Mechanics Of Solids
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Lecture – 34
Tutorial 2

Welcome back to the course mechanics of solids. So, in the last lecture we have solved 1 numerical problem regarding this stress strain relation, now in this lecture will be taking another couple of problems. Now this is the second problem all right in this chapter and problem says.

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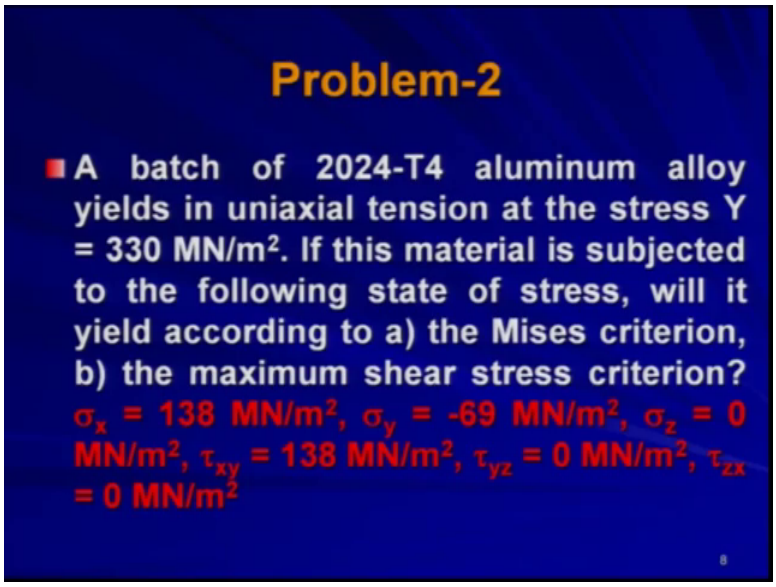

A batch of 2024 T 4 aluminum alloy yields in uniaxial tension at the stress Y equal to 330 mega newton per meter square and by this time I hope that you still recall that what is your yield I mean stress right, that is the value of Y if this material is subjected to the following state of stress, will it yield according to the mises criterion and the maximum shear stress criterion and these other things are given sigma x, sigma y, sigma z, tau x y, tau y z and tau z x. So, these 6 stress components are given and you need to find out that whether this material will yield under this combination of stress. So, different stress components are there under the action of this state of state, state of stress the material will yield as per as mises criterion and as per maximum shear stress or the crista criterion.

So, let us see so now, as you know that your maximum shear stress criteria or the crista criterion or mises criterion both the criterion I mean have been developed based on the principle stress components right. So, in this problem sigma x there is stress components are given in xyz coordinate system. So, we are going to find out the stress components in principle coordinate system there is 1 2 3 coordinate system. So, let us start the problem. So, first find out the principal stress using the Mohr circle. So, will be using the Mohr circle to find out the principal stress and the, the stress if you look at that it is given sigma x is equal to 138 mega newton per meter square, sigma y is given minus 69 mega newton per meter square, sigma z is 0 tau y z and tau z x both are 0.

So, hence we can conclude that this problem is a plane stress problem right you have sigma x, sigma y and tau x y. So, the stress is in xy plane.

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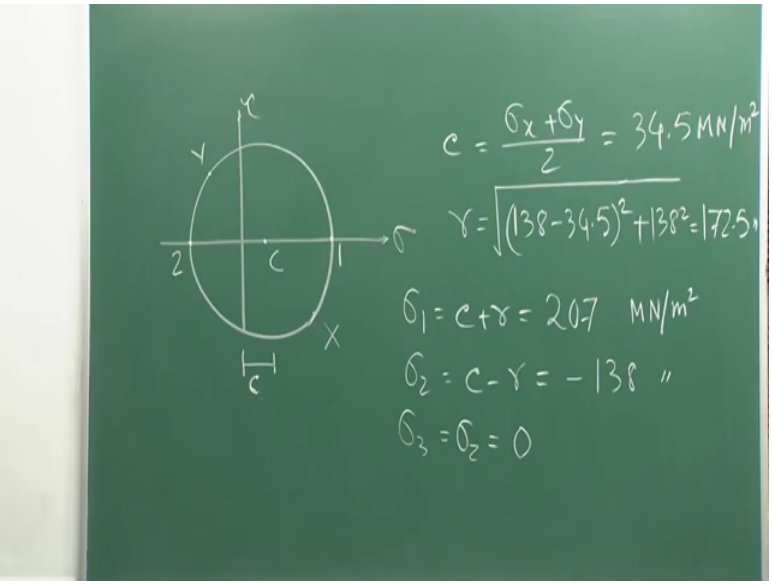

