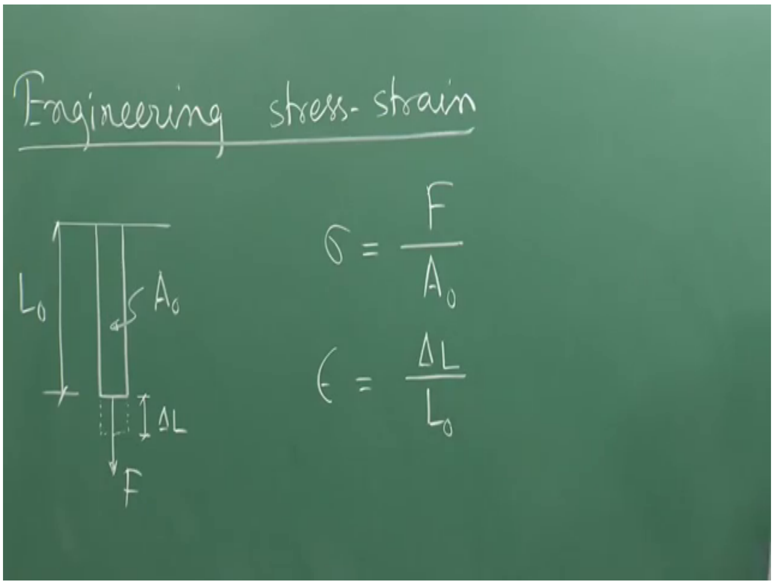
Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 33
True Stress Strain

Welcome back to the course mechanics of solids. So, in the last lecture if you recall we talked about the failure criteria the misses, 1 misses failure criteria as well as crista failure criteria. So, those 2 things we have discussed and there we, I mean at the end actually we just talked about engineering strain and true strain right. So, basically what does it mean?

(Refer Slide Time: 00:43)



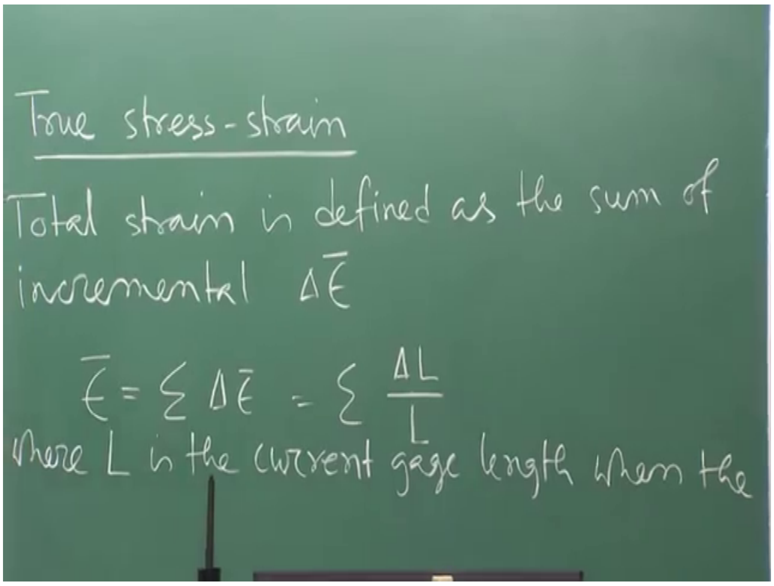
So, first if we look at the engineering stress strain, basically what does it mean? Why they are different? Why you are getting some different terminologies like engineering stress strain and true stress strain? So, what exactly does it indicate?

So, suppose you are doing some or you are taking some uniaxial member and you are applying some force, tensile force here and due to that action you will be getting some elongation in that member right, that is the common phenomena in the normal body and the original length of the member say L naught and original say cross sectional area of the member, say A naught. Now, if this increase in length is say delta L then basically your sigma as per we are discussing so far we have discussed whatever. So, that sigma that is the normal stress right, normal stress is nothing, but F by A naught right and your normal strain is given by delta L by L lot, right.

Now, the way you are getting the stress that tells you that we are not changing the force F during that during the experiment or during the test, but; however, as you know due to the poison effect when you are pulling it, when you are getting some increase or elongation in the in the member you will be getting the reduction in the area, cross sectional area a naught will be getting reduced, but that we are not considering. We are considering, when we calculating the stress we are considering the stress based on the original area A naught right and similarly when we are calculating strain at that time delta L is the final elongation of the member, but this L naught I mean this strain is nothing but a continuous process right, you are applying the load, you are increasing the load and the elongation is going on increasing right, but still we are considering the original length of the member and based on that we are calculating the strain.

