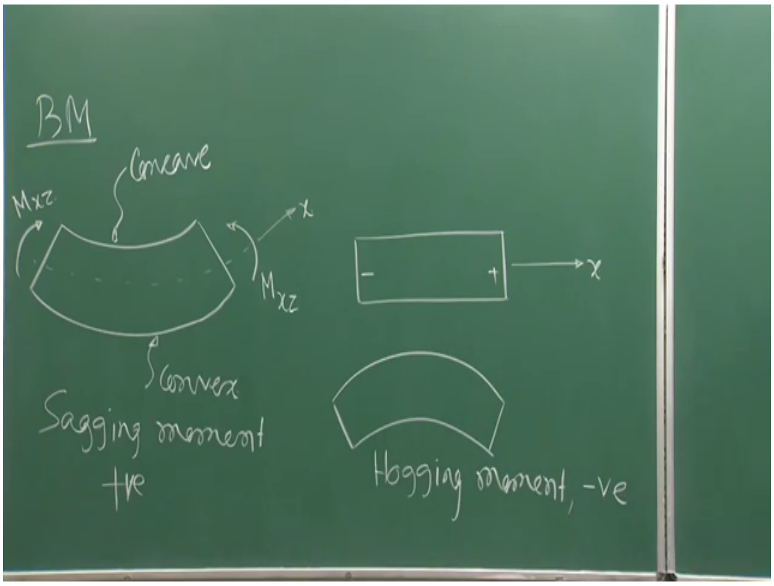
Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 36
Bending Moment

Welcome back to the course Mechanics of Solids. So, in the last lecture, we were talking about the analysis of the slender member and we have seen different force components and the moment components which will be acting in different way in the slender member, the axial force, shear force bending moment and twisting moment. Then, we defined the sign convention for the axial force. So, if it is tension, then it is positive. If it is compression, then it is negative. Then, we define the sign convention for the shear force by following the same sign convention whatever is used for our axial force.

So, similar kind of sign convention we proposed for positive shear force. That means, the left part should come down under the action of the shear force. Now, today we will be talking about the sign convention for bending moment.

(Refer Slide Time: 01:08)



Now, if you look at say this is my x axis, this is some exerted figure of course. So, even you will see later on I mean when we were talking about this bending thing in more detail, you say you will appreciate that the plane section be remaining plane even after bending. So, this is some exerted figure. After bending this will take some, this kind of shape here this is my say x axis say this is my x axis. So, I am exerted and it is still horizontal. So, this is some exerted figure or whatever I mean if I try to show like that this is slender member, a part of slender member. So, this is my x axis, ok.

So, this part, this face is positive, this face is negative. Negative x axis negative x plane and this is positive x plane. Now, if the bending moment if it acts like that as per our definition, this is my positive bending moment, right because on the positive x direction or on the positive x plane, the moment whatever is acting that is Mxz is positive in nature, right. Similarly, on the negative x plane, the direction of the moment is positive in nature. So, therefore this kind of bending moment will give you the positive bending moment.

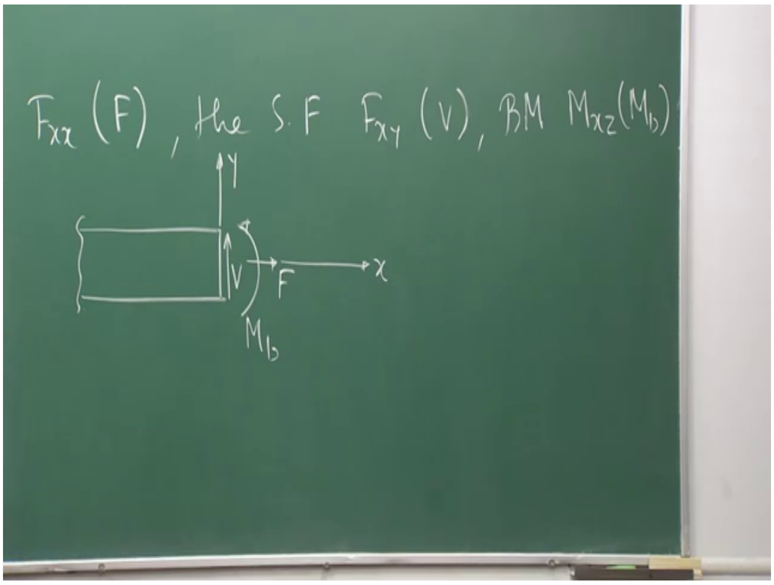
Now, if you look at this shape, basically this top part is your say concave, right. This is something like if this is a slender member under the action of bending moment, I get this kind of shape, right. The top part is concave and the bottom part is convex. So, this is nothing, but it is defined it is known as sagging moment. You have the sag, right. So, this is known as sagging moment and this will be defined as positive. Similarly, if you have this kind of say configuration due to the application of moment that is nothing, but the concave is coming. Now, below at the bottom fiber and convexity is coming on the top fiber of the slender member.