So, therefore, we can draw the Mohr circle for that, so let us draw the Mohr circle this is your sigma tau space. So, your sigma x is given as 138 mega newton per meter square and tau x y is also given as 138 mega newton per meter square. So, your say x point is coming here say and tau x y is positive so that will be coming below your sigma x. So, this is your x point and this is somewhere it will be here it will be your y point. So, with this I can draw a Mohr circle. So, this is the Mohr circle, the center of the Mohr circle is somewhere here that is your C and this distance is small c and this point will indicate

major principal stress this point will indicate minor principle stress. So, our objective is to find out sigma 1 and sigma 2.

So, for that you know what are the things you are supposed to do, first you find out c value that is nothing but sigma x plus sigma y by 2 that is coming as 34.5 mega newton per meter square. Similarly, r can be calculated as 138 minus 34.5 square plus 138 square. So, that comes as 172.5 mega newton per meter square. So, you have got c and r so therefore, your sigma 1 will be c plus r that will be equal to 2 0 7 mega newton per meter square and sigma 2 will be c minus r that will be coming as minus 138 mega newton per meter square and of course, your sigma 3 that is nothing, but your sigma z by default sigma z is your one of the major principal stress that is 0. So, you have got all the principal stress components sigma 1, sigma 2, sigma 3. So, now, you can you can apply your first will apply our mises criterion.

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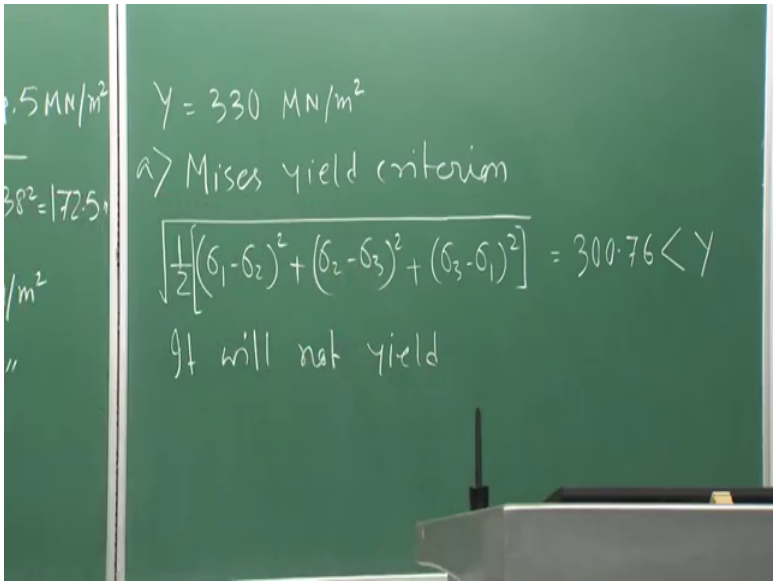

So, the in the problem it is said that y value is equal to, that y is equal to 330 sorry 330 mega newton per meter square. Now, as per misses yield criterion half sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square right. So, that is coming as 300.76 which is less than y.

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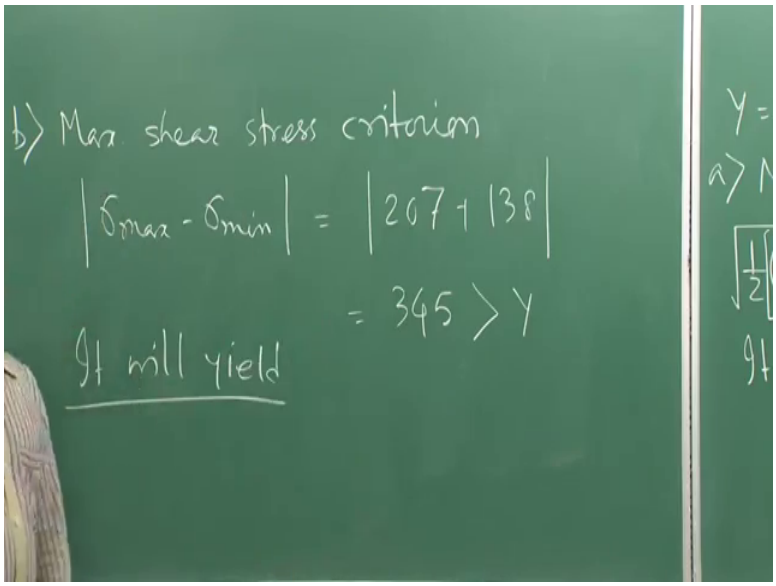

