Mechanics Of Solids

Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 38
Differential Equation Relationship

Welcome back to the course Mechanics of Solids. So, in the last lecture we just talked about the distributed load if you have the distributed load over whole beam then how we can analyze. But most of the times see if you see that instead of taking the distributed load I mean actual distribution distributed load in into consideration we generally take the resultant of the distributed load and we analyze the problem, but you need to remember that this resultant when you are taking that this resultant will be only used to calculate the support reactions and other forces it cannot be used the resultant force cannot be used or conceptual resultant force cannot be used to get the shear force and bending moment the internal forces and moments cannot be obtained by considering the resultant force only.

So, we need to know that if you have the distributed load then how conveniently you can convert that into the resultant force and how you can find out the support reactions. Now let us talk about that.

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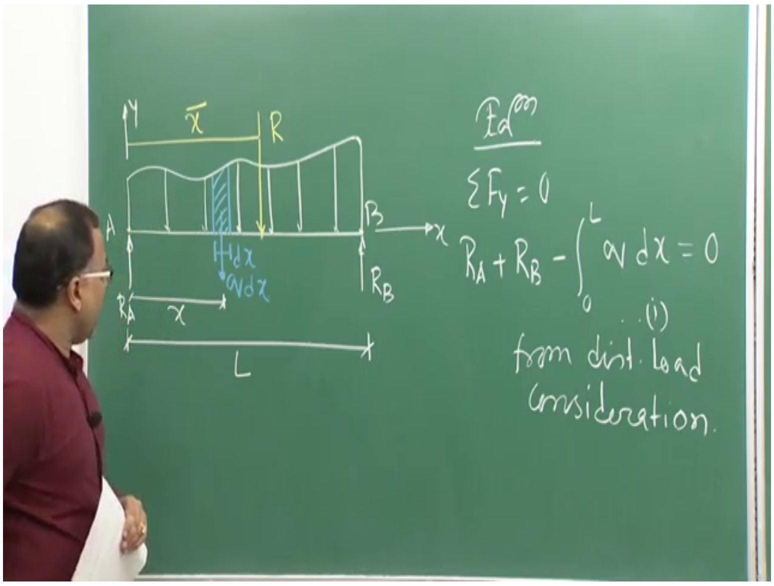

Suppose if you have this kind of say distributed load R A. So, over the beam where R A and R B are the support reactions this is your say x direction this point is B this point is A and you have the distributed load like that and you are considering one small element dx and this total load is say q e x where q is your uniformly distributed load intensity. So, this is your say y direction and this distance say x and the total length of the you say L and q is the load intensity. And if I considered the whole distribution the resultant of the whole distribution or the whole distribution of load is nothing, but say R which is acting say x distance x bar distance away from say y axis.

Now, what is my objective my objective or what I am saying that instead of considering the concept of the distributed load that the way we have solved last problem if you recall triangular distribution load and all there we considered a small element and we considered the load intensity multiplied by the small length and that will be nothing, but concentrated force. And that will be integrated over the whole length we did like that right since you are doing that much of complicated thing I can simply consider this resultant, resultant R means resultant of this whole distribution and the distance of this resultant from the y axis which is nothing, but x bar if we can calculate these two things that will be well enough to find out the support reactions R A R B instead of taking or instead of considering the distributed load ok, let us do that.

Now, considering the equilibrium if you consider the equilibrium; that means, summation of F y equal to 0 I can simply write R A plus R B minus 0 to L q dx. So, equation one and that is from your from distributed load consideration agreed or not. So, that is coming from distributed load consideration. So, whatever we did earlier whatever we learned so far when you have the distributed load. We are not talking about the resultant and all those things to if you considered the distributed load that is q dx is a concentrated force on the invited small ceiling dx and I am integrating over 0 to L that is giving me the total load and based on that I can consider this is my equilibrium condition of course, this is 0.