So, this is nothing, but hogging moment and therefore, as per our definition, this will be defined as negative bending moment. So, whenever you get positive bending moment that will be because please try to understand, you need not remember all those things because this is pretty convenient to remember that sagging moment is positive, hogging moment is negative. That is very convenient way to remember, but the thing is that it is actually coming from your moment direction.

If you have the positive moment on the positive x plane and the positive moment on the negative x plane, then you will always get this kind of sagging nature of the bending. Similarly, on this plane if you have this kind of moment, that means on positive x plane negative moment and negative x plane positive. I mean negative I mean positive moment if you get, then ultimately that will give you the negative bending moment, right. So, as per our definition because see I am calling this is z direction is the 2 or I mean novel to the board.

So, basically if you consider this moment is always positive, right in as per our right hand thumb rule, so this moment is always positive, right. So, this is coming from the same sign convention. However, to remember it very well we are introducing some physical interpretation like sagging moment and hogging moment, fine. So, now if you see that if plane of loading is only the xy plane because they are in the slender member whatever we have seen in the last class that we have we considered all the force components and the moment components in xyz coordinates system, right. All three free now, but if the loading is in xy plane which is pretty common like if you consider a continuous beam, long beam, this is the loading is along the xy plane, right.

(Refer Slide Time: 06:42)

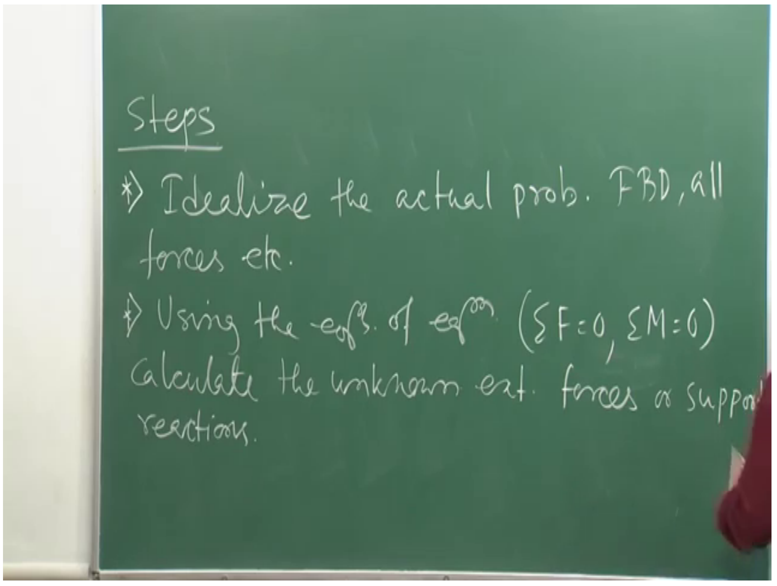


So, in the z direction; that means, normal to the beam is very unuseful. So, in that situation you have only three components left out to define the nature of slender member. What are those? Fx x that is nothing, but the axial force, the shear force F x y that will be defined as v and bending moment that is M x z and that will be called as Mb. So, if I considered this is my slender member, this is my say x direction and this is y.

So, therefore, this is your v, this is your F and this is your Mb. So, all components axial force, shear force and bending moment. All are shown positive on this configuration. So, if the loading is in the xy plane, then only you will be having these three components, the axial force F, the bending moment Mb and the shear force v. That is all because you will be not getting another shear force component, another bending moment component or of course, twisting moment is not there because we are not giving this kind of loading.

So, therefore, twisting moment is not there, now for most of the times we will be I mean in this particular chapter we will be considering loading is in xy plane and we will be trying to solve this kind of slender member and we will be trying to obtain the unknown axial force, shear force and bending moment, ok.