So, therefore, it will not yield, now as per maximum shear stress criterion let us see what happens.

So, you have sigma max minus sigma min that will give you the maximum shear stress that is coming as 207 plus 138 modulus of that I am because you are we are only interested the, the positive sign. So, fine, so that is coming as so 207 is your maximum major principal stress and sigma 2 is your minor principal stress that is minus 138. So, that is giving me 345 which is greater than y. So, therefore, it will yield.

So, in that, in that way actually you can find out whether the material will experience yielding or not, by following these 2 criteria under certain combination of stresses.

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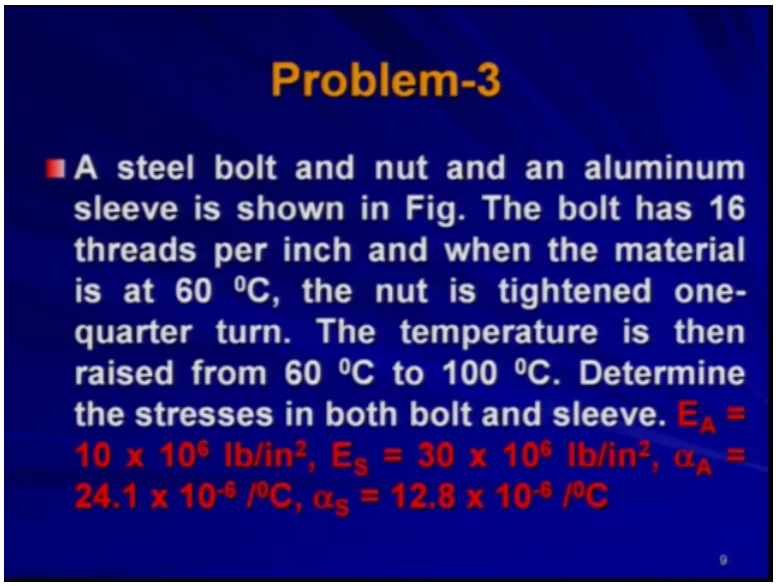

So, will take the next problem, the next problem says a steel bolt and nut and an aluminum sleeve is shown in the figure. So, I will draw the figure right now, the bolt has 16 threads per inch and when the material is at 60 degree centigrade temperature. The nut is tightened to 1 quarter turn the temperature is I mean see the bolt has sixtee16 threads per inch that is, but when the material is at 60 degree centigrade then the nut is tightened to 1 quarter turn then the temperature is raised from 60 degree centigrade to 100 degree centigrade. Now, determine the stress in both bolt and sleeve given that is here is Young's modulus of aluminum is given, Young's modulus of steel is given yes the thermal expansion coefficient of aluminum is given alpha a and alpha s that is the thermal expansion coefficient of steel.

So, now the problem is something like that you have an assembly of nut bolt and sleeve in that assembly you have given some 1 fourth or 1 quarter turn of the bolt, I mean nut due to that turn I mean 1 quarter turn of the nut the bolt as well as sleeve will be experiencing some mechanical strain at 60 degree centigrade. Now, that assembly will be explained in the mechanical strain and that mechanical strain we can calculate, now once this assembly is under equilibrium and then you are raising the temperature form 60 degree centigrade to 100 degree centigrade at that time only you will be observing the thermal strain due to the thermal expansion ok.

