Mechanics Of Solids

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Lecture – 40
Tutorial 4

Welcome back to the course Mechanics of Solids.

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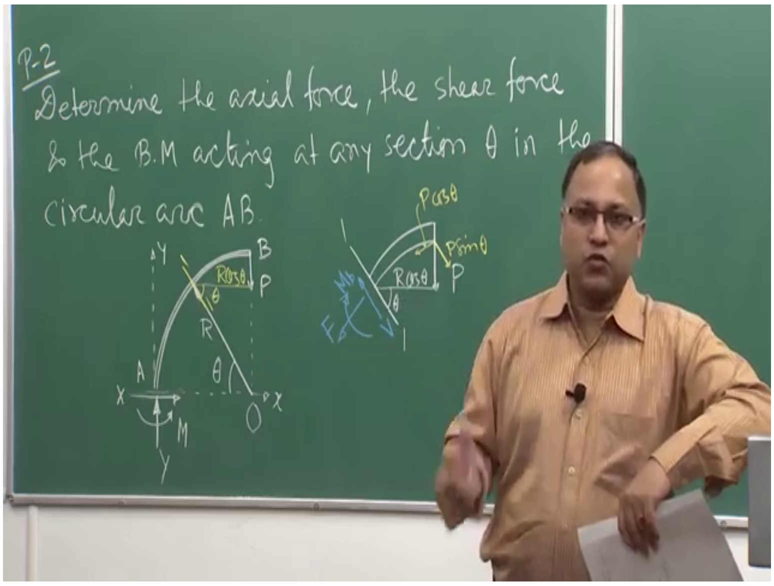

So, in the last lecture, we solved one numerical problem on the bending moment and shear force calculation and their diagrams. Now, another problem another numerical problem we will be taking today that is the problem says determine the axial force, the shear force and the bending moment acting at any section theta in the circular arc AB. So, AB is a circular arc as shown in this figure. So, at the end B, you are applying one vertical force P due to that action of external externally applied force p, you have to calculate some bending moment shear force and axial force at any section theta as shown here. So, O is a point I mean this point actually you can consider that is a centre of the arc circular arc.

So, what is my first job, if you want to solve this problem what is my first job first job is to draw the free body diagram. So, let us draw the free body diagram. So, we will remove this support and we will replace this support by applying the forces. So, y-direction force, x-direction force and moment because so you can have this is a y and this is x. So, these are the support reactions because that support was fixed support. So, therefore, we will get all the three components of reaction fine. So, this is the free body diagram of that particular say circular arc.

And as per the geometry actually. So, if I consider one section here, so this is my one section - section 1 1. So, this angle is theta. So, therefore, this is R cos theta from the geometry I can get it. So, if I draw now this is the free end basically this is a kind of cantilever theorem right this is a free end, there is no support here, the support is here only. So, it is always convenient for any cantilever structure it is always convenient to come from the P end. So, it will be coming also from the free end.

So, let us take the right hand side of the section 1 1 and draw the free body diagram of that. So, this is my section 1 1, and this is P sin theta and this is P cos theta component of P because this is R cos theta as shown here this is your section 1 1 this angle is theta. And you are getting the development of your shear force like this and bending moment like this and the axial force like this fine. In this case actually your load is not only in the I mean loading and all those things are not exactly in the x y (Refer Time: 04:33), therefore I mean it is in the x y plane that is ok, but the thing is that it is not I mean you do not have I mean the situation is not like that that you are not having a axial force. The earlier cases whatever you have seen that you did not have the axial force because the x-direction force was not there, but here the case is not like that you will be getting the axial force also we will see that.

So, therefore, the section will be associated with the shear force, bending moment as well as axial force. As we discussed that in x-y plane, if you have the loading you will be having all three components internal forces, internal forces two internal forces and one internal moment the bending moment fine.