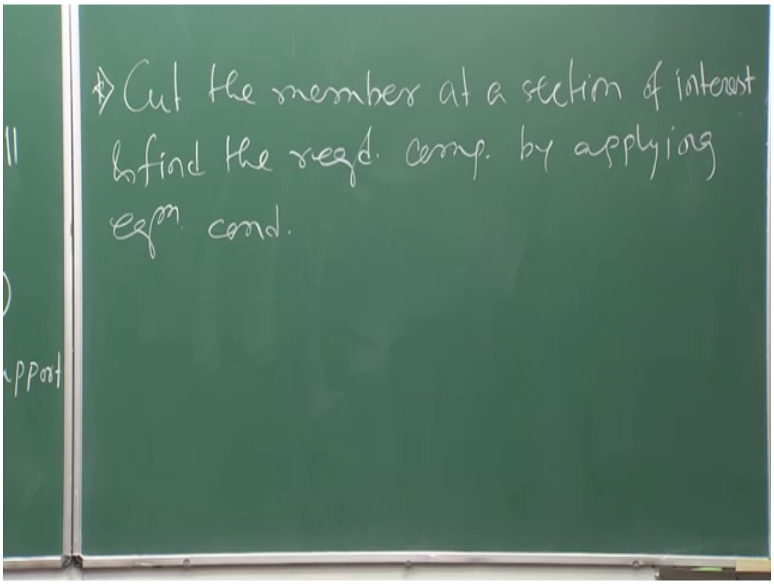
(Refer Slide Time: 08:38)



So, now let us take one I mean before taking one small example, we should know what are the steps involved in this analysis. So, let us do those steps involved in this analysis of the slender. Remember the first one. Idealize the actual problem and that means FBD all forces etcetera, ok.

Next step, first you idealize the actual problem. Next step is that using the equations of equilibrium that you know summation of F is 0, summation of M is 0, calculate the unknown external forces or support reactions.

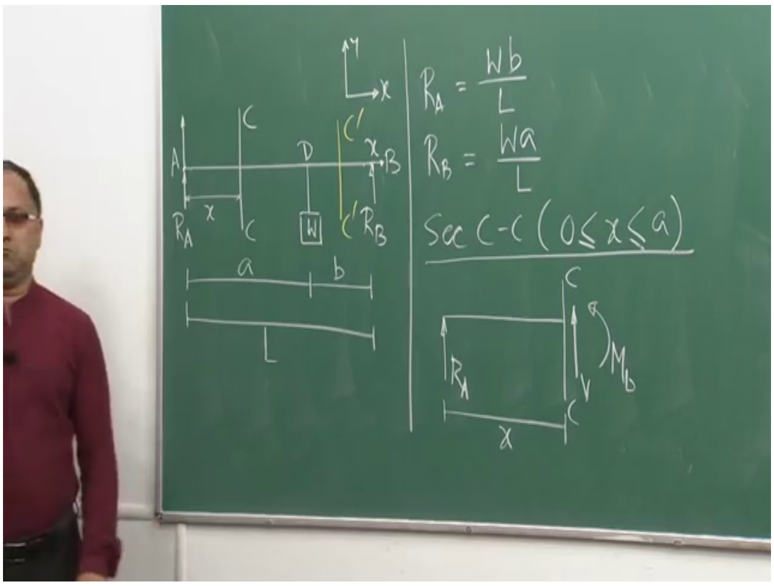
(Refer Slide Time: 10:11)



In the third step, that is the last day basically cut the member at a section of interest and find the required component by applying equilibrium condition, ok.

Now, what does it mean? So, we will take one small example, simple example, so that you will understand that the steps how they are involved in the analysis process.

(Refer Slide Time: 11:15)



So, this is a small or simple example problem. This is a beam simply supported like this. On this supports is a slender member. Of course, I am showing one line that does not mean it is a kind of linear. This is having proper cross-section square or circular whatever, ok.

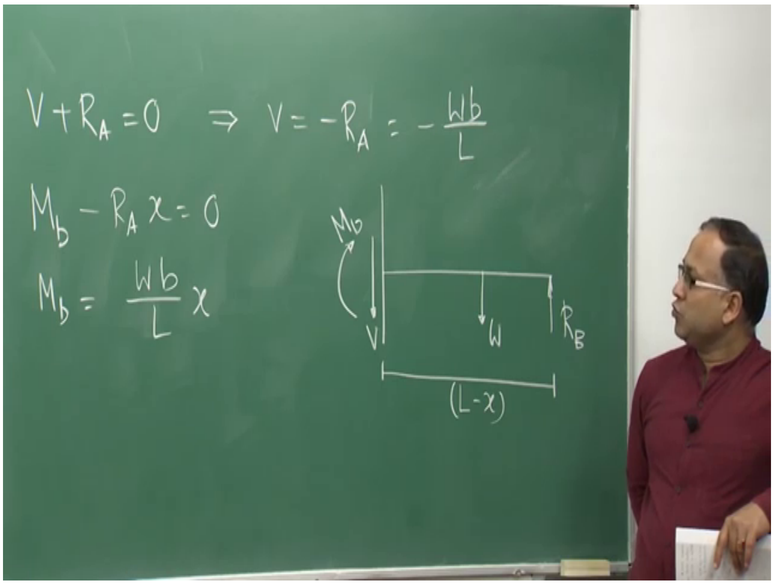
Now, 1 t load W is acting here. Now, the distance from here to here is A, from here to here is B, this support is A distance B and the total length of the beam is L. Now, I am taking one section cc at this location that I will be coming later on, fine. So, this is a problem. So, as per our steps whatever we just discussed, we have to idealize the system. Once you idealize the system, then basically we will draw the free body diagram. So, if I want to draw the free body diagram, this support should be replaced by the force reaction force RA and say R V only the vertical force will be there because this is simply supported and there is no horizontal force.

