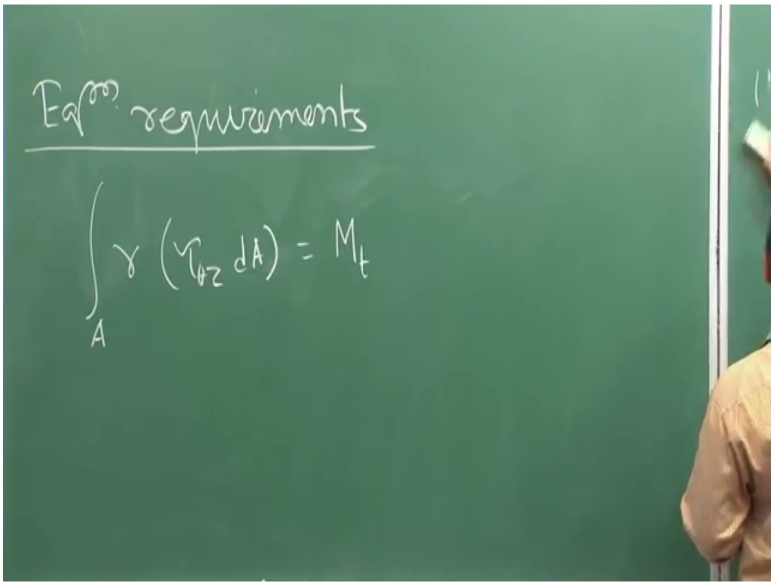

So, this shear stress is nothing, but the internal say adjustment this internal force shear force which is developed at each cross section that will try to balance the externally applied twisting moment M t right. So, that we are getting from this figure. So, here actually what you are getting you are getting tau theta z acting on the plain. So, this is one this the say cross sectional plain. So, therefore, to balance as you know. So, cross shear will be always equal, therefore, you will be having the shear stress like that. Now the same thing we are just showing in this particular segment say this is the variation of tau theta z this is minimum at the center and maximum at the periphery.

Now, we are considering a small area say delta A on the cross section. So, on that small area delta A your tau theta z is acting right. So, therefore, the shear force developed on that area is nothing, but tau theta z into delta A that is a shear force acting on that small element delta A area. So, this shaded area is nothing, but delta a. So, therefore, the shear force is nothing, but tau theta z into delta A.

Now if you integrate over the whole area that will try to balance the applied twisting moment agreed or not that is very simple right. This shear stress is getting developed due to the application of the external moment M t. So, if you do the integration over the whole area then whatever shear force will be getting that will try to balance the externally applied twisting moment M t. So, then the equilibrium requirements is nothing, but considering the equilibrium we can simply write over the whole area A r into tau theta z d A is equal to M t agreed. So, tau theta z into d A is a shear force acting on the small area delta A right and if you multiply that with r. So, that will give me the twisting moment caused by that shear force and if you integrate over the whole area whatever twisting moment you will be getting that should be equal to the externally applied twisting moment M t fine.

So, this is your equilibrium requirement I am coming from the equilibrium condition or satisfied from the equilibrium condition you can develop or you can satisfy or you can get this equation fine. Now we are going to develop the stress and deformation relation stress and deformation in a twisted circular shaft.

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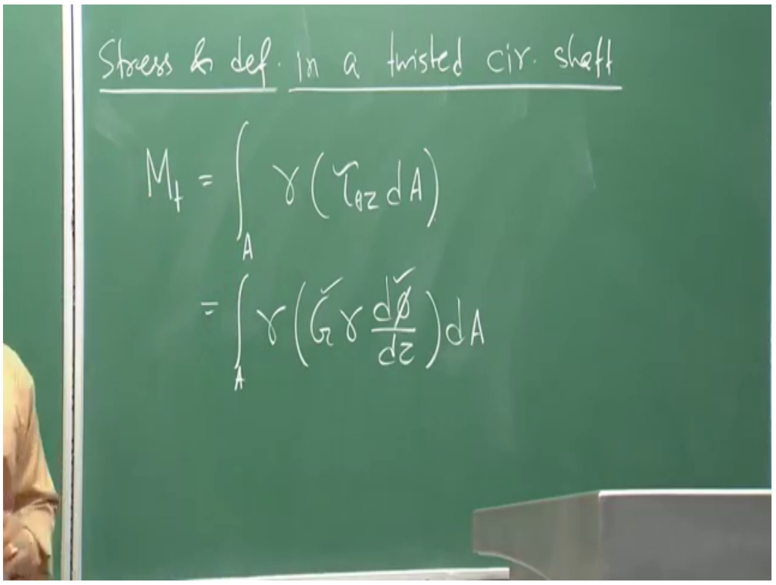