So, this strains whatever or this stress whatever we are getting these stress strain both are known as engineering stress strain, then what is the true stress strain.

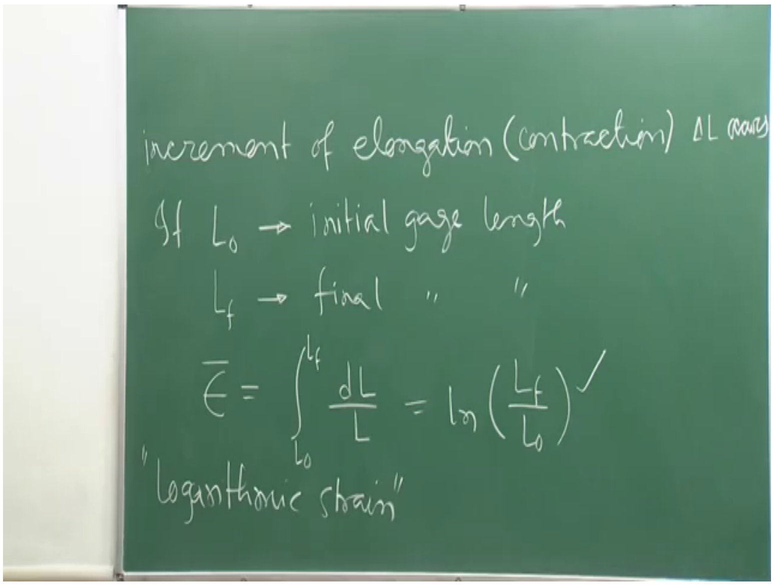
(Refer Slide Time: 03:47)



So, when we are talking about the true stress strain, because why I am saying true why so; that means, an engineering stress strain are not true they are basically some kind of I mean based on the original configuration right. So, they are not giving you frankly speaking, they are not giving you the actual response or actual figure what is happening during the process of the test right, during the process of the application of load, but that if you want to capture at that instance what is happening if you want to capture that thing then you have to talk about true stress strain.

So, in the true stress strain basically the strains may be large right, if you are considering strain is large and then if you, if you calculate engineering strain basically that will not give you the correct picture right, if the as long as your strain is very small then you can still think of the original I mean you can still think of calculation of the strain with respect to the original plane. But if your strain is very high then this will be giving very very approximate value right. So, the 2 stress strain basically the strains may be very large and if the strain is very large then basically you can consider the total strain is defined as the sum of incremental strain, incremental strain that is say delta epsilon bar. So, we are defining the true strain epsilon bar. So, there for your true strain epsilon bar is nothing, but summation of summation of your incremental strain that is your delta epsilon bar, which I can write summation of delta L by L, what is L? L is the length L is the current gauge length. So, at that instant when you are considering that particular increment right.

(Refer Slide Time: 06:34)

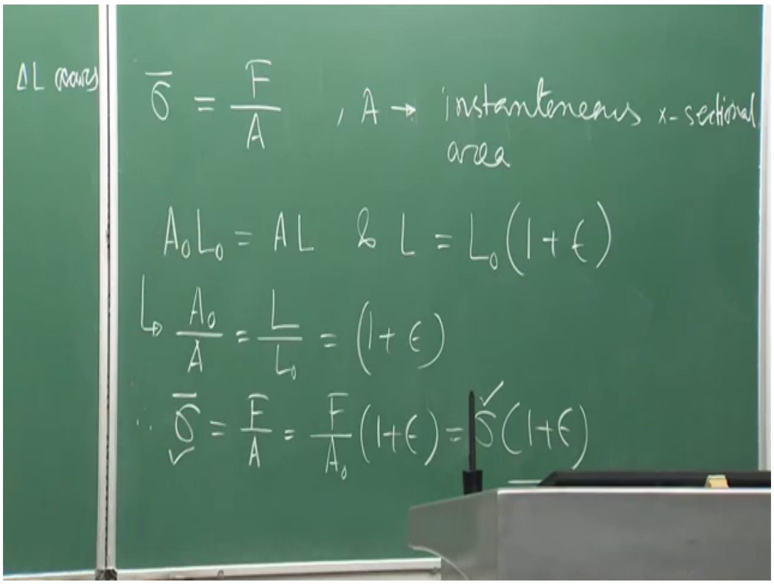


So, we can write down that where L is the current gauge length when the, when the increment of elongation or contraction whatever you consider depending on the type of loading, delta L occurs. So, while L is the current gage length then at that particular instance what is the gage length you are, you are defining the strain based on that current gage length when the increment of elongation or contraction depending upon the load whether it is tensile load or a compressive load delta L occurs.

