**Mechanics Of Solids**
**Prof. Priyanka Ghosh**
**Department of Civil Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture - 37**
**Shear Force and Bending Moment Diagrams**

Welcome back to the course Mechanics of Solids. So, basically in the last lecture if you recall we are talking about the forces as well as the moments internal moments which are getting developed in a particular slender member and they are we have seen that if you have the loading in x y plane in the slender member and basically you will be having axial force, shear force and the bending moment right. And then we took one example problem like this so one simply supported beam A B is resting on supports A and B based on that we calculated reaction R A and R B.

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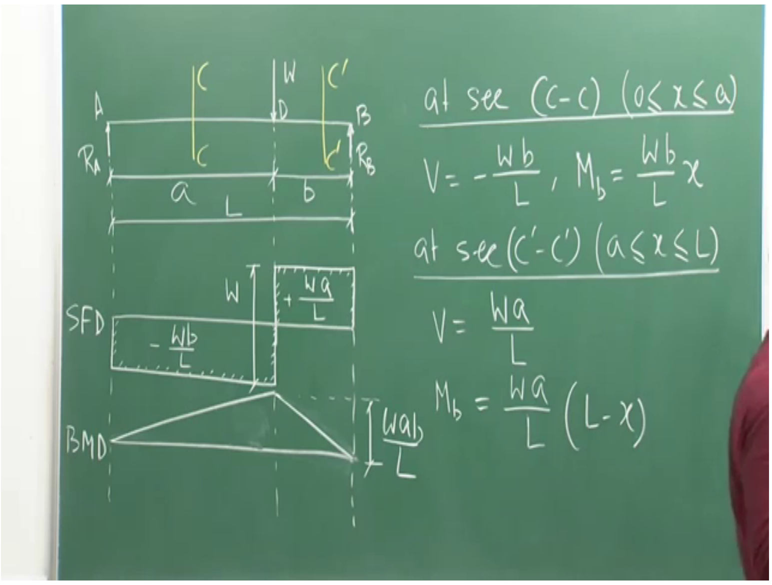


And of course, the beam is experiencing one concentrated force W at the point D and we considered two different sections to define the shear force as well as the bending moment for the whole beam section C C and section C prime C prime. So, section C C is bending from 0 to a for a section C prime C prime is bending from A to L right. And we have got the shear force and bending moment by satisfying the equilibrium condition and we have got V and M b. So, like this, so V is equal to minus W b by L that section C C or as M b equal W b x by L and similarly the shear force and bending moment can be expressed in C prime C prime section like this.

Now, if you look at the expression for the shear force and bending moment basically the variation is linear right, if you try to plot the shear force versus I mean shear force in the x axis and along in the y axis sorry. So, shear force along y axis and the along the x axis if you plot x axis means the beam is spanning in the x direction right. So, what I mean to say that if you try to get the variation of the shear force along the whole beam then that variation will be very much linear as you can see because these two sections will define the state of shear force as well as the bending moment completely for the whole beam right. So, if you look at the shear force expression, so that will be also linear in x. Aimilarly the bending moment expression is also linear in x right.

So, now it is very much advantages most of the times and therefore, people do that people follow this procedure that you get the variation of shear force and bending moment for the whole beam and that variation is known as shear force diagram and bending moment diagram, in short will be calling that thing as SFD that is shear force diagram and BMD that is bending moment diagram. And these two diagrams will be very much handy for a design engineer which will talk about the complete variation of the shear force and the bending moment for the whole beam.

Now, let us I mean because all the times you cannot or you may not like to express or find out the value from this expressions from these expressions of shear force and bending moment rather you try to get the they the picture of the variation how it varies how the shear force is varying over the whole beam, how the bending moment is varying over the whole beam, so this picture will give you much more exhaustive say detailed about the analysis. Now let us draw the shear force diagram first. So, this is my SFD. So, this is my baseline this solid line is my baseline. So, now, if you look at for section C C, so section C C is varying or session C C is valid rather from this point to this point right from A to D because it is varying from 0 to a small a right that is shown here. So, from this point to this point what is the value of your shear force that is given by minus W b by L constant.