So, this I am calling as the equation 1. So, please stick to the equation number we will be referring this equation number subsequently. So, you have got from equation 1 we have got M t is integration over the whole area r into tau theta z d A this can be further written as integration over the whole area r instead of tau theta z I can simply write G r. So, that is the relation already we have established d phi d z into d A fine, that can be further written as because G is constant this is constant this is also constant because rate of twist or the twist per unit length that is also constant.

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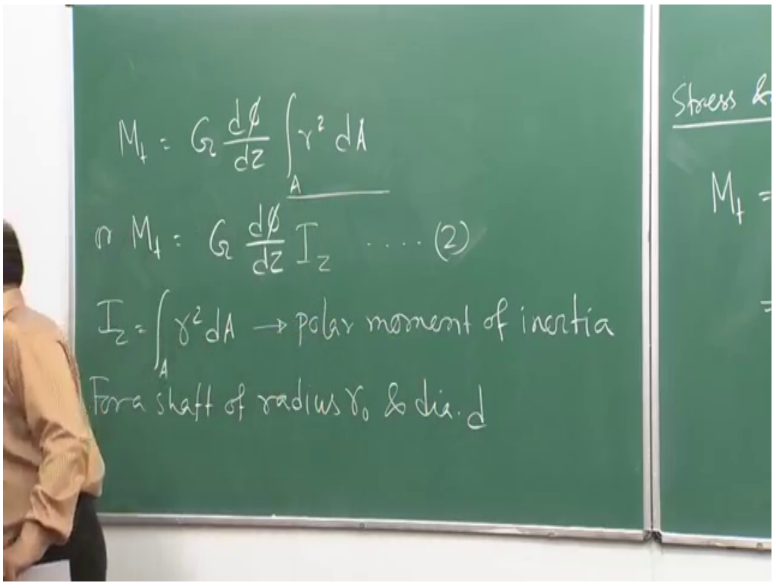

So, therefore, we can simply write.M t equal to G d phi d z r square t a integration over the whole area. So, or we can write M t equal to G into d phi d z.

Now please tell me because this is from your earlier knowledge right what is this, integration over the whole area r square d A this is nothing, but the polar moment of inertia if you go back to your I mean physics class or I mean earlier mechanics class or whatever you have gone through. So, if you look back. So, this expression is nothing, but the expression for polar moment of inertia. So, therefore, I can simply write I z because with respect to z axis you are considering the moment of inertia and that I am calling as equation 1. So, I z we know that is integration r square d A and that is your polar moment of inertia since we are considering the circular shaft. So, we know that for a shaft of radius r naught and diameter say d your I z is simply equal to, your I z equal to pi r naught to the power 4 by 2 or in terms of diameter pi d 4 it is 4 by 32. So, this you can calculate or you can determine by using the basic principle I am just giving you the expression for polar moment of inertia for the circular shaft.

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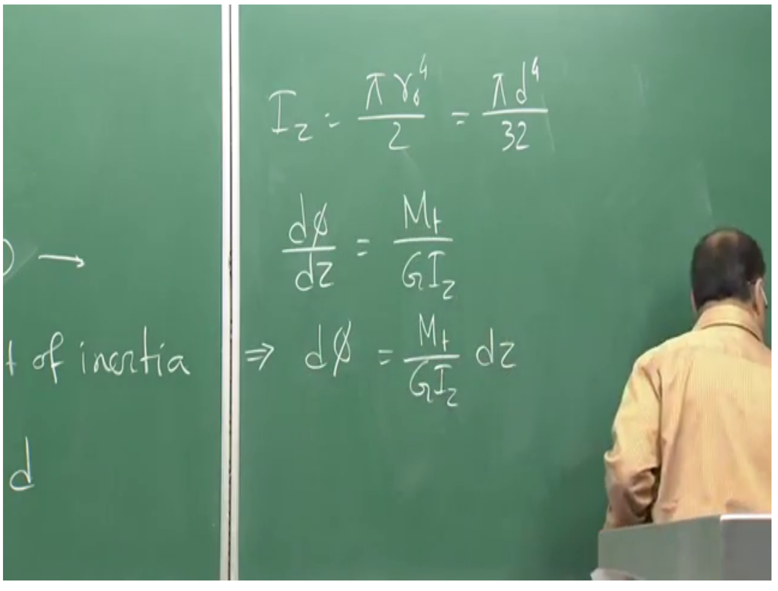