So, that we can write down so if you are L not is your initial gauge length and say L f is your final gage length, then basically your true strain can be expressed as the integration L naught to L f, d L by L right very clear, I mean there is no doubt further. So, that if you do you will be getting that will be L n, L f by L naught. So, because logarithm base e is coming therefore, sometimes in some book it is called as logarithmic strain logarithmic strain. So, logarithmic strain is nothing but true strain, in some book you will be getting this term logarithmic strain. So, true strain or logarithmic strain both are same understood. So, this is the value of or the expression of true strain it is not very simply delta L over L naught. So, it is a continues I mean continuously or continually you are getting the increment or the elongation and based on that you are calculating the strain.

Similarly, you can calculate the stress; that means, true stress.

(Refer Slide Time: 09:05)



So, true stress that sigma bar we are saying, true stress is given by F by A, not A naught in case of engineering stress we calculated engineering stress equal to F by A naught, but here it is F by A. Now, what is A? A is instantaneous cross sectional area as I told you that when you are pulling in the bar or the pulling the I mean say member right your cross sectional area is I mean reduced getting reduced right, if you are compressing the bar or member your cross section would be will be getting increased. So, depending on that at that particular instance what is the cross sectional area if you considered that and based on that if you calculate the stress then that stress is nothing, but your to true stress.

So, it is not based on your original think. So, it is taking care of the instantaneous picture of the system or under the load. So, that your since you are this plastic strain or whatever strain you are I mean whatever say strain you are observing due to during that period actually your volume is not getting changed right, I am not changing the volume of the material right when you are applying the force your volume is not getting changed if your volume is not getting changed then I can write A naught L naught where A naught as well as L naught are the original dimensions, must be equal to A into L at any instant during the test whatever cross sectional area you are getting or whatever length you are getting gage length you are getting they must be same because we are not changing or we should not expect any change in volume right.

So, form and your L is at particular instant your L is given by L not l naught plus epsilon into L l naught right. So, that is your gage length at any instant. So, that can be written as this can be from here I can write A naught by A is equal to L by L naught is equal to 1 plus epsilon, that is strain. From this I can simply right now this is your true stress therefore, true stress is nothing but F by A can be written as in place of A I am writing, I am writing in terms of A naught, F by A not 1 plus epsilon. Now, what is F by a naught? F by a naught is nothing, but your engineering stress. So, this much is the difference coming between the true stress and the engineering stress understood.

So, this is your true stress this is your true stress and this is your engineering stress. So, this is the factor should multiplied with the engineering stress to get the true stress. Now, if you take the true stress versus I mean true as well as engineering stress versus true as well as engineering strain then how it will look like stress strain curve.

(Refer Slide Time: 13:12)