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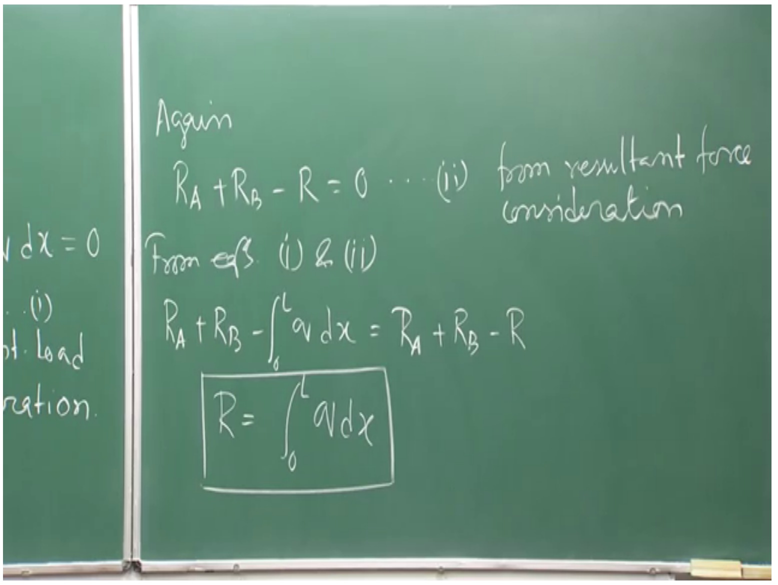

Now, if I consider only the resultant force then I can simply write R A plus R B minus R equal to 0. So, that is from resultant force consideration. So, from equation 1 and 2 what I can write - R A plus R B minus 0 to L q dx is equal to R A plus R B minus R because both must give me the same effect as I told you. The main thing is that what we are we are replacing the distributed load the complicated thing by the resultant force of the distributed load R. So, in this process this equation if I say this equation is satisfying my equilibrium condition and this equation is also satisfying the equilibrium condition from the resultant force consideration then they must be same if they are same then I can simply write R is nothing, but q dx. So, your resultant force is nothing, but this and now based on that once you get the resultant force you just put it in this equation you will be getting the expression for R A and R B fine.

Now, taking the moment equilibrium, now if you take the moment equilibrium this suppose M A moment with respect to A point is 0 summation of moments all moments, now what I can write R B into L minus 0 to L x into q dx equal to 0. So, that is a equation 3 from distributed load consideration if you consider the distributed load, so this is the equation coming from the moment equilibrium condition.

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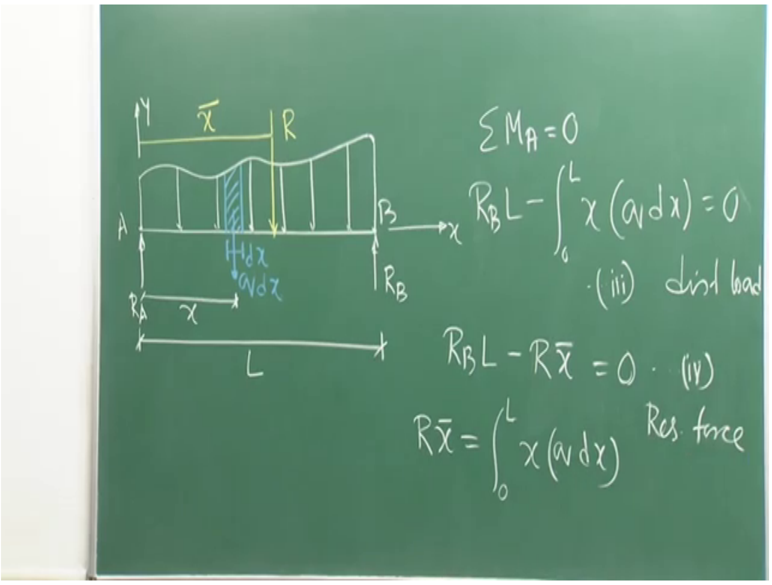

So, if you consider the resultant force then what will be getting R B into L minus R into x bar that is it, so equation 4. So, that is coming from resultant force consideration agreed and they must be same again by following the same logic whatever we did earlier. So, they must be same, so therefore, I can simply get R x bar is equal to 0 to L x q dx. So, from there I can simply write from there I can simply write x bar equal to 0 to L q x dx by therefore, what we can comment whatever we have got just.