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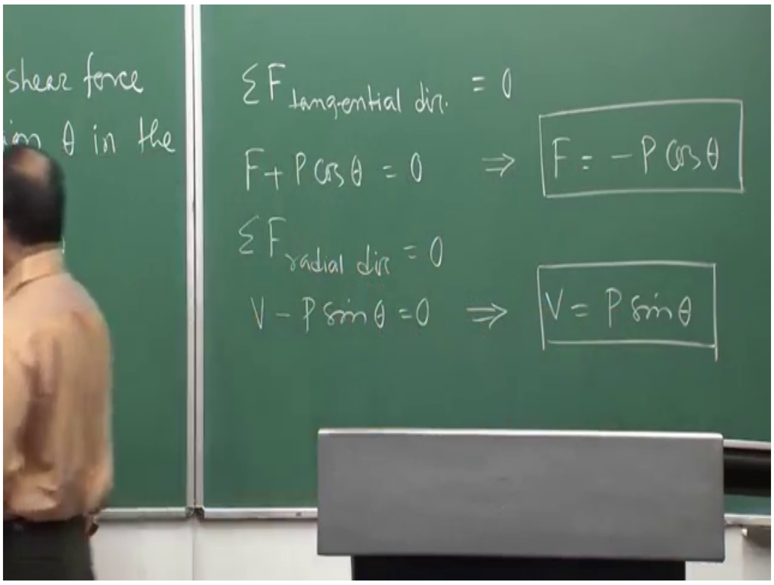

So, now, if you try to resolve the first is that if you try to consider the equation of equilibrium for F tangential direction is 0. So, whenever it is not mandatory that all the times you will be considering the equilibrium, equilibrium means it will be valid for any direction. So, I am considering the summation of all forces in the tangential direction is 0, I can do that. So, what are the forces that acting in the tangential direction if you look at this figure, this F that is axial force as well as this P cos theta, so F plus P cos theta is equal to 0. So, these are the forces acting in the tangential direction.

So, from this I can get the axial force F equal to minus P cos theta; minus sign tells that whatever direction we considered for F is not correct, fine. Then I am considering F summation of F in radial direction is 0, summation of all the forces in the radial direction is 0. So, now what are the forces are acting in the radial direction this V of course, this is the radial direction that is a shear force and this component of P that is P sin theta is also acting in the radial direction. So, therefore, I can simply write V minus P sin theta is equal to zero from which I can get V equal to P sin theta fine. Any issue? I hope it is right.

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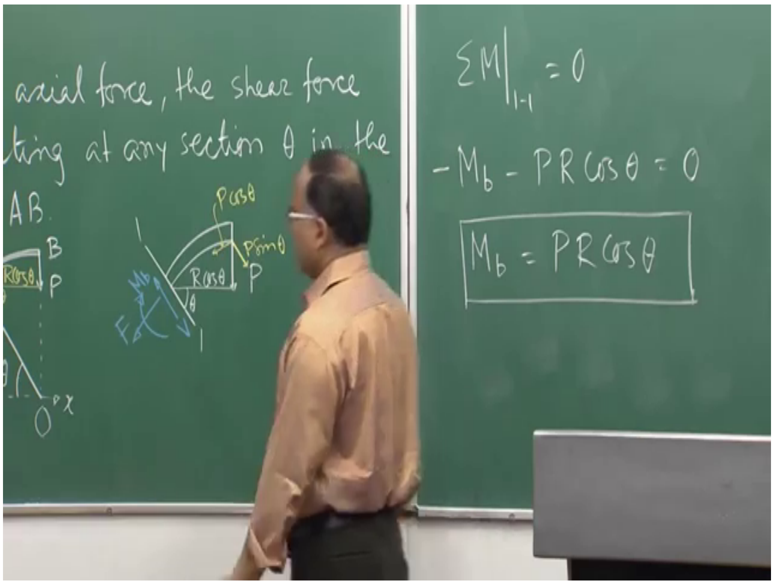

So, now, if we considered the moment, moment equilibrium now we are going to consider the moment equilibrium that is summation of moment with respect to section 1 1 is zero. So, there the moment M b is acting is fine. So, clockwise movement I am considering negative as per our sign convention and then this P I am not considering the components rather this P into R cos theta will give you the clockwise movement. So, minus M b minus P R cos theta is equal to 0. So, from this I can get b equal to P R cos theta. So, you have got shear force, axial force as well as bending moment at any particular section which is making an angle theta with the x axis fine.

So, with this I conclude your bending moment and shear force chapter. I hope that you have understood the concept. This is basically the new concept you are getting because bending moment and shear force and this is nothing but the backbone of the analysis or things of your mechanics of solids, I mean for any cylinder member. So, we will see that later on when you will be finding out the deflection of the cylinder member when you will be finding out the stresses developed in the cylinder member, so for all the cases you need to obtain the bending moment and shear force at any particular location or particular section. So, with this I conclude, now we will be moving to the next chapter that is torsion of circular shafts and thin walled tubes fine.

