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## Lecture – 39 Tutorial 3

Welcome back to the course Mechanics of Solids. So, in the last lecture basically we talked about several things about the analysis of the slender member to find out the bending moment and shear force right. So, today you will be taking couple of numerical examples on the same topic like determination of shear force and bending moment as well as shear force diagram and bending moment diagram subsequently.

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So, let us take this problem today. The first problem that is draw the shear force and bending moment diagrams for the following case, so this is the beam A B C is a beam which is having the overhanging part B C it is supported at point A and point B simply supported with overhanging part B C and under the action of the uniformly distributed load W naught. So, now, we need to find out the shear force diagram and bending moment diagram for this particular beam.

Now, what should be my approach the first step for this problem will be to draw the free body diagram right. So, if you want to draw the free body diagram basically. So, you generally do or generally replace the supports with the reaction forces like R A and R B there is no horizontal reaction at A because there is no horizontal external forces. So, now my next job is to find out the reaction or the reaction forces R A R B by satisfying the equilibrium equations. So, if you satisfy the equilibrium equation if you considered moment with respect to 0 you will find out R B equal to W naught L plus a whole square by twice L and similarly if you consider summation of all the forces in y direction. So, this is your say y and x. So, loading is in x y plane. So, if you consider the summation of all the force in y direction is 0 then basically you will be getting R A equal to W naught L square minus a square by twice L. So, these two things we are getting from the equation of equilibrium fine.

So, now, what is my next step we will be choosing few steps I mean few sections few sections by which we can define the shear force and bending moment equations or the expressions for the whole beam. Now how many sections we need how many minimum number of sections we need for this problem to taken, by saying this problem it is very clear that two sections will be required - one section will be between A and B and the second section will be between B and C right. So, that is quite obvious. So, I am taking two section say section 1 1 and section 2 2 and then I am drawing the free body diagram for section 1 1. So, I am considering the left hand part of section 1 1. So, that will look like, so section 1 1 which is varying 0 to L. So, this is the free body diagram this is my shear force V this is my binding moment M b and this is. So, this is your section 1 1 fine.

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the following Sec I-1 (0 
$$\leq x \leq L$$
)  $\leq F_{y} = 0$   
 $f_{x} = 0$   
 $f_{x$ 

Now, if you want to find out shear force and bending moment you can find out by satisfying the equation of equilibrium. So, equations of equilibrium if you satisfy them basically first one is your F y equal to 0. So, I can simply write R A plus V minus W naught into x equal to 0. So, from there I can get V equal to W naught twice L x minus L square plus a square by twice L. So, this is the expression for the shear force which will find out or which will determine the shear force between 0 and L for section 1 1. So, section 1 1 is varying between 0 and L 0 and L right in the formula.

So, now at x equal to 0 your V at x equal to 0 is equal to simply this part will be 0 simply minus R A which is nothing but minus W naught L square minus a square by twice L and at x equal to L your V is equal to you put x equal to L there you will be getting W naught L square plus a square by twice L.

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So, you have got the magnitude of your shear force at two extreme points at x equal to 0 and x equal to L fine. So, that will require to draw the shear force diagram later on.

So, now if you look at this equation or this values right at x equal to 0 V is simply minus of this; that means, the negative value and x equal to L the magnitude of these positive value right. So, the magnitude of shear force is changing from negative to positive from 0 to 1. So, there must be some point between 0 and L where shear force is becoming 0 that is not sure because if you want to travel from negative to positive you must cross the 0 point right. So, we need to find out where the shear force is becoming 0 between 0 and

1. So, to do that expression of shear force we can put simply that thing equal to 0. So, if V is 0 then I can get W naught twice L x minus L square plus a square by twice a L equal to 0 from there I can get x equal to L square minus a square by twice L so that means, s equal to L square minus a square by twice L basically your shear force becoming 0 fine.

So, now these are the informations we have extracted from that expression whatever we have derived based on summation of F y equal to 0. Now will be considering the moment equilibrium. So, we will be considering the moment equilibrium.

 $\frac{Sec I-I(0 \le x \le L)}{\sum \omega_{b}} = 0$   $\frac{\sum \omega_{b}}{\sum \omega_{b}} | x = 0$   $M_{b} + \omega_{b} x \cdot \frac{x}{2} - R_{A} x = 0$   $M_{b} = \frac{\omega_{b} x (L^{2} - a^{2} - Lx)}{2L}$   $M_{b} = \frac{\omega_{b} x (L^{2} - a^{2} - Lx)}{2L}$   $A + x = 0, \quad M_{b}|_{x=0} = 0$   $A + x = L, \quad M_{b}|_{x=1} = -\frac{\omega_{b} a^{2}}{2}$ 

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So, let us take the moment equilibrium equation summation of m equal to 0. So, we can simply write M b if you look at here M b plus W naught x into x by 2 minus R A into x equal to 0. So, from this I can get M b equal to W naught x L square minus a square minus L x by twice L. So, at x equal to 0 your M b is equal to 0 from this expression fine we are going to find out the bending moment at two extreme points at x equal to L this is if you put x equal to L here you will be getting minus W naught a square by 2.

