Mechanics Of Solids

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Lecture – 41
Torsion

Welcome back to the course Mechanics of Solids. So, in the last lecture we started the new topic that is the torsion or the twisting moment in the circular slender member right and there the important conclusion we had drawn if you recall that the important conclusion was that based on your symmetry argument right, the plane section remain plane right that is the first argument or the conclusion we had drawn. And the second conclusion what we had drawn that is the straight diameters will be remaining straight to satisfy the symmetric condition.

So, based on these two conclusions ultimately if you consider a circular say slender member, I mean in the situation under the action of twisting moment say M t like whatever is shown in this figure.

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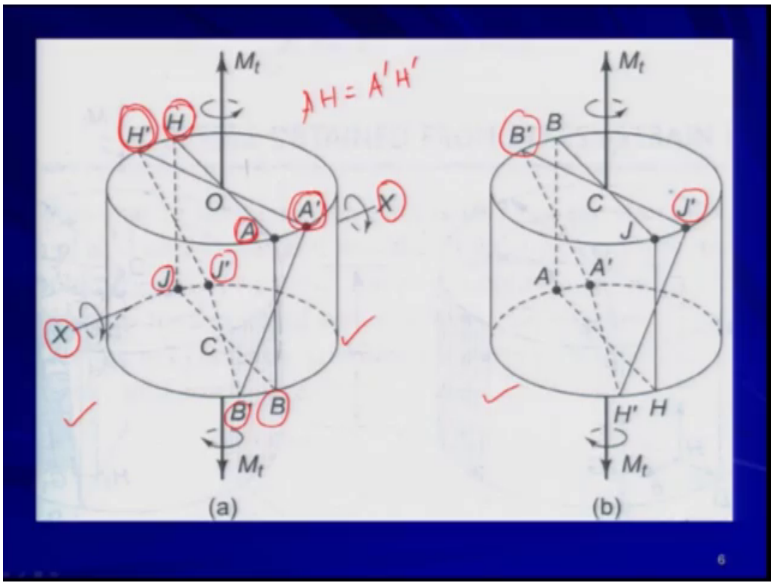

So, your diameter in this particular figure if you look at, so diameter say A H after application of the twist it is going to take the position A prime H prime and similarly B J on the bottom surface is taking the position at B prime and J prime right.

So, I mean you see this A H was a diameter before application of the twist and after application of the twist still it is a straight line A prime H prime. So, it is not the curved one because curved diameter has been ruled out right based on the symmetry argument whatever we had drawn or whatever we had concluded in the last lecture. So, and if you look at this kind of say the straight diameter remains straight. So, that argument basically will satisfy the symmetry argument as well, I mean for that I mean how we can prove that suppose we are doing the same thing. So, this is the say part say figure one in this figure one whatever we have got after the deformation now we are giving the rotation with respect to X axis. So, this B J B prime J prime is going on the top surface now the bottom surface is going to the top surface and top surface is coming back to the bottom surface right.

So, this orientation of B prime J prime is such that that it will match say it will match the lower part of the upper half I mean to say if you have two segments say this is the circular say circular say slender member you are considering two segments right. So, you have to match at the interface right. So, that matching at the interface is getting satisfied by considering the straight diameter even after application of the twisting moment.

So, now, what we can conclude from this. So, in this particular say kind of say twisting or the torsion chapter basically we will be considering only the deformation like this; that means, you are getting some kind of say displacement in the diametrical position. So, A H they are going to a prime H prime, but they are not getting elongated they are not getting curved shaped and all those things are ruled out by following the symmetry argument.

So, this type of configuration will be valid and we are going to take this kind of or we are going to analyse rest of things based on this configuration. So, now, the important thing is that we are going to talk about something about the strain happened during this kind of say torsion. So, we are going to assume something and there is some basis of such assumptions first one is that there is no extension or contraction in z direction.