So, you will be getting the stress developed due to the mechanical strain and stress developed due to the thermal strain and these 2 stress components must be added algebraically to get the stress develop total stress developed in the components. So, this result, so let us do let us see the problem how we can solve it, first will draw the figure

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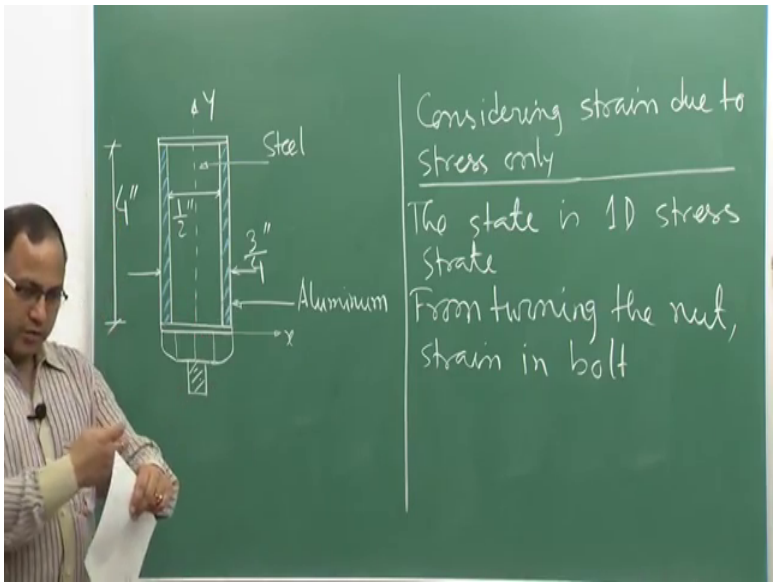

So, total depth of the arrangement is 4 inch. So, this is your aluminum sleeve all round aluminum sleeve something like your cylinder, inside that you have the steel nut and this is the bolt sorry steel bolt and this is the nut and this total outer dimension is 3 fourth inch and the inner dimension is 1 half inch. So, we are we are solving this thing this problem in aps unit whenever less that is not a problem well.

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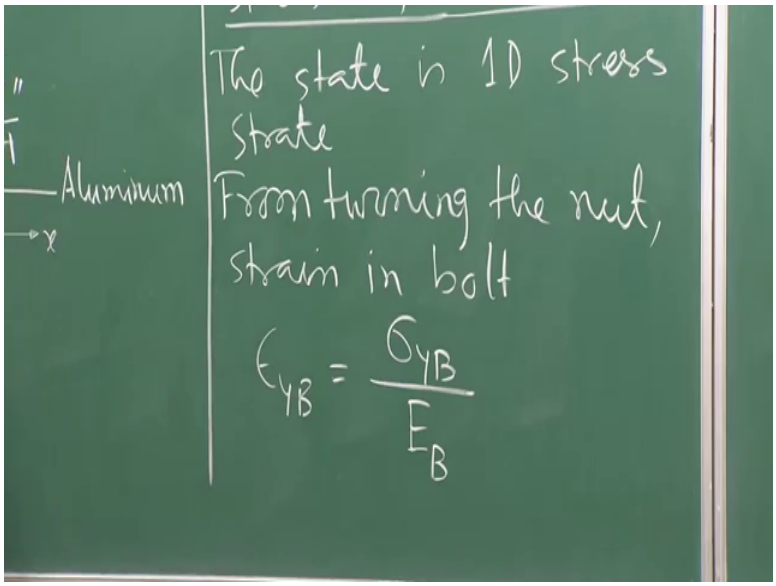

So, now the first thing is that considering, Considering strain please try to understand this saying conceptually, it is very very conceptual thing what is actually happening here considering strain due to stress on; that means, mechanical, mechanical strain we are considering first due to the tightening of the nut and other things. So, the state is 1 dimensional stress state that is very clear from the figure this is unidirectional in x direction there is nothing left out. So, only y direction you will be getting in the movement or deformation whatever.

So, from turning the nut the strain in bolt will be how much? When you are turning the nut in the bolt the bolt will be under tension right because you are applying here you are you are giving the turn. So, it is trying to push that thing push the sleeve. So, therefore, the nut I mean this bolt will be under tension right. So, that strain basically is nothing, but say epsilon y b, epsilon y is the strain in the y direction and b stand for bolt is equal to the stress developed in bolt in y direction divided by e b that is the Young's modulus for bolt that is nothing, but your steel e b is nothing, but steel right.