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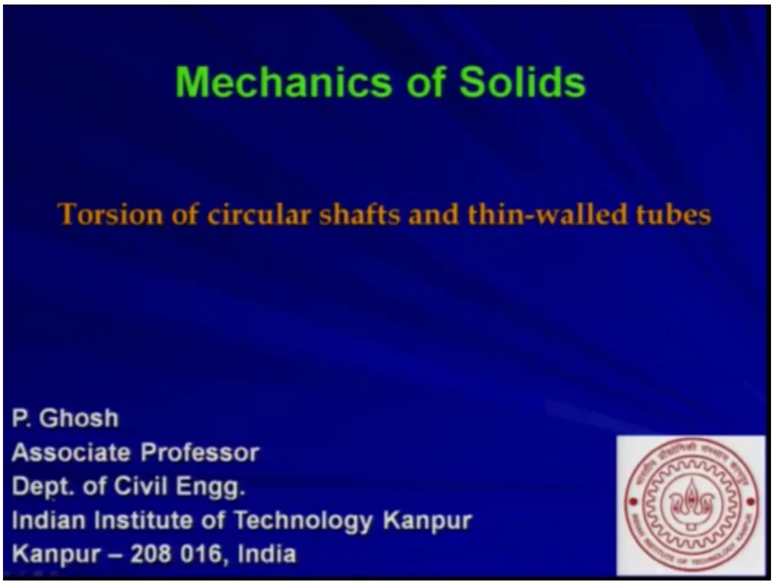

So, now we are going to talk about the torsion circular shafts and thin walled tubes. So, as I told you in the very first class that we are not going to talk about the torsion in the rectangular shaft because that is beyond the scope of this particular course.

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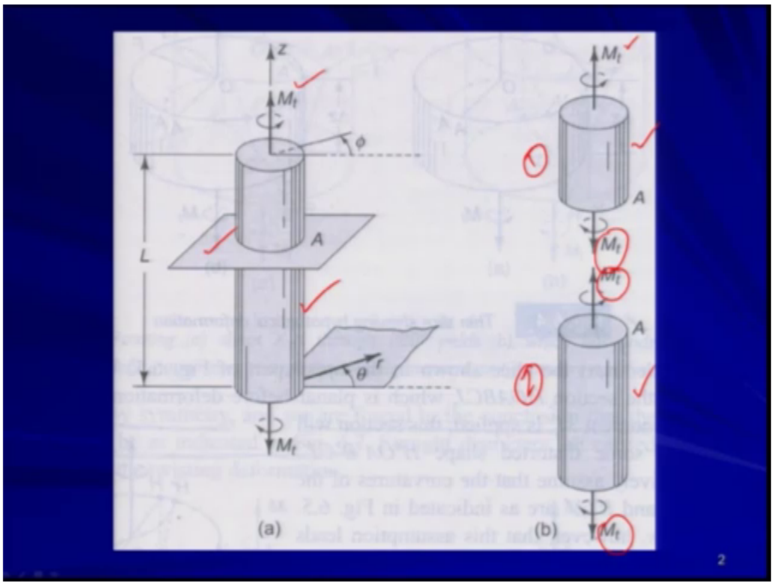

Now, if you consider this kind of say circular shaft, so for the time being, we are considering the solid shaft. And this shaft is basically under the action of the torsional moment M t, the circular shaft loaded only by twisting moment M t at its ends at both ends you are having M t that means, you are having a circular shaft you are giving the twisting moment like that. And the equilibrium requires that each cross section of the shaft must transmit the twisting moment M t is not it? Because if you consider two parts this is one part and this another part say and you are cutting at say this plane you are cutting this shaft and you are separating out these two parts this is one say and this is part two. You will see that two satisfy your equilibrium here if you have the M t, you should have M t here also and you have you should have here also and here also so that means at the equilibrium requires that each cross section of the shaft must transmit the twisting moment M t that you have got it.