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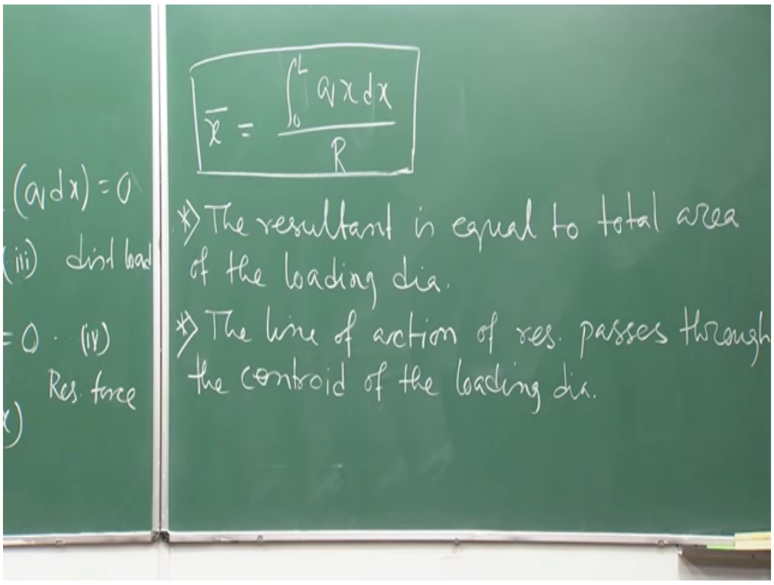

Now the resultant is equal to total area of the loading diagram right R what is the expression for R resultant force e is equal to 0 to L q dx. So, therefore, the resultant is equal to the total area of the loading diagram right. And the second conclusion what we can draw from this the line of action of resultant passes through the centroid of the loading diagram that you are getting form there right. The line of action of the resultant passes through the centroid of the loading diagram. So, this R will be always passing through the centroid of the loading diagram.

So, these two statements or these two conclusions we can draw from this analysis. Now one thing you will appreciate that this resultant or the concept of this resultant is valid only when you are talking about the or you are trying to find out the reactions external reactions.

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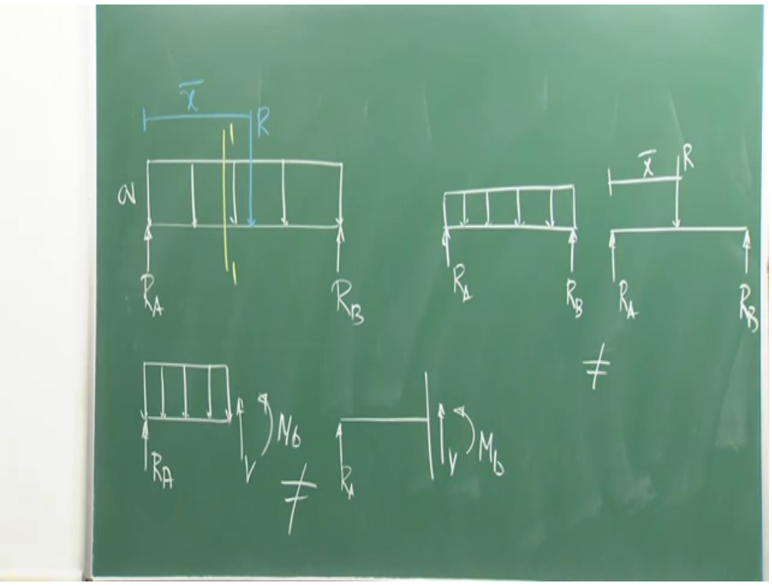

Now what I mean to say suppose this is my beam simply supported some beam R A R B and you have the say distributed uniformly distributed load with load intensity q say. Now to find out R A and R B this resultant business will be fine enough; that means, if you want to find out R A and R B see in this case both R A and R B will be same, so R A equal to R B equal to the area under this loading diagram. But if you want to find out the bending moment and shear force then suppose you are taking one section here section 1 1 say and you know the resultant of this loading diagram the whole resultant of this whole loading diagram say R which is passing through the centroid of this loading diagram that comes from the second I mean statement.