If you see here if you put M b equal to 0 if you put M b equal to 0 in this expression then from this equation it is very much clear that there are two values of x y where the bending moment is becoming 0 yes or no one point already we have obtained that is at x equal to 0, but another point is a to obtain yes or no. So, if you look at this expression, so that if you put bending moment equal to 0 here at x equal to 0 the bending moment is becoming 0 that is that is already obtained, but another point you must get from this for again the bending moment is 0 right. So, let us find out that point. So, we are just simply putting W naught x by twice Laughter, L square minus a square minus L x equal to 0.

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So, from this I can get M b at x equal to 0 0. So, from this I can get x equal to 0 or L square minus a square by L square so; that means, M b at x equal to 0 and M b at x equal to L square minus a square by L is also equal to 0 fine.

So, now what you are getting you are getting it is starting from 0 and then it is gradually building up and then again it is becoming 0 and then it is going to the other side that kind of say I mean picture some hussy picture not very clear picture you are getting some hussy picture getting right, so that this could be the possible variation of M b. So, there must be some point where M b is becoming maximum is not it, there is some there must be a point that is there must be some value of x where the bending moment is becoming maximum because during this variation some where the bending moment will be touching the maximum value let us find out that.

So, if I want to find out that. So, for that we simply do as you know to get that maximum bending moment. So, therefore, your bending moment maximum is happening if you do that you will be getting x equal to L square minus a square by twice L. So, at that x value basically your bending moment is becoming maximum if you do that I mean very simple thing right because derivative of that equation. So, the maximum value of your bending moment therefore, will be at x equal to L square minus a square by twice L is equal to W

naught L square minus a square by whole square by eight L square. So, this is the maximum value of bending moment which is occurring between 0 and 1. So, these are the information should be required when we trying the bending moment diagram.

Now, so this is all about your section 1 1. So, we have got the maximum information from section 1 1, now we need to solve the section 2 2. So, let us solve that thing.

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So, your section 2 2, at section 2 2, so for section 1 1 we had taken the left hand side of our section right for our convenience as you know so this convenience how I mean how your problem will be getting a little bit easier. So, that thing you have to judge and this judgment will be coming from more and more practice and more and more say solving of the problem.

So, for section 2 2 will not be coming from left rather we will be taking from right because that will be more convenient to find out or to analyze. So, let us take the right hand side of section 2 2 this is yours section 2 2 this is your V all directions are positive this is your section 2 2 this is your W naught and this distance is L plus a minus x agreed because section 2 2 is second x distance away from a. So, this distance will be L plus a minus x fine. So, again and this section is valid between L and L plus a. So, again will be doing the same thing will be taking summation of F y is equal to 0 that gives me minus W naught L plus a minus x equal to 0 from which I can get V equal to minus W

naught L plus a minus x. So, at x equal to L plus a your B is simply 0 and at x equal to L your V is equal to minus W naught into a.

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$$at x=L, V \Big|_{x=L} = -\omega_0 a$$
  

$$\leq M=0$$
  

$$-M_b - \omega_0 \left[ (L+a) - x \right] \frac{[L+a-x]}{2} = 0$$
  

$$M_b = -\frac{\omega_0 \left[ (L+a) - x \right]^2}{2}$$

Now we will be taking the moment equilibrium we will be considering the moment equilibrium if you consider that. So, minus M b minus W naught L plus a minus x into L plus a minus x by 2 is equal to 0. So, from which I can get M b equal to minus W naught L plus a minus x whole square by 2.

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So, for this at x equal L plus a your M b is equal to how much simply 0 and at x equal to L your M b is how much minus W naught a square by 2 fine. So, these are the information we have got from section 2 2. Now let us draw the bending moment and shear force diagram. So, first we will draw the shear force diagram. So, shear force diagram we are drawing.

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So, this is your baseline this is A point this is B point and this is C point. So, your shear force diagram will be looking like this, this is negative this is positive and this is again negative this is the value already we have calculated L square minus a square by twice L this distance where shear force is becoming 0 that also we are calculated if you look back the solution. And this part already we have calculated for section 1 1 it was coming W nit L square plus a square by twice L and this part will be coming from section 2 2 that was minus W naught to a.

So, this is your typical shear force diagram similarly I can draw the bending moment diagram let us draw the bending moment diagram this is say A this is B and this is C. Generally we draw shear force and bending moment diagram just below the actual say free body diagram of the beam; however, you do the space restriction in the board I am drawing like this in separate wise originally we show one by one. So, first fuel diagram of the beam and then just below see SFD and then below of that BMD fine anyway. So, that is not an issue. So, generally we show like that ok.

So, now for bending moment at A what was the value of bending moment simply 0 and that has gone some maximum value and becoming 0 and getting some value at B and again it is at C it is 0 right. So, this is your maximum value which was calculated where the bending moment was maximum that was calculated as L square minus a square by twice L and this maximum value was calculated as if you recall W naught this is positive and this is negative W naught L square minus a square by 2. So, this is your typical BMD. I hope you have understood the process involved in this kind of say problem. So, you have got SFD and BMD for this simply supported with over hanging part beam.

So, I will stop here today. So, in the next lecture will be taking one more say numerical problem and then will we switch over to the next chapter that is torsion, torsion in circular shaft. So, I will stop here.

Thank you very much.