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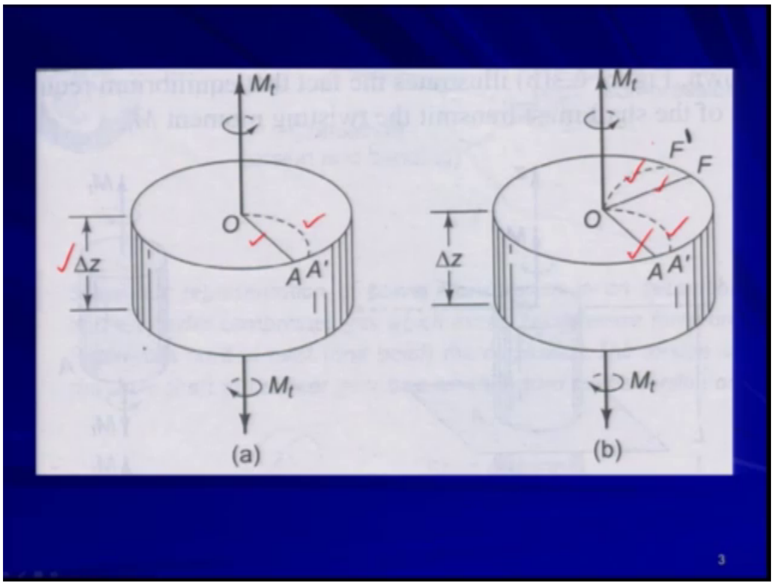

Now, we are going to talk about the symmetry argument. Now, what is that? Now, basically if you look at if you if you consider one particular cross section say this is one cross section some I am taking some slices. So, I have the complete shaft, I am taking some slices of say depth delta z. And there we are considering that the originally straight radius OA before application of the twisting moment, the OA was the original radius. Now, this OA is taking the shape of some curved surface OA prime after giving the twist. What did I say, the OA was the original radius before application of the twisting moment. Now, after application of the twisting moment that OA takes the shape of some curved surface or curved line that is OA prime; and because this is a kind of say isotropic material. So, I mean there is no reason that you will be getting the different configuration for different radius. So, if OA is experiencing the twisting moment and getting the shape of OA prime, so therefore, OF another radius on the same plane will be taking the shape of OF prime and so on the curved shape. So, the initial straight radius is becoming the curved lines after application of the twisting moment.

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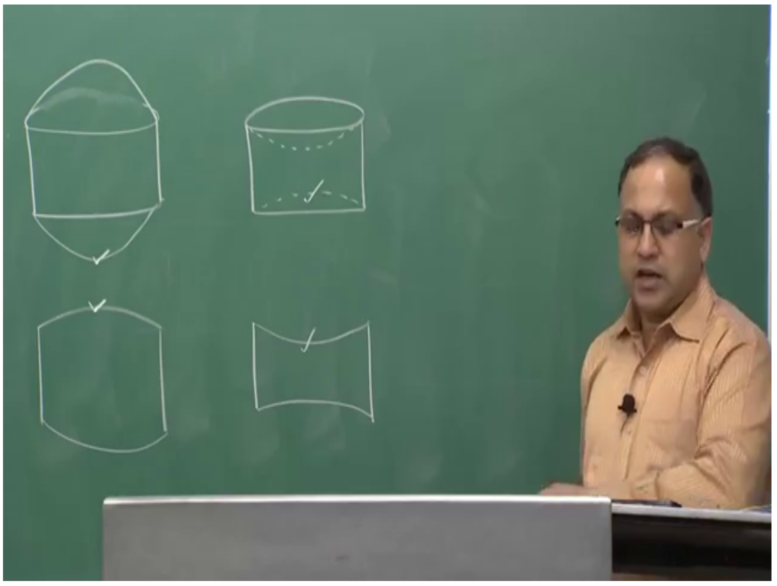

Now, suppose that one end of the slice, now suppose the one end of the slice, so this is the thing. Now, one end of the slice is getting bulged up after the due to the application of the twist. Let us say I mean suppose I do not know what kind of shape it will take just we are assuming that after the application of the twist or the torsional moment that this surface is getting bulged out. So, therefore, to satisfy equilibrium condition, the lower part also should bulged out is not it, it will also should bulged out or suppose it is not bulging out rather it is getting destined in some kind of depression you are getting. You may get the bulging out or destined situation or the configuration. I do not know which configuration it will take, I am just assuming. I am assuming that I am after the application twisting moment, it might take this kind of shape like your muffin or it might take this kind of shape where you are getting the depression like that.

Now, if you consider the another segment of the shaft that will also will be having the bulging out or some depression like that. Now, do you think now basically this is a continuous shaft right, and when you are applying some twisting moment, do you consider that segments are getting say separated out no, that is a continuous cylinder member that is a continuous circular shaft. So, if it gets if the upper part is getting bulged out, the lower part will also be getting bulged out. So, this part and this part will never match together right. So, it will be impossible to fit a number of such deformed slices together to form a continuous shaft.