Now, from equation 2 I can simply write d phi d z from equation 2 I can simply write d phi d z is equal to M t by G I z which can be further written as d phi is equal to M t by G I z d z which can be integrated over this limits which can be integrated over this limit say 0 to phi d phi equal to 0 to L M t G I z d z.

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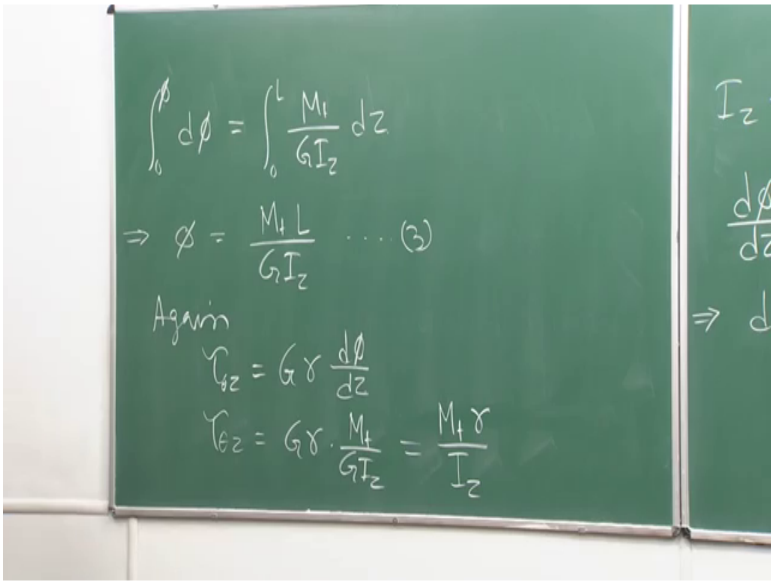

Now what it gives me? So, it gives me say phi equal to M t L G I z. Now here if you look at if I consider say I am considering the length of the say some number is L in the z direction the length of the random number is L. Now if I consider the twist I am relative that is a relative twist right, so bottom I mean say plain bottom most plain is experiencing say 0 twist and the top most plain is experiencing say twist of phi then this d phi is basically getting the range from 0 to phi and that is ranging from 0; that means, at the bottom plain to the top plain that is L. So, what I am getting from this expression. So, this expression I am saying say equation 3.

Now, what I am getting from this? I am getting that angle of twist between two ends; that means, that means the bottom end and the top end right between two ends is given by this. So, if you want to find out how much twist is happening in that slender member due to the application of this twisting moment M t then this is the expression this is the expression for finding out the twist. Now from this it very very much cleared that as you increase your L that is a length of the slender member your twist will be more is not it, as you increase the length of the slender member your twist will be more with a same amount of twisting moment M t.

Now again we are just going back to this expression tau theta z already we have seen G r d phi d z. So, from this I can write tau theta z equal to G r instead of d phi d z I am writing this M t G I M t G I z fine. So, that gives me M t r by I z. So, what I am getting? I am getting the shear stress developed due to the application of M t in terms of the geometrical parameters. Now if you look at as you go towards the because that is; obviously, true because as for a constant amount of twisting moment as you go towards the periphery you will be experiencing maximum shear stress that is already known.

Now, if you know how much twisting moment you are applying if you know where exactly you are going to find out the shear stress at the periphery or in between or at the same I mean at the same time it will be 0 because r is 0 means that is 0. So, if you know if you know the magnitude of or the value of say radius r where exactly you want to find out the shear stress you know the polar moment of inertia of the circular shaft you can find out how much shear stress is getting developed at that particular location fine.