So, by using your and this is your say xy coordinate system and the loading is in only the xy plane. So, from by exploiting the equilibrium condition, you can get RA and RB and you know how to do that. So, I am not going to do that thing again. So, by considering in the moment equilibrium as well as the force equilibrium, you can find out RA and RB. So, that will be coming RA as WB by L whereas, RB is equal to WA by L, ok.

Now, as I told you that I am considering one section cc in such a way that it is lying between A and this point say if I say this point is A D point. So, it lies between A and D. So, I am considering one section cc which is greater than 0, less than a and once I draw the section and then, I want to draw the free body diagram of the left side of the section.

So, how to look like? So, this is the beam part r a is acting here and this is my section cc. So, when I am cutting it when I am taking it, this is my second step. Ig you recall I am cutting I mean third step, right. Cut the member at a section of interest and find the required component by applying equilibrium condition. So, I am cutting as per my requirement. So, I am cutting and now what is my requirement. I will come to that point later on. So, how I am choosing section cc, that will be discussed later on. For the time being you just believe me that I am taking one section cc which is between 0 to a. That means between a and d when I am taking this part only. So, I should replace the rest of the part by providing some shear force and bending moment because action force will not be there. I am not showing the axial force because you do not have any axial force in these direction. So, if there is no force, external force in the x direction, therefore axial force is 0. That is a very trivial solution that I am not doing again, ok.

(Refer Slide Time: 15:59)



So, now considering equilibrium or considering equilibrium you can simply write V plus RA equal to 0. From this I can simply write V equal to minus RA is nothing, but minus WB by L. I can write it and I can take the moment equilibrium. So, this distance is nothing, but x because I am choosing one section x distance away from point A. These distance is x because I mean my coordinate system is starting from here. So, this is my origin side, A is my origin. Generally we take n. I mean left hand side n point as the origin fine. So, this is x, where x is varying from 0 to a time there is no issue for that.

So, now I will be taking the moment equilibrium. So, moment equilibrium if I want to take, so MB is there that is anti-clockwise and this RA is acting clockwise RA into x. So, that will be coming as RA into x equal to 0. That is all because we will not be participating because I am considering the moment with respect to this section. So, if the component or the contribution of V will not be coming in to the picture, fine. So, from this I can simply get MB equal to WB xy L, ok.

Now, you may ask me that why I have taken only the left hand side of section cc. You will ask me that who has told me to consider the left inside of section cc. Now, there is no rule for that. It is purely based on your convenience. You can choose even the right hand part of the section cc and you can check it that the same solution you will be getting if you can see the right hand side part of the section cc. How would it will look like? This is my section cc. If this is a beam here, it is RB and here you have the W, force W and this distance is not x L minus x because x is starting from A, right. So, therefore, this will be L minus x and if I want to have because I considered shear force as positive, but eventually it comes negative. So, therefore, the direction of shear force whenever I have chosen here that is wrong. It should be other way round. So, that is different issues.

So, we will not think about that thing previously. So, when we will be starting the problem, we will not go by any intuition. We will consider all the things positive or all the thing is negative as they come and based on that we can choose our actual direction. So, here also we will be choosing V as positive. So, if I choose V as positive here, this is the negative plane. So, V will be positive in this direction and MB will be positive in this direction as per our sagging moment positive.