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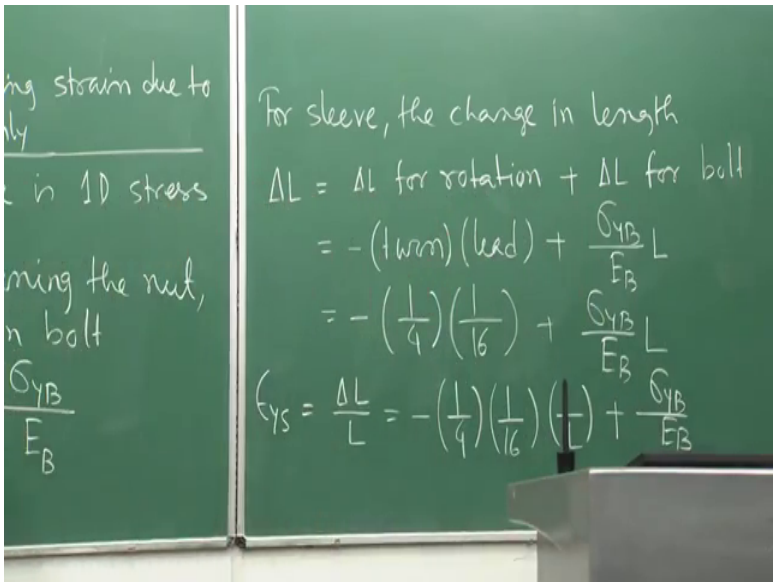

Now, for sleeve what is happening for sleeve for sleeve the change in length say delta L is equal to now delta L for rotation plus delta L for, delta L for bolt. Now, what is the meaning? Now try to understand the concept try to understand the mechanism what is happening. So, when you are giving one-fourth turn off the nut yes nut then basically it will try to compress the sleeve. So, due to that rotation of the nut you are getting the compression in the sleeve, but that will, that will create some force on the sleeve and that sleeve will try to expand again due to this because your bolt is getting extended and we are not considering any gap in between right; that means, we are not allowing any gap in the assembly or we are not allowing any kind of say discontinuity in the assembly.

So, the amount of turn of the nut will try to compress the sleeve and at the same time this bolt will try to get extended, as I told you due to this rotation of the nut and that extension will try to extend the sleeve also right. So, therefore, this this delta l total the change in length of the sleeve will be consisting of the contribution from the rotation of the nut as well as the extension of the bolt and this part as given in the problem that is that will be coming negative because that will creating the compression on the sleeve. So, that is nothing, but the turn how much turn you are giving in to the lead that. So, this generally comes from your screw jack or this kind of threaded material.

So, this is the rule right this much of this I mean distance or this much of length or the change in length is happening, but that should be minus because that would be the

compressive in nature plus your sigma y B by E B into L sigma y b e b is nothing, but your epsilon YBThat is the strain in the bolt the same amount of strain would be happening in the sleeve as well otherwise you will be getting the discontinue in the gap right and and this much of deformation or the change in length is happening due to the rotation of the nut. So, this has been given as 1 fourth turn and 16 it is given in the problem, the bolt has 16 threads per inch. So, 1 by 16 that is the lead Plus sigma y B E B, L therefore, epsilon y is what is that strain in y direction in sleeve. So, strain in sleeve is nothing, but this delta L by this delta L in change in length is sleeve divide by the whole length that is coming as minus 1 by 1 by 4, 1 by 16, 1 by l plus sigma y B by E B ok.