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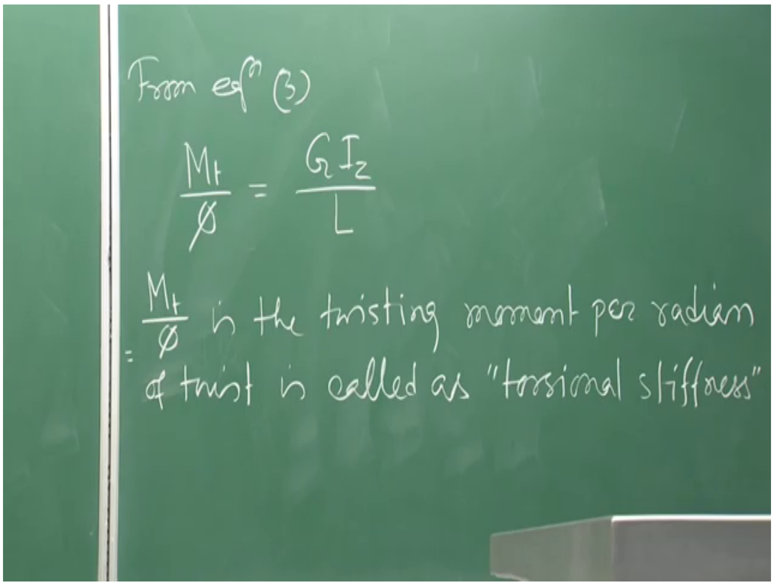

So, now, from equation 3 we can again write, from equation 3 we can write M t by phi is equal to G I z by n that is also very important relation where M t by phi it is very much similar to your say spring constant. If you recall what is that the moment or the twisting moment required per unit twist right it is very much similar to if you try to draw or try to map with I mean try to map this with your say spring constant spring whatever what is the definition of spring constant force required for per unit displacement right. Similarly this is the twisting moment or the moment required for per unit of twist.

So, this M t by phi is the twisting moment per radian of twist and is called as torsional stiffness like your spring constant that is a part of torsional stiffness. So, now, one thing is very clear from this discussion that if you deal with a particular material then G is constant, if you deal with steel, steel shaft or aluminium shaft or say copper shaft your G is constant because G is the material property. And if you are dealing with a particular length of the shaft then L is also constant right then basically your torsional stiffness will be dependent directly on your polar moment of inertia I z if you increase the polar moment of inertia your torsional stiffness also will increase fine.

So, now we will look at the torsion in hollow circular shaft, torsion of hollow circular shaft.

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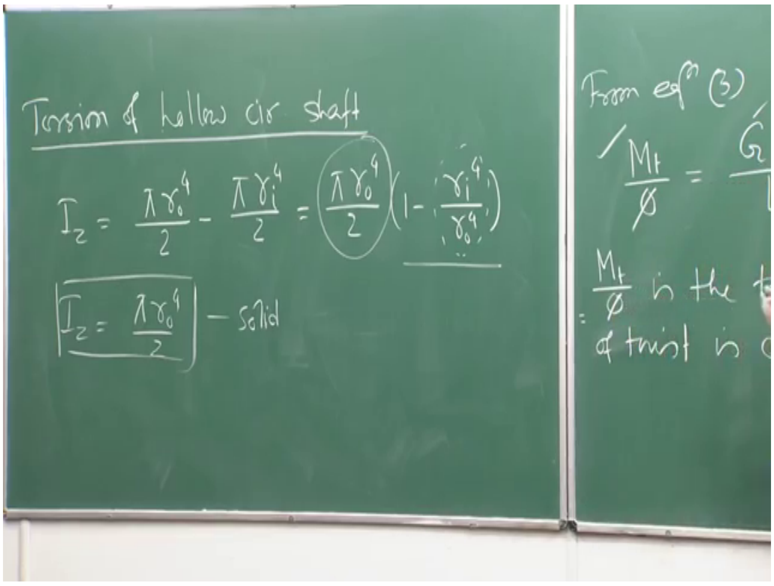

So, please come back to this figure. So, this is one hollow circular shaft where r i is the inner radius r o is the outer radius this is the variation of say shear stress tau theta z it is under the action of externally after twisting moment M t.