Now if you try to find out the shear force and bending moment under this kind of distributed load that is not exactly same if you consider this kind of thing, they are not same what I mean that these two things are not same as long as you are trying to find out the bending moment and shear force. So, therefore, this concept of resultant and the line of action of the resultant force will not work when you are going to find out the bending moment and shear force in a particular section.

So, there actually you can even again you can consider that thing by considering the resultant an resultant and the line of action of the resultant for this much of loading here actually you have to modified. So, you cannot consider the global resultant and the line of action of the global resultant into the consideration of your bending moment and shear force calculation understood. So, these two things are not same as long as we are going to find out the internal forces and moments that is nothing, but shear force and bending moment.

So, now we will try to establish the differential equilibrium relation. Let us see how we can establish that.

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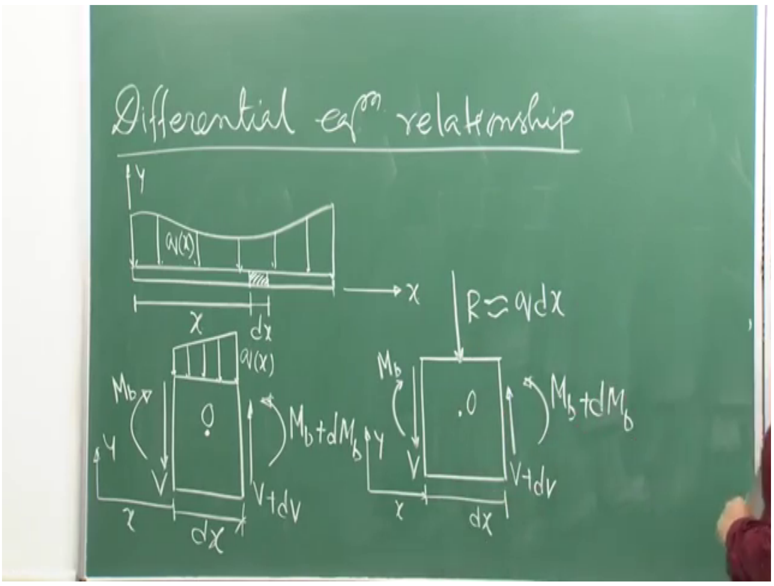

So, differential equilibrium relations, suppose this is my beam. So, this is the say x direction this is say y direction I have some distributed load over the beam that is nothing, but say intensity you say q x I am considering one small segment on the beam dx at x distance from the origin and I am trying to blow it up this small segment and I try to draw the free body diagram of this small segment.

So, if I try to draw the free body diagram of the small segment dx this is some exact figure you have load intensity on top of that say that is q x, so this is y x and this is your dx this is the midpoint of the segment say O. So, therefore, on this side you will be having the shear force V and all are positive M b on this side you will be having some variation in the shear force that is V plus d V due to the travel along the distance dx and you will be having M b plus d M b . So, now, this thing again I can draw in terms of my resultant concept whatever we have just learned this is midpoint o this is the dx x distance away and this resultant say R A is approximately can be written as q into dx fine enough and all the forces will be remaining same p M b V plus d V M b plus d V.

So, this R as per our definition of the resultant force R should pass through the midpoint of this element through O. So, now, if I apply the equilibrium condition say F y is 0 for this element small element I can simply write V plus d V minus V minus q dx equal to 0 agreed or not fine.