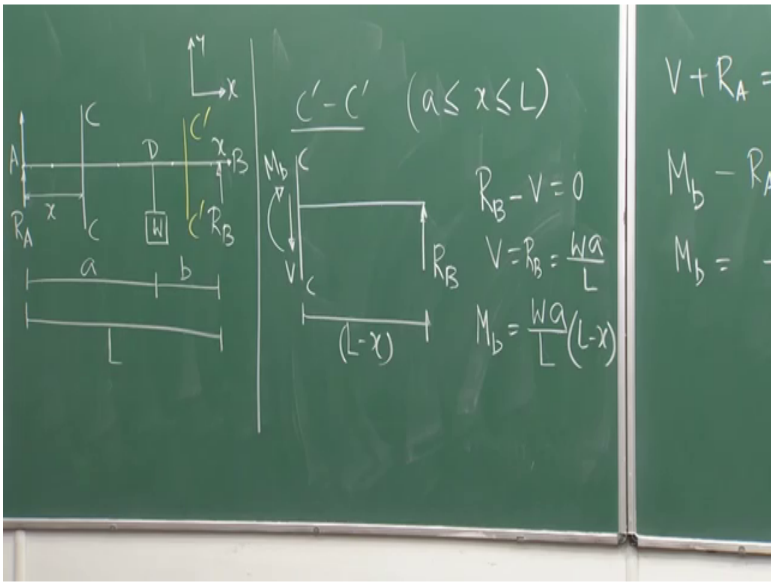
Now, you can check it whether you are getting the same solution of MB and V. So, therefore, whether you consider left hand side of the section cc or right hand side of the section cc, it does not matter from where or whether you are coming from A to section cc or you are coming from B to section cc. It does not matter as long as you are consistent with your sign convention and other things, fine.

So, now if you look at this problem, these problems give me the shear force. Now, this is the shear force value. Now, this shear force is valid from which point to which point? This shear force is valid from point A to D. Why because, this section is valid within this zone within this range 0 to A. So, whatever information I am getting from that particular section that will be valid only from 0 to A. Now, if I want to go beyond A, that means from D point from D to B if I want go, then I have to take one more section and that is section c prime that is all.

So, what you can conclude from that is, whenever you are getting some change in externally applied force, you must take another section to define the whole thing because from A to D, there is no change in externally applied force. Here RA is acting and there is no externally applied force between these two points. So, therefore I have taken only one section and that is defining the whole part, but as long as you reach the point D, immediately you have the externally applied force W here, right. So, therefore this cc section will not be able to capture the behaviour of DB part.

So, therefore I have to take one more section c prime which will take care of this DB 1. is that clear? So, now if I take section cc prime c prime, if I take another section c prime c prime as I have shown and consider the free body diagram.

(Refer Slide Time: 22:08)



So, this c prime c prime section will be A to L. This is ranging from A to L. So, 0 to A we have got now A to L. That will completely conclude the whole structure.

So, now here we will be taking the right side of section cc c prime c prime. Why? This is purely based on my concave because here you see this is little bit because the problem is very simple. So, that is why you will not appreciate whether it is difficult or complicated, but here if you come from right to left, right to section cc, right that is little bit complicated than if you come from left to right, but here in this case if you come from left to right, that will bit complicated.

Then, if you come from right to left, therefore I am considering right to left. So, this is my L minus x and this is my MB and this is my V. All I am showing positive. So, here I will be getting RB minus V is equal to 0. So, therefore, V is equal to RB is equal to WA by L and MB I can find out and get it WA by L into L minus x. So, therefore, what you have got? You have got the complete information. Company information means the shear force as well as bending moment for the whole beam.

Now, at each and every point if you consider on the beam, you will be getting the shear force and bending moment. Isn't it? So, suppose I am here what will be my shear force bending moment? I can find out from whatever is there in section c c. If I am here I still cannot find out bending moment and shear force by considering section c c. If I am here I can find out bending moment shear force information from section c prime c prime. So, that means where you go on this beam, you can find out the bending moment and shear force information from this calculation, ok.

Now, if you look at these calculations here, V is minus. If you consider cc section, this is the expression for V, this is the expression for MB and this is the expression for B and MB. When you consider section c prime c prime, but if you look at this variation of MB is linear in x with x basically the variation of MB is linear. If you see this is the mathematical equation that will be linear. So, of course if you see from say 0 to A, it is constant.

Then, again from A to L, it is taking different value. So, now every time if you calculate from this expression, that may not be that much convenient or that may not give the clear picture about the variation of bending moment and shear force across the whole slender member. So, therefore what people do, they try to plot the variation of bending moment and shear force along the whole beam or the slender member and that variation is known as bending moment diagram and shear force diagram. It is very important in mechanics and if you are going to design something, so this bending moment and shear force diagram will be very prime, say information for a particular design engineer, right.

So, as an analysis engineer, you have to provide these bending moment and shear force diagram. Based on that the design engineer will decide where the bending moment is coming maximum, where the bear share force is coming maximum, based on that they will try to design the section.

So, in the next lecture, we will take this problem again and we will try to explore the bending moment and shear force diagram and how they look like and what are the informations we are viewing, we are getting from that diagrams. So, I will stop here today.

Thank you very much.