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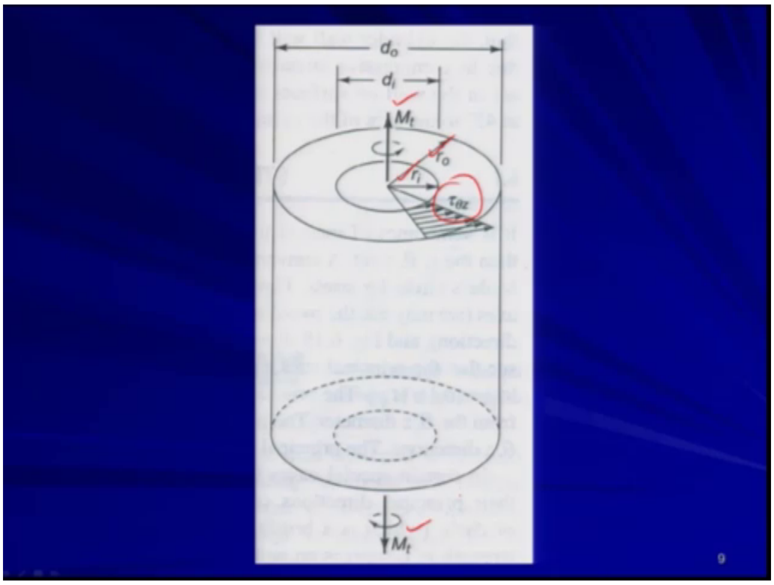

Now, for this hollow circular shaft your I z is simply equal to as we have calculated 2 minus pi r i to the power 4 2 right that gives me 1 minus r i to the power 4 r o to the power 4. Now what you are getting from this actually, your polar moment of inertia for a hollow circular shaft is given by this expression fine.

Now if you look at this term this term is basically giving me the fourth power of r i by r naught radius I mean ratio right, fourth power of r i by r naught ratio. Now basically the as I told you that I mean if your G and L is I mean they are constant then your torsional stiffness is complete dependent on the polar moment of inertia. Now if you look at this expression if you apply say solid circular shaft then I z will be simply for solid now for hollow this I mean this polar moment of inertia is taking the expression like this now this part is very very negligible is not it, r i by r naught to the power 4 that ratio or with the fourth power. So, that is why it is negligible.

So, this basically this hollow if you consider the hollow circular shaft then basically your polar moment of inertia is not getting affected much that I mean to say your actual polar moment of inertia for the solid circular shaft is this, but when you are considering the I mean hollow shaft then your polar moment of inertia is this one. So, this is the part which is getting multiplied with this factor. So, this factor is almost equal to one because this ratio is taking to the fourth power. So, if you consider hollow circular shaft your polar moment of inertia is not getting much affected. So, if your G that is a shear modulus if you are dealing with same material say you are having one solid circular shaft with say steel made of steel and one hollow circular shaft again made of steel then your G is same for both the shafts and if you are considering the same length of the shaft then L is also same for both the shaft.

Now, after doing this analysis basically you are also saying or you are also getting the idea that your polar moment of inertia is not getting affected much right. So, always your idea should be that you use higher torsional stiffness right for any material any kind of shaft if you get higher torsional stiffness that will always be better right, but that you are not getting properly from this because I z z for both the cases are I mean almost comparable. So, you are not getting much enhancement in the torsional stiffness if you use a hollow circular shaft then what is the advantage of using so hollow circular shaft.

Now advantage is that you are getting almost same amount of polar moment of inertia and if you are using the same material like G I z L all are say same then basically you are getting the same amount of torsional stiffness. So, whether you are using the solid circular shaft or the hollow circular shaft you are getting the almost same amount of torsional stiffness, but at the same time you are getting reduction significant reduction in the weight of the shaft because you are taking the inner part or the inner material of the solid circular shaft out right. So, therefore, this hollow circular shaft is giving you the almost comparable or the same amount of torsional stiffness at lower amount of weight.

So, therefore, when I mean in different mechanical system or may be some other say mechanical say machines where the weight is an issue because this shaft will be carried by the bearing and all those things right when the weight is an issue then in that situation you will be suing the hollow circular shaft because your torsional stiffness will be remaining almost say comparable.

So, with this I will stop here today. So, in the next lecture will be continuing with the same discussion and will be solving couple of numerical problems.

Thank you very much.