So, if I say this is my base line. So, below the baseline it will be negative that that is my convention I am using and above the baseline whatever will be plotting that is positive.

So, it is coming negative, so shear force is coming negative minus W b by L and that is constant. So, this is W b by L that is the variation of shear force from A to D constant variation of course, linear and then at I section C C, C prime C prime; that means, that is varying from D to B right from this point to this point and what is the value of shear force W a by L positive right again constant. So, W a by L worse and total will be W you can add it and you can find it because here actually you are getting the abroad jump because of the application of this concentrated force when shear force is becoming I mean all most constant I mean shear force is becoming constant from this point to this point and suddenly you are applying some concentrated forces or shear forces will be getting sudden jump and then this will be the variation this will be the again constant variation from D to B.

So, this is known as shear force dagger. In some book you will see that people I mean to understand in a very better way they catch it, but I am not putting the compulsion to you it is up to you if you do not show that thing there will be no problem. So, they try to show this hatching in some book you will be getting that oval. So, this is your shear force diagram.

So, now if you look at this shear force diagram basically the designer will understand if you if you give this shear force diagram for this beam. So, this are all immediately understand. So, this is the variation of the shear force for the whole beam and according they will they will take or they will consider the design parameters.

Similarly now if you try to draw the bending moment. So, this is my BMD bending moment diagram, now if you put x equal to 0 if x is equal to 0; that means, you are at point at that time what is the value of your bending moment that will be coming from this expression if you put x equal to 0 bending moment is 0 and if you put x equal to a right bending amount will be W a B L a B by L right. So, that x equal to a it will be W a B by L rather I will write later on. So, I am just using the point like this. So, and the variation is linear. So, I am just connecting these two points and again at x equal to say L your bending moment will be coming from this expression if you put x equal to L it will be coming 0. So, from this point to this point again it will be linear, so it is not looking like linear. So, let us get the actual linear line. So, this is your bending moment, so this W a b by l. So, this is your bending moment diagram understood. So, in this way we can we can get the complete picture of shear force variation and bending moment variation for the whole beam.

Now, if you look at this bending moment diagram you are getting the complete picture about the variation. Now the maximum bending moment is happening at this point and the amount the magnitude of the maximum bending moment is nothing, but W a b by l. So, it is completely 0 at this support and 0 at this support and in between that it is linearly varying. So, this shear force diagram and bending moment diagram are very very important aspects in the analysis of your slender men.

So, now we considered here only the concentrated force right, now instead of that I mean sometimes you may have the distributed load. Suppose one if you consider one beam and on top of the beam you have the slam on the roof right roof will be generally giving the distributed load over the whole beam. So, if you have the distributed load over the whole beam instead of some concentrated force you may have the concentrated force you may have the distributed load any kind of loading you can have on the slender member right. So, if you have the distributed load on the slender member then how you can analyze that thing let us take that example and let us take, let us talk about that thing now. So, distributed load.

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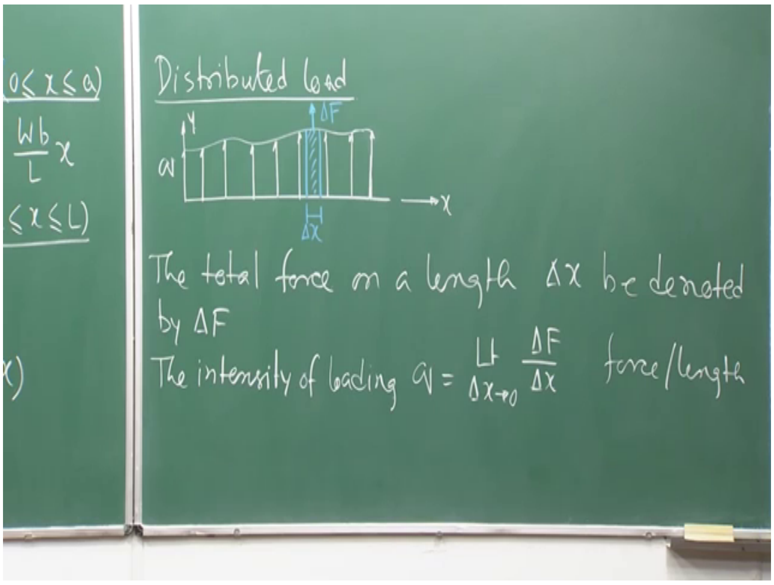