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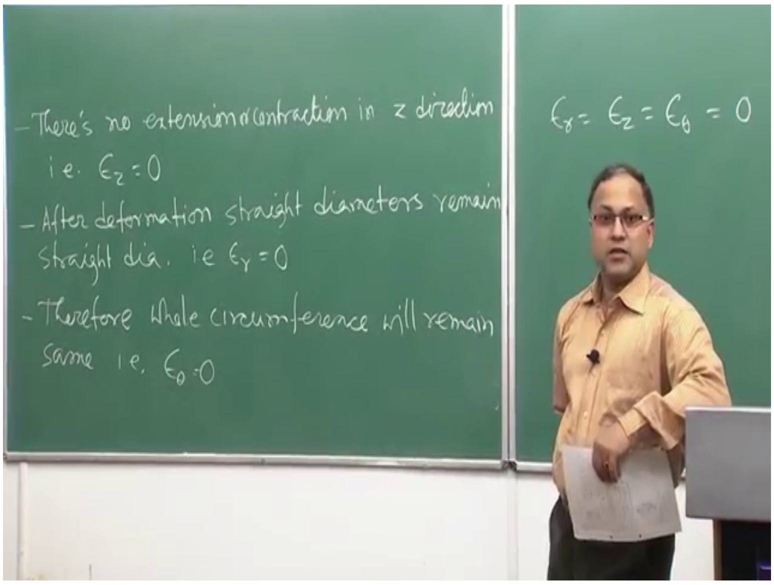

What does it mean? So, as I as I told you that z direction is say if this is my say this you can consider this chalk as one of the circular as slender member now I am applying the twisting moment here. So, the z is the vertical direction along the axis of the slender member now the statement is I mean that is quite obvious right if you consider a small strain problem right if you are giving a small strain in the z direction you will not observe any elongation or contraction in this in this member right if you if you apply some twist are you getting any elongation or contraction in this chalk no right.

So, therefore, in the z direction there is no extension or contraction. So, if that is show then what we can assume, we assume that the strain in the z direction is 0 simply. So, that tells me epsilon z is 0 agreed. Now the second assumption that after deformation straight diameters remain straight, straight diameter right, already we have seen from the symmetric algorithm that the straight diameters remain same straight even after deformation.

So, if the starlight diameter remains same remains straight even after deformation then basically if you look back this figure this A H was a diameter initially right, now this A H is I mean they this diameter is taking the position A prime H prime, now what you can conclude from that. So, A H is nothing, but equal to A prime H prime what does mean then; that means, the straight diameter remains straight that is fine, but there is no change in length in the diameter because we are dealing with a small strain problem. So, therefore, we are not considering any change in length in diameter; that means, the deformation is not happening in the diametrical direction. Therefore what we can conclude from that. So, that is epsilon r is 0; that means, in the radial direction your strain is 0 because diameter is not getting changed.

Then the third assumption is if the diameter is not getting changed if your strain in the radial direction is 0 then we can say therefore, whole circumference will remain same because in the polar coordinate system you have three direction right one is a z direction one is a r direction that is radial direction another one is theta direction that is along the circumference right. So, if there is no change in the radial direction then there must be no change in the circumference also because if diameter is not getting changed then why the circumference will change right it should not change.

So, therefore, whole circumference will remain same if that is remaining same then we can say epsilon theta is also 0 agreed. So, what we are getting from this discussion that epsilon r is equal to epsilon z is equal to epsilon theta all are 0 in this particular situation when you considering circular slender member under torsion and you are also considering small strain problem. So, under that circumstances you would be getting epsilon r equal to epsilon z equal to epsilon theta equal to 0 fine.

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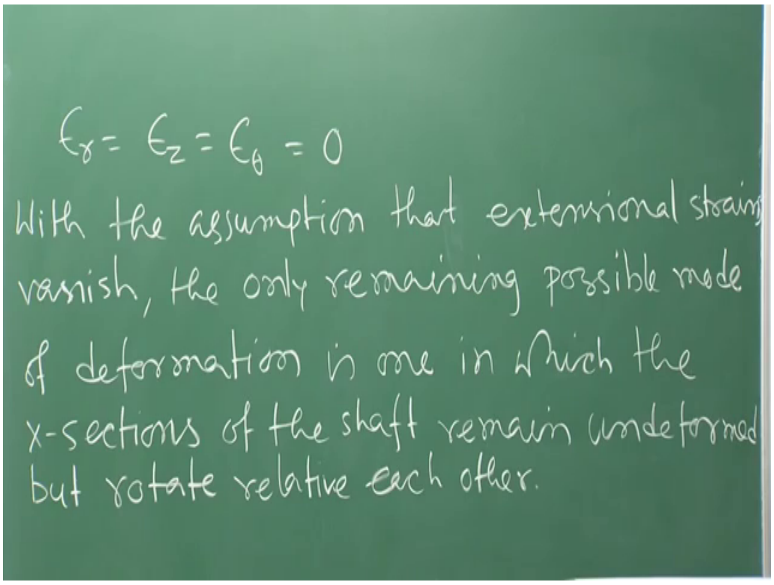