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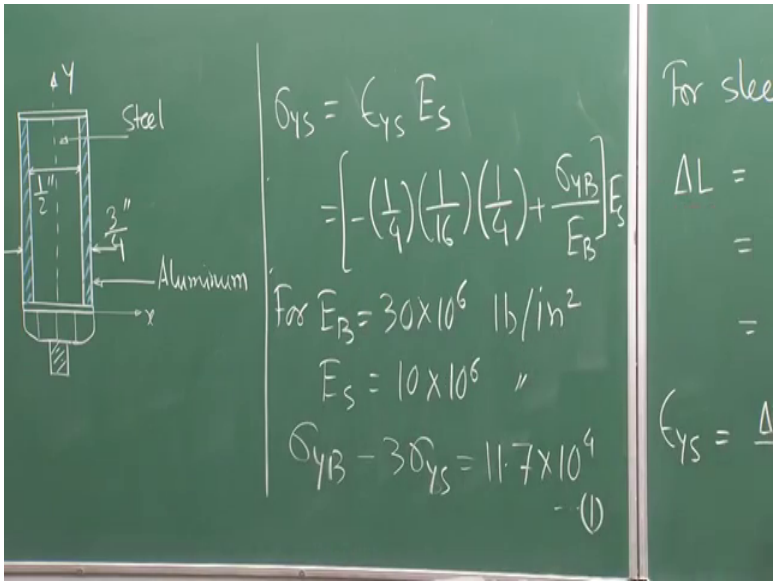

So, from this I can get sigma y s is equal to epsilon y s into e s that I can write from the stress strain relation which is nothing, but from this equation I can write minus 1 by 4, 1 by 16 and L is nothing, but 4 inch that is given plus sigma y B by E B into E s, this E s stands for e sleeve please try to understand e sleeve is for e aluminum e sleeve sleeve is made of aluminum. So, e s is not like I mean e of steel this is E of E of sleeve that this is a turn is I mean e stands for sleeve and this b stands for bolt do not get confused with that.

So, now from this as given in the problem E B is nothing, but for your bolt that is given as 30 into 10 to the power 6 pound per inch square and E s is 10 into 10 to the power 6 pound per inch square. So, by putting these values in this equation I can simply get

sigma y B minus 3 sigma y s is equal to 11.7 into 10 to the power 4 you say this is equation 1, by simply putting these values here I am getting 1 relation between sigma y b and sigma YSThat I am keeping equation 1.

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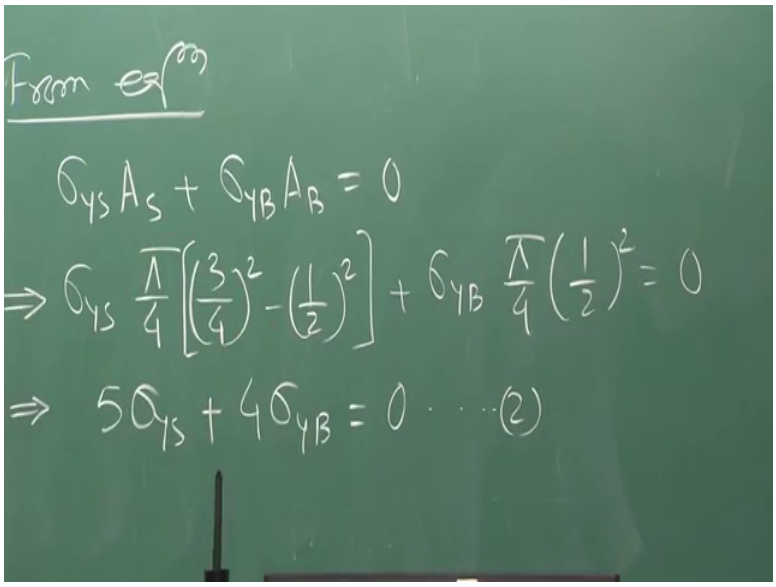

Now from equilibrium I can write sigma y s, A s plus sigma y b a b equal to 0 sigma y s is means the force in sleeve and sigma y b a b is nothing, but the force in bolt right. So, the vertical force that is a force in y direction sleeve as well as bolt the summation of all the forces must be equal to 0 because there is no external force applied in this assembly right I am not giving or not applying any external force. So, that must be 0 so this can be written as sigma y s, now this is a cross sectional area 3 by 4 whole square that is the outer diameter minus inner diameter. So, that gives the cross section area of sleeve plus sigma y B into pi by 4 into half. So, that is nothing, but the diameter of bolt. So, that gives me 1 equation 5 sigma y s plus 4 sigma y b equal to 0, say equation 2.