So, if you have the continuous distributed load say this is your say beam this is your x direction this is your say y direction you have the continuous beam and you have the distributed load like that. So, this is the distributed load over the whole beam and we are now considering one small segment say delta x and this distributed load over the link delta x I can consider some concentrated force say delta f, now what does it mean let us let us talk about that.

Therefore the total force on a length delta x be denoted by say delta time that is the total force. So, what a small link is delta x the total force is nothing, but say delta f. So, therefore, the intensity of loading say q can be written as limit del x tends to 0 del f del x that is force by length. So, q is your intensity, intensity of loading. So, intensity of loading can be given by this expression very simple. So, now, whenever you will be getting this kind of distributed load. So, how you will analyze the system now let us talk about the most common type of distributed load that is nothing, but the uniformly distributed load and in short generally will be using that word that is UDL uniformly distributed load.

So, if you have the uniformly distributed load over the whole bean.

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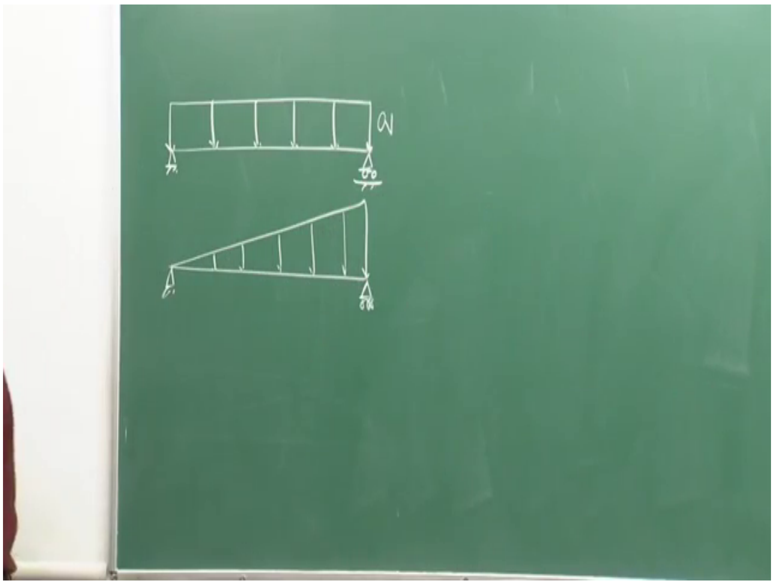

Then how it varies, it either it varies constantly either the distributed load say this is the beam either you have this kind of distributed load constant distributed load with constant load intensity q per unit length or you may have this kind of loading linearly varying loading. So, generally most of the times will be using or will be dealing with this kind of distributed load now if you have this kind of distributed load how you will analyze the system. So, let us talk about that thing will take one small example to understand the process of analyzing this kind of distributed load.