So, now, with the assumption that extension, that extensional strain vanish strains extensional strains means this epsilon r epsilon z epsilon theta those are all normal strains that is extensional strains either extension or contraction right. With the assumption that extensional strains vanish the only remaining possible mode of deformation is one in which the cross sections of the shaft remain undeformed, but rotate relative to each other. So, this we can conclude from this discussion that with a assumption because we are assuming this, because these are things these are the basis of those assumptions with a assumption that extensional strains vanish because epsilon r, epsilon z epsilon theta all are 0 the only remaining possible mode of deformation is one in which the cross sections of the shaft remain undeformed, but rotate relative to each other.

Now, what does it mean? Say extensional strains are 0 that is fine, but only thing you see here actually you are not getting any deformation in the diameter right. So, diameter is not getting any change right A H remains equal to A prime H prime. So, therefore, what is happening actually that your deformation I mean the shaft remain I mean the cross section of the shaft remain undeformed right cross sections of the shaft remain un deformed I mean it is not happening like the circular cross section is becoming elliptical after the deformation it is not happening like that the cross sections remain un deformed.

Only you see happening that the rotate relative to each other; that means, A point is going to A prime point and H point is going to H prime point similarly B point is going to B prime point and J, J point is going to J prime point; that means, the points on the circumference are only changing with respect to each other that is all. This is the kind of strain or the deformation you are getting in the material. So, your cross section is not changed cross section remains circular and the if I mean not only circular because epsilon r is 0 therefore, you are not getting any change in the circumference; that means, the whatever diameter was there initially that will be remaining same and at the same time the circular cross section will be remaining circular because it will not be becoming as elliptics or elliptic cross section right that is ruled out.

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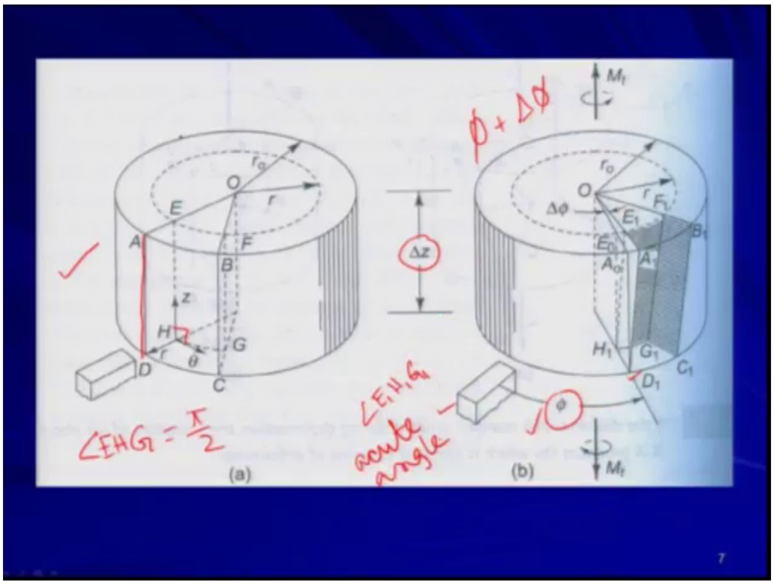

So, now if we look at this shear and if we try to analyse the prior diagram, if you look at this figure this figure the bottom section actually this is one segment we are taking one segment from the circular shaft and this is the say depth of the segment delta z fine. Now what I am saying the rotation of after due to the application of the twisting moment the oration of the bottom section is say phi which is the say initially it was here and after that it is going here right.

So, this rotation of the bottom section is say phi and the rotation of the top section will be it will be more than phi and that is nothing, but phi plus delta phi. So, what is happening this A D this line initially this was the line now this line is coming here. So, this is basically coming here A 1 D 1. The A D is taking the position A 1 D 1. So, D 1 from D to D 1 the rotation is say phi whatever we have just talked about and the rotation between a point and a one point is nothing, but phi plus phi plus delta phi agreed.

So, the top section will be rotating by an amount phi plus delta phi. Therefore, the relative rotation causes the rectangular element. So, earlier this was rectangular element say this is say E F, E F. So, I mean (Refer Time: 16:42) rotation is causing that is a rectangular element E F. So, this is the rectangular element E F G h. So, if you see this rectangular element is becoming one parallelogram E F 1 E F 1 G 1 H 1. So, this relative rotation is making that rectangular part E F G H to a parallelogram E 1 F 1 G 1 H 1.