So, this part and this part will not fit at all at all; even this part this distain or the depressed part will not be fitting to this part also, am I right. So, if you get this kind of configuration after deformation, after the application of the twisting moment then basically your geometric compatibility is not getting satisfied. So, to satisfy this, we can conclude that when a circular shaft is twisted its cross section must remain plane.

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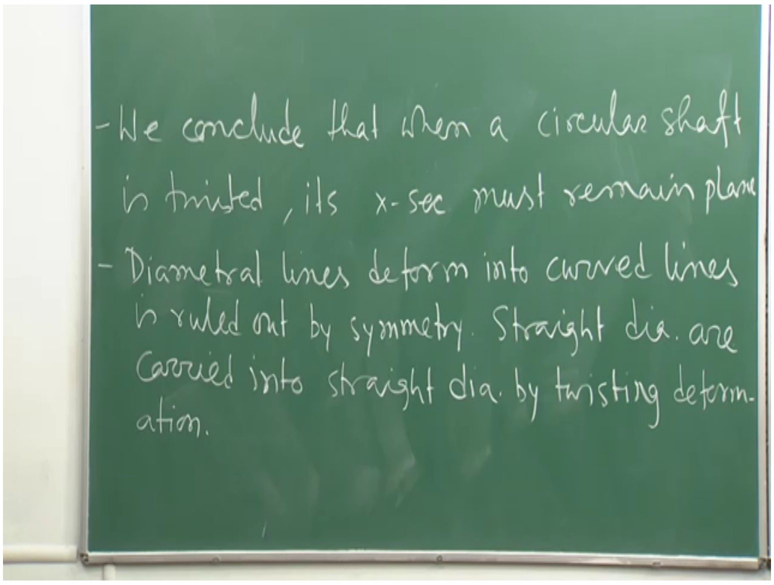

So, what we can conclude, we can conclude that we conclude, based on this discussion we conclude that when a circular shaft is twisted; its cross section must remain plane. So, this kind of bulging out or distinct configuration is not acceptable, they are ruled out. So, only the plane section will be remaining plane even after the application of the twist. So, that is the first conclusion we are drawing from the symmetry argument. So, now what we can say because we still do not know how the configuration or how I mean deformation will take place, how the configuration of or the deform shape will be there, so that we can find out from this figure.

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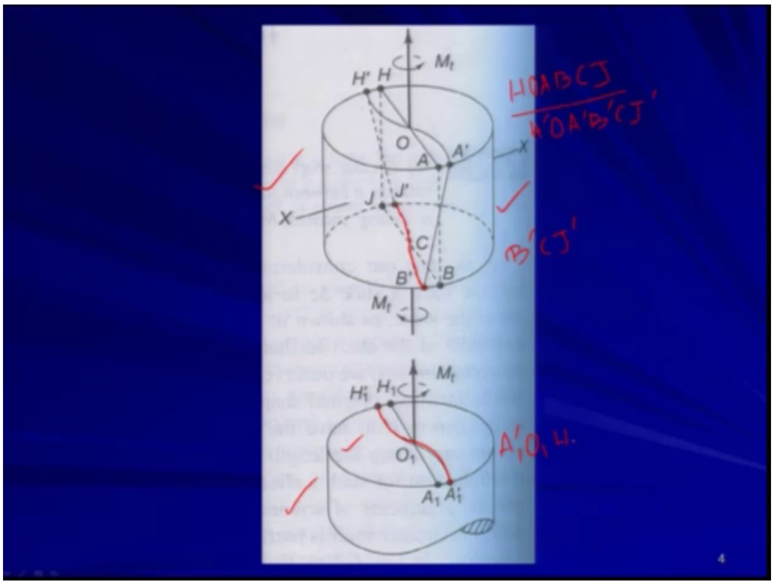

Say in this figure, we have before the application of the twisting moment, we have this plane HOABCJ, so this is a plane. Can you visualize the plane? So, before deformation I am considering one plane in this segment under the action of this twisting moment now after application of the twisting moment this plane is taking the shape H prime, OA prime, B prime, C, J prime. So, after application of the twist let us say it is taking the shape like that I mean from our bare eye or without any say I mean say discussion we can think of like that.