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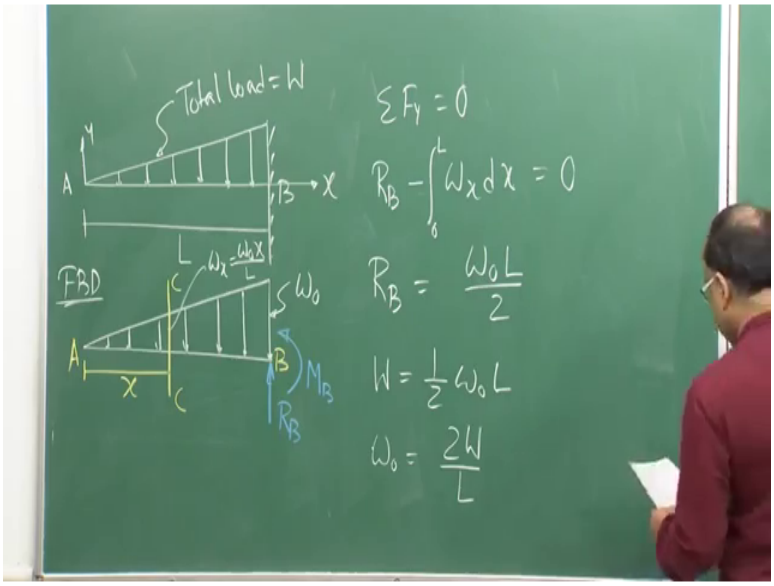


So, examples is one cantilever beam with linearly varying distributed load this is A this is B the total load equal to capital B the total load means area of this triangular distributed distribution the area under the triangular distribution is W and the length of the beam is say L. Now if you want to draw few diagram you just replace the supports here already one support is there, so you replace the support this is my free body. So, there will be one reaction say R B and there will be one moment say M b, I am not considering the horizontal force that is exhortational force here right this is this is your say y at x. So, there is no reaction in the exertion because there is no extra force applied in exhortation. So, there for I can ignore or I can consider that is 0.

So, these are the reaction forces and moments and this extreme interlude intensity is say W naught small W naught. Now if I want to analyze this thing what is my first step, I will be considering one section right I will be considered one section, section say C C at some distance x as a from the free end. So, this is your free end this is your fixed end fine, this is some cantilever beam you know what is my cantilever beam right one end its completely fixed another one is hanging kind of thing. You might have seen this kind of thing in several civil engineering structure like your gerga or may be the cantilever slab in flight I mean aero plane you generally get the wing of the I mean flight. So, that kind of structure is nothing, but your I mean cantilever structure.

Well, I am considering one section here which is existence from the free end free end say this is my B now how we can get the reaction forces. So, the first your first job is to find out the reaction force R B and reaction moment M b, how will find out that you get from your equilibrium condition. So, for that I can simply satisfy summation of F y equal to 0. So, that gives me R B minus 0 to L, and this is varying and this is my say W x. So, W x is nothing, but the load intensity at x distance away from point free end a and that W x if you express in terms of this extreme load intercity that will be W naught x by L fine, so that we can express like this. So, this R B minus 0 to l, so we will integrating that thing W x D x now if you look at this what is W x D x, that is the concentrated force over a distance over a small distance D x as we just discussed now right we just now discussed right. If you have a concentrated force delta f over a small distance delta x right then the loading intensity is your limit delta x tends to 0 delta f by delta x right.

So, from that say condition you get this right. So, if you integrate from 0 to L you will be getting the total intensity. So, that is nothing, but 0. So, that gives me if you put the value of W x from this expression you will finally, getting this is W naught L by 2. Now what is this W naught L by 2? That is nothing, but the area of the whole triangular load is not it. So, now, let us because it this is given actually total load is given. So, if I know the total load I can express W naught in terms of capital W. So, let us do that. So, your capital W is nothing, but half into W naught into L this is area of the triangular distribution and that is nothing, but the total load. So, from there I can find out W naught is nothing, but twice W by L fine. So, this is the load intensity at the extreme point; that means, at the fix fixed support in the support.

Now, as per our previous convention now will be considering the free body diagram of the beam which is coming left of this section C C. So, if you consider that is left part of the beam and if I draw the free body diagram that will look like this.