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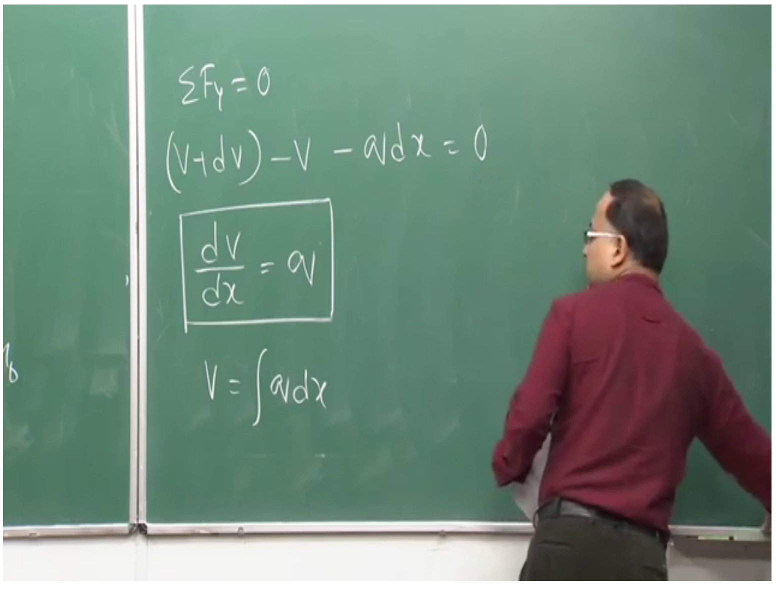

So, from this I can simply write d V dx equal to q this is the differential relation between the shear force and your load intensity q. Similarly I can consider or from here again we can write say V is nothing, but the institution of q dx over the whole length that is my shear force that also I can write. I can consider the moment equilibrium as well if I considered the moment equilibrium moment with respect to midpoint O then I can write M b plus d m B plus V plus d V into dx by 2 plus V into dx by 2 minus M b equal to 0. So, from there I can write d M b plus V dx plus d V dx. So, this term is very very small, so therefore, we can neglect it.

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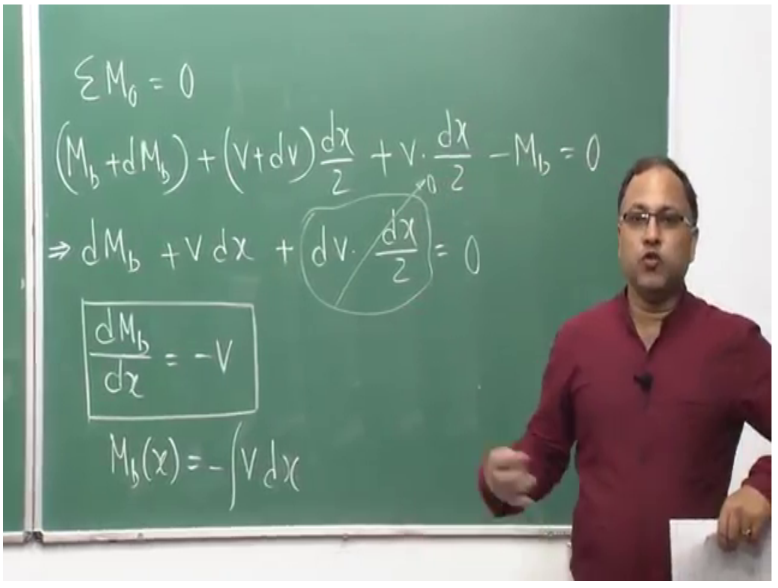

So, from this I can simply write d M b dx equal to minus V this is another important differential relationship differential equilibrium relationship; that means so that is why you might have seen when I was talking about the bending moment is always one order higher than the shear force right. At the time I was I was just giving the hints of this relation so; that means, if you do or if you take the differentiation of the bending moment right you will be getting the shear force or in the other way you can simply get M b x is V dx, but opposite in sign. So, that depends on the loading if you consider the loading in the vertical downward direction that will be like that.

So, what I mean to say a shear force will be always one order lower than the bending moment and the difference if you have the equation of the bending moment you differentiate that equation you will be getting the shear force.

So, with this I will stop here today and this is the chapter for your bending moment and shear force calculation for the slender member then in the next lecture we will be taking couple of numerical problems and that will conclude this chapter and then will move to the torsion or the twisting moment in for the circular shaft.

Thank you very much.