Now in this part if you look at the originally right angle. So, now, the originally this was a right angle right E H G. So, this was originally a right angle E H G angle E H G it was 90 degree. Now, after rotation after this relative rotation what is happening, now it is taking E one H one G one right and that is nothing, but some acute angle not it does not remain 90 degree agreed because the rectangular portion E F G H is becoming the parallelogram and therefore, this angle if you try to visualise this angle E 1 H 1 G 1 they that does not remain 90 degree that becomes some acute angle which is less than 90 degree because some shear strain is happening now what is the magnitude of shear strain as you know that is change in angle right.

So, therefore, I can write because that is happening in theta z plane if you see that is happening in theta z plane. So, therefore, I can write gamma theta, z that is strain in theta z plane that is simply equal to limit del z tends to 0 E naught E 1, if you look at the figure you will get it what is E naught E 1 that is the part which is the extra the additional part due to the rotation at the top section. So, E naught E 1 by H 1 E naught agreed.

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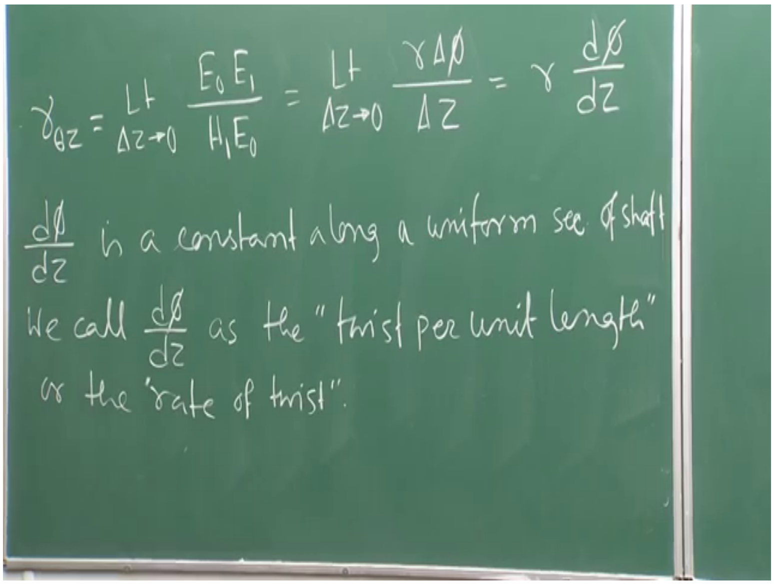

So, this part can be written as E naught E one if you see and the radius is nothing, but r that is that is shown here. So, if I consider this circumference this circumference the radius is r. So, r into delta phi is nothing, but E naught E 1. So, r into delta phi is E naught E 1 divided by delta z H 1 E naught is nothing, but the delta z that is the depth of the section in the z direction. So, that gives me in the limiting condition that gives me r d phi d z. Very important relation we have established between the shear strain and the radius of the (Refer Time: 20:28) member. Now if you look at this d phi d z there is a special thing for the d phi d z, now what is that. So, d phi d z is nothing, but the twist per unit length is not it twist per unit length and that is always constant because the thing is that we are considering the isotropic material we are considering small strain.

So, therefore, if you consider any per length whatever behaviour you will be getting in the twist I mean say because you have I mean all the sections as you have seen n the last class where all the sections are experiencing the same amount of twist then why, there is no reason that you will be getting different twist part unit length right because material is same and that is also isotropic material. So, the twist will be also same per unit length right. So, therefore, this d phi d z is a constant along a uniform section of shaft and we call d phi d z as the twist part unit length or the rate of twist.

So, in some book it I written as twist per unit length in some book it is written as rate of twist. So, both are same do not get confused with that ok fine. So, therefore, if d phi d z is constant then I can simply write gamma theta z is directly proportional to radius r agreed because if d phi d z is constant, therefore, I can simply write that gamma theta z is directly proportional to r.