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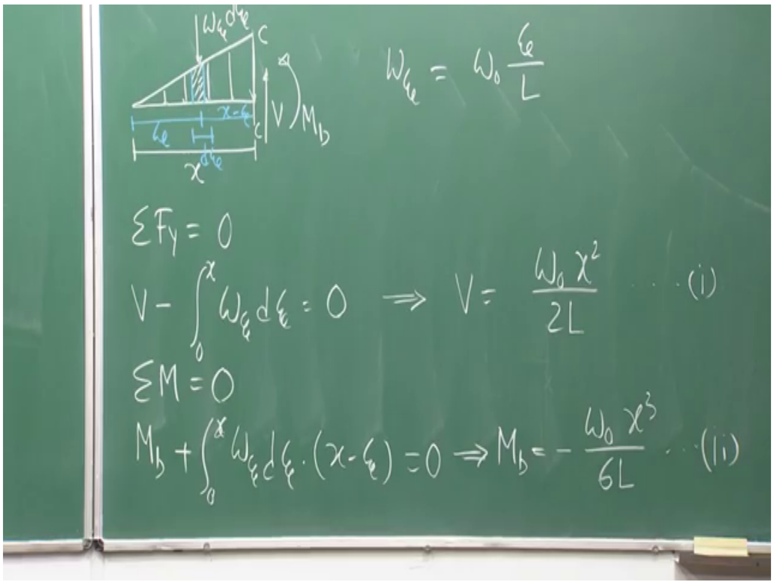


This is V this is M b that is bending moment and shear force and this is say x distance this is your section C C existence I from the free end. Now what we are considering to define this thing in a better way we are considering one local coordinate system says zai and we are defining this is my zai this is a small distance d zai and this is your zai therefore, this distance is x minus zai some local coordinate system we are going to introduce that is zai. Within x I mean this distance x is nothing, but your global coordinate system, but we are we are introducing one local coordinate system that is zai and this in this area your load intensity is W zai, d zai where W zai can be obtained us W naught zai by L because everything we are expressing in terms of W naught because W naught we have got in terms of capital W that is the total load of the triangular distribution. So, we have got the expression of W zai in terms of W naught.

Now from there if we take or if we satisfy summation of f y equal to 0 we will be getting V minus 0 to x W zai dot zai equal to 0 from there I will be getting V equal to W naught x square by twice L that is the shear force expression right, that is the expression for shear force in terms of x. Similarly I can find out if I considered the moment equilibrium I will be getting M b plus 0 to x W zai D zai into x minus zai which is equal to 0. So, from there I will be getting M b equal to minus W naught x cube by 6 L.

Now one interesting thing you might note I mean already in the previous problem also we have noted it whatever expression you are getting for the shear force the bending moment is one order higher in terms of x right. What is the expression of shear force here? W naught x square by twice L that is quadratic equation whereas, if you look at the bending moment expression that is cubic expression one order higher. In the previous problem whatever we had taken in that problem the shear force was constant right it was not dependent on I mean it was constant. So, extra not there, but when we are talking about the bending moment then that was linear in x. So, therefore, we are getting one order higher in terms of x fine. So, that is very very interesting say phenomena and later on will be seeing that thing that derivation of your bending moment will give you the shear force something like that. So, that relation would be establishing. So, what I mean to say that whatever equation you will be getting for the shear force for bending moment that equation will be one order higher in terms of x.

So, now, you have got the expression for M b a now I want to find out what is the value of M b capital M b from this expression. So, capital M b capital M b means this the support reaction the support reaction is nothing, but the bending moment at x equal to L agreed or not your M b, the support reaction right moment reaction is nothing, but the bending moment at x equal to L. So, if you put x equal to L there then you will be getting capital M b is equal to minus W naught L square by 6 clear fine.