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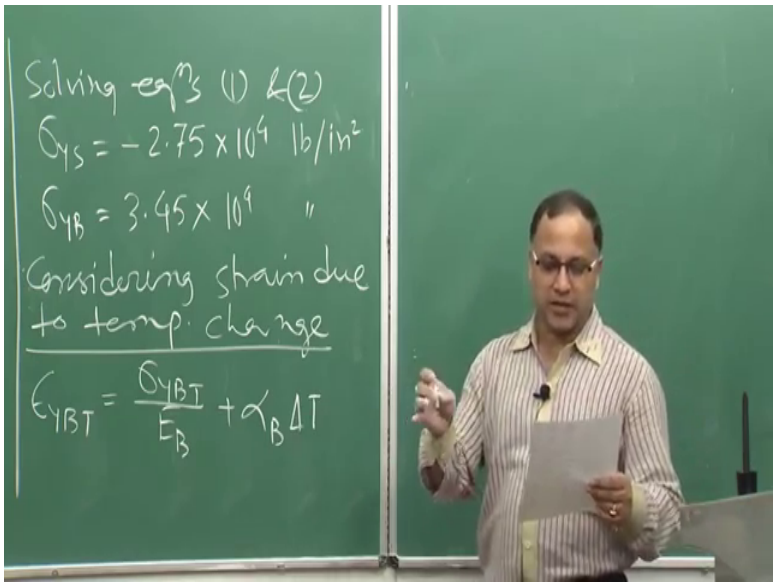

So, now solving equation, equations 1 and 2 we get sigma y s equal to minus 2.75 into 10 to the power 4 pound per square inch and sigma y b equal to 11.7 sorry 3.45 into 10 to the power 4 pounds per now from here you can see that due to the mechanical I mean strain your sleeve is experiencing the compressive force compressive stress and the bolt is experiencing the tensile stress fine. So, this is this is coming due to the mechanical strain now you have to consider the thermal strain. So, that will be considering considering strain due to temperature change.

Now, for that epsilon YBT, so t is stand for temperature. So, epsilon YBT is equal to sigma YBT by E B plus alpha b delta t. Now, if you put the all the values you know e b e b is nothing, but the e value of steel alpha b you know that is thermal expansion of the steel and delta t that is the change in temperature that is 100 minus 60 that is 40.

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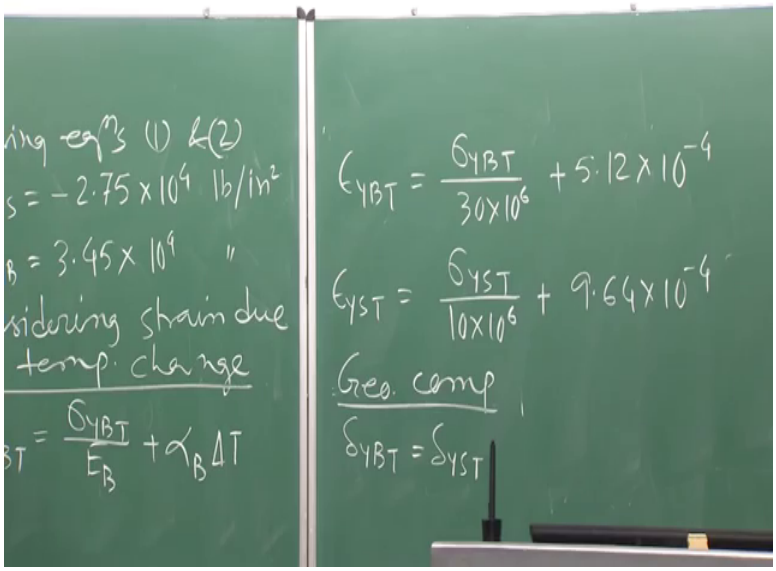

So, if you put all the values you will be getting epsilon YBT equal to sigma YBT 30 into 10 to the power 6 plus 5.12 into 10 to the power minus 4 similarly epsilon YST in a similar fashion you will be getting if you put all the things you will be getting sigma YST by 10 into 10 to the power 6 plus 9 point 6 4 10 to the power minus 4 now your geometry compatibility says that delta YBT is nothing, but delta YST.

So, anyway so, I will stop here today in the next class we will be continue in the same problem. So, geometric compatibility says that delta YBT equal to delta YBT; that means, the deformation in the sleeve must be equal to deformation in the bolt. So, I will stop here today in next class will continue the same problem.

Thank you very much.