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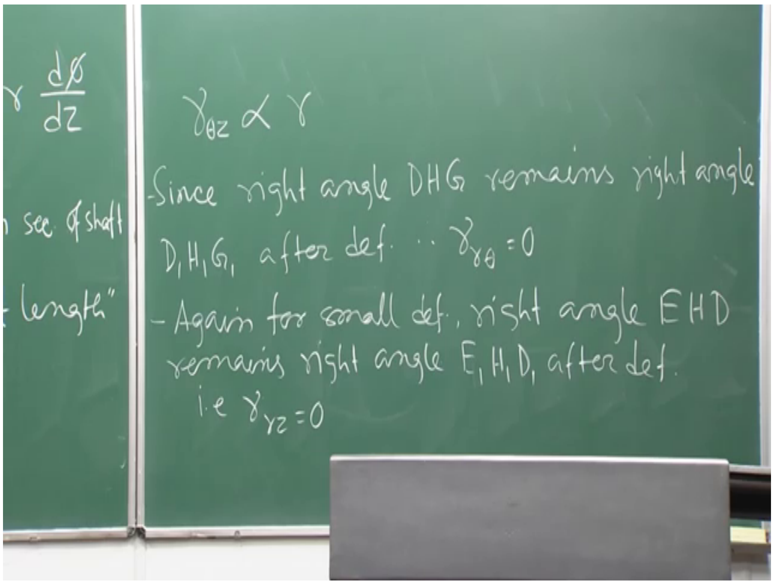

What does it mean then? That means, if you come back to this figure it means that at the center when r is 0 you will be getting your shear strain I mean this gamma theta z is simply becoming 0 right and that will be maximum at the circumference; that means, at the outer periphery. So, as r increases as your, I mean radius increases your shear strength is increasing right. So, this is the variation of shear strain right from the center to the extreme periphery fine.

Now, I have I mean from this discussion what we have got, we have got that three normal strain components are simply 0 right one shear strain component that is gamma theta z is nothing, but r into d phi d z that already also we have established, but other three other two shear strain components we need to specify or we need to find out right. Now those are basically since for those actually shear strain components we can conclude something you can see that since right angle D H G you just look at the figure remains right angle D 1 H 1 G 1 after deformation.

Therefore your gamma r theta is simply 0 because there is no change in angle now what did I say D H G, D H G, this angle remains ninety degree in the deformed shape also right there is no change in this angle. So, therefore, there is no shear strain right, gamma r theta will be must be will be simply 0. Similarly again for small deformation problem right angle E H D remains right angle E 1 H 1 D 1 after deformation right therefore, there is no change in there is no change in angle.

So, therefore, we can simply write that is your gamma r z is equal to 0. Now what is this E H D angle. So, this is nothing, but this angle right this angle remains unchanged in E 1 H 1. So, that will be remaining 90 degree. So, therefore, there is no change. So, we have got all the strain components for this circular shaft under small deformation condition. So, we can simply write down epsilon r is equal to epsilon theta is equal to epsilon z is equal to gamma r theta is equal to gamma r z equal to 0, fine and only existing strain components that is gamma theta z is nothing, but r d phi d z fine.

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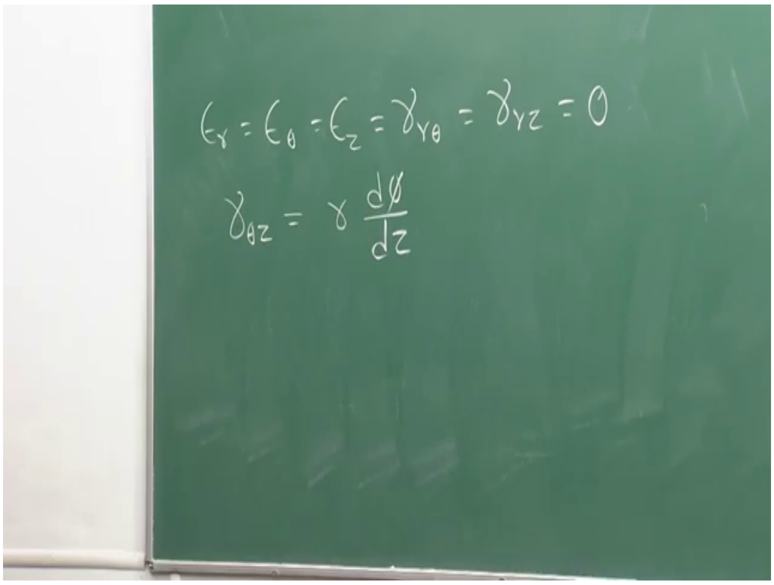

So, I will stop here today. So, in the next lecture we will be talking about the stress developed from this strain and we will try to establish the relation between the twisting moment and I mean other things, I mean your stress developed due to the twisting moment and all those things we will finding out in the next lecture.

Thank you very much.