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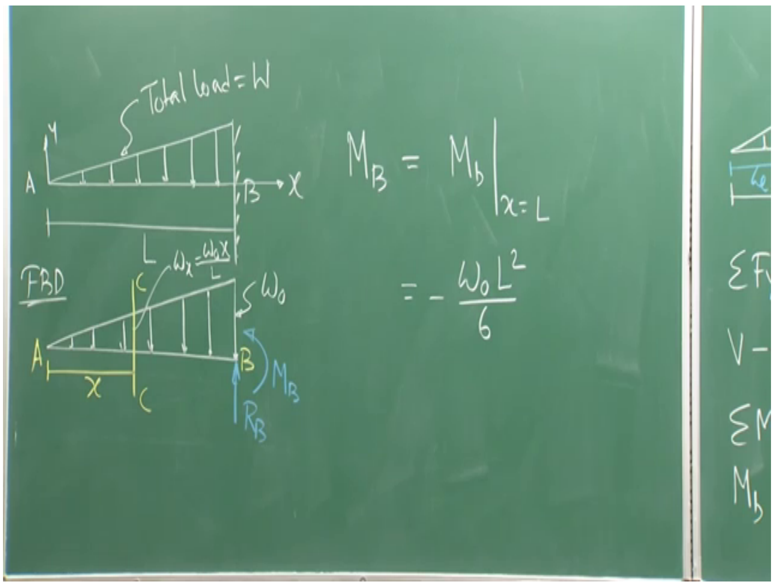


So, now we are going to draw the shear force and bending moment diagram. So, how it will look like?

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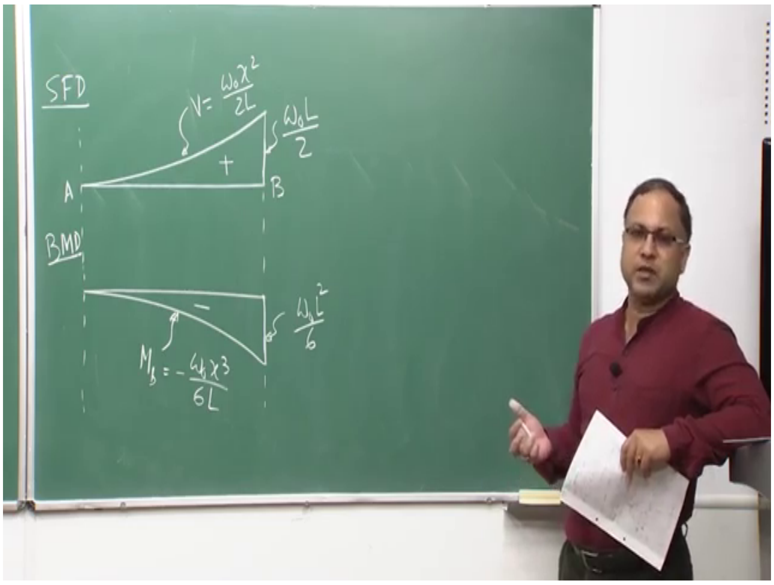


So, we are going to draw the shear force diagram SFD. So, this is your base line, this is your base line point A B. Now if you see the expression, so only one section will be sufficient to define the complete variation of shear force because here actually the load is I mean you are not getting any change in load this is a continuous road right. So, there therefore, only one section will be sufficient enough to define the whole bending moment and shear force characteristics right. So, what was the expression for V? The expression for V was your in the in that was the quadratic equation whether that is a that was a quadratic equation that was W naught x square by twice L.

So, if you plot that see you will be getting. So, where it will be W naught L by 2 and that was positive and this curve is nothing, but V equal to W naught x square by twice L. This is your shear force diagram. Similarly if you try to plot your bending moment diagram say this is my baseline your bending moment equation if you recall the bending moment expression that was in quiver form right. So, therefore, at x equal to 0 that was 0 at x equal L that should be minus W naught L square by 6 and in between it will be negative. So, this is your quadratic equation M b equal to this is minus this is plus minus W naught x cube by 6 L and here this is nothing, but your M b that is W naught L square by 6. This is your bending moment diagram. I hope you have understood that if you have the distribution distributed load like this then how you can analyze or how you can find out the shear force diagram and a bending moment diagram for a beam.

So, I will stop here today in the next lecture will be talking about the resultant force and then will be seeing that how we can establish the differential relation between bending moment and shear force and shear force and the loading. So, I will stop here.

Thank you very much.