Now, if the if the curvature or this line, so what is a shape of B B prime C J. So, this is the shape of B prime C J line C J prime line. So, this is the shape of B prime C j prime line at the bottom surface. Now, just below that you are having another segment, you can consider like this which will be behaving very similar to this segment. If it behaves very similar to this segment then of course on the top surface, this is at the top surface. On the top surface you will be getting this kind of deformed shape A 1 prime O 1 H 1 prime, A 1 prime O 1 H 1 prime. And this top surface will be connected or will be touching the bottom surface of this segment the upper segment. Now, if their radius or the rotation of the radius is such that that it makes B prime C j prime line in the upper part; in the lower part it makes A 1 prime O 1 and H 1 prime. Are they matching? This is the first question, they will never match, so they will never match, is not it? This line the line on the bottom surface is not matching with the line on the top surface of the next segment.

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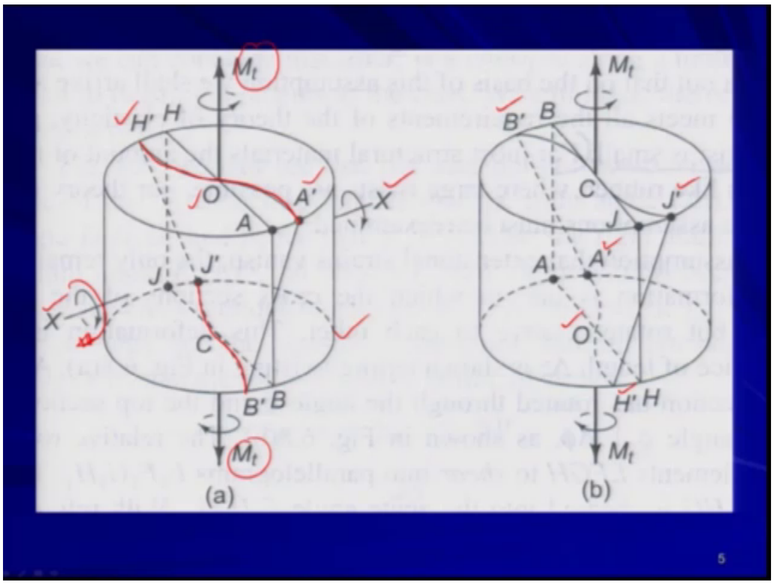

So, therefore, this is not matching at all. So, therefore, there is some ambiguity which has to resolved. Now, if you look at, if you consider that B prime C J prime is not taking that kind of curvature it is taking this kind of curvature. Now, if it makes this kind of curvature, so then that will match with the lower part fine. But now if I rotate this is my x-axis, x-x axis here if I rotate 180 degree if I give rotation 180 degree, so this B prime C J prime will be coming at the top fine and this A prime O H prime will be going at the bottom because I am just rotating the drum. So, this is a drum this is the segment say I am just giving the rotation like that. So, top surface is becoming the bottom surface and bottom surface is becoming the top surface. I can do that I am doing that.

So, after that we just look at what I have done, I have not done anything everything is remaining same. This segment is under the twisting moment M t and everything is remaining same, the same surface rotation is remaining same only thing is that under the same loading, I am just rotating the whole drum. I am making the top surface as the bottom surface and bottom surface as the top surface that is all. But by making that what I am getting, before rotation you are having this kind of say deformed surface on the top surface deformed line on the top surface, but after rotation you are getting this kind of deformed line on the top surface they are not matching at all right, they are not matching.

So, from the symmetry argument, we can say that the curved lines due to the application of the twisting moment, the straight radius will be remaining straight, they cannot be the curved radius, because curved lines are not satisfying the symmetry argument, agreed. So, what we can conclude, we can conclude that the diametral lines that is my second conclusion. Diametral lines deform into curved lines is ruled out by symmetry. The straight diameters are carried into straight diameter by twisting deformation. So, these two segments are very, very important which are coming out from the symmetry argument.

First conclusion is that when a circular shaft is twisted its cross section must remain plane. So, cross section if it is circular, it will be remaining circular and remaining plane it will not be bulging out or destined and diametral lines deform into curved lines is ruled out from the symmetry argument already we have discussed. So, therefore, the straight diameter are carried out are carried into straight diameter by twisting deformation. So, straight diameter will be remaining straight even after application of the twisting moment or the twisting deformation. So, I will stop here today. So, in the next lecture, we will be discussing about the stresses developed due to this or stresses as well as strain developed due to this application of twisting moment.

Thank you very much.