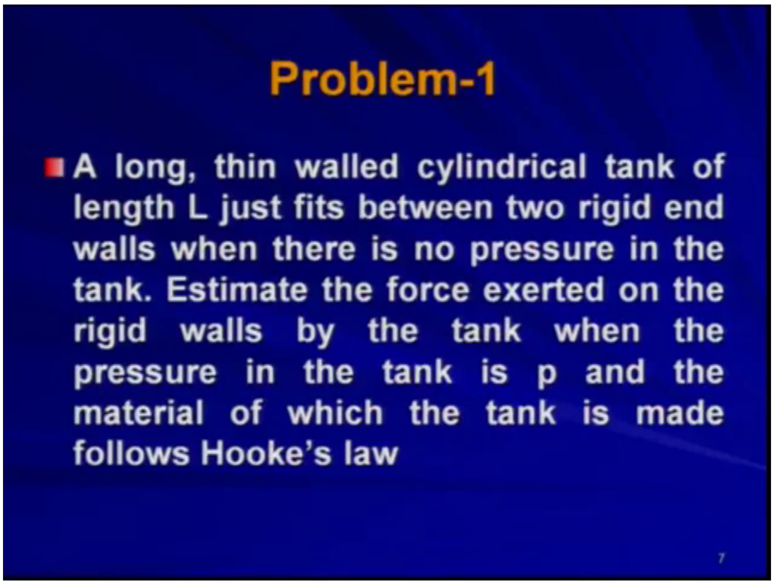
So, this is your stress strain this is your stress stress. Now you, this is your true stress strain curve, this is your engineering stress strain curve and you are getting the facture intention suppose you are you are applying the tensile force. So, you will be getting facture that point and here you will be getting fracture at that point. So, now, if you look at so, up to this point say both you would engineering strain; that means, as long as you are dealing with a small strain problem then engineering strain and true strain will not makes much difference right, as long as you are within this range of strain you see that same curve true strain and engineering strain both are predicting almost similar behavior or similar amount of strain right or stress strain behavior, but after this point suddenly your engineering stress strain follows this path whereas, the true stress strain is following this path.

Now, if you look at this region actually if you define your stress strain behavior based on your engineering stress strain behavior then after this point basically you are not getting this much of extra stress happening in the material. So, that approximation engineering stress strain approximation is basically giving you some conservatives some at a particular strain you are getting much higher stress if you talk about true stress strain right. So, that is why I mean in in this particular course also we will be sticking and I mean basically we are also considering the small strain problem in this particular curse. So, therefore, we will be sticking to the engineering stress strain. So, we are not talking will not be talking about much about the true stress strain I hope it is clear.

So, with this I conclude this chapter that is your stress strain and temperature relation. Now, we will take a few numerical problems to understand whatever we have learned so far in this particular chapter.

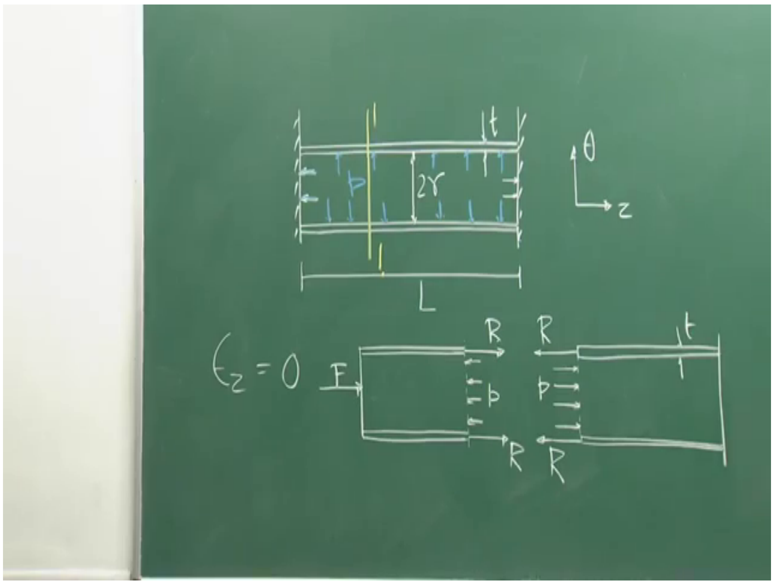
The first problem we are taking here so if you look at the, the slide.

(Refer Slide Time: 15:58)



The first problem tells that a long thin walled cylindrical tank of length L just fits between 2 rigid end walls when there is no pressure in the tank. Estimate the force exerted on the rigid walls by the tank when the pressure in the tank is p small p and the material of which the tank is made follows Hook's law. So, let me draw the figure.

(Refer Slide Time: 16:32)



So, basically you have 2 end walls and you have a very thin walled tank which is just fitting between the end walls and now this is the thickness of the wall, this is the length of the tank as given in the problem L and this direction say this is z and this is theta theta is something like your circumference that is a hoped direction and the 2 r is the inner diameter of the tank and this is under internal pressure internal pressure p. So, problem says that if you, if you go back to the problem, problem says that the tank is I mean tank just a fits between the 2 rigid end walls when there is no pressure, it is just fitting between 2 2 walls now estimate the force exerted on the rigid walls. So, this is the force exerted on the rigid walls as shown here, when you are applying some internal pressure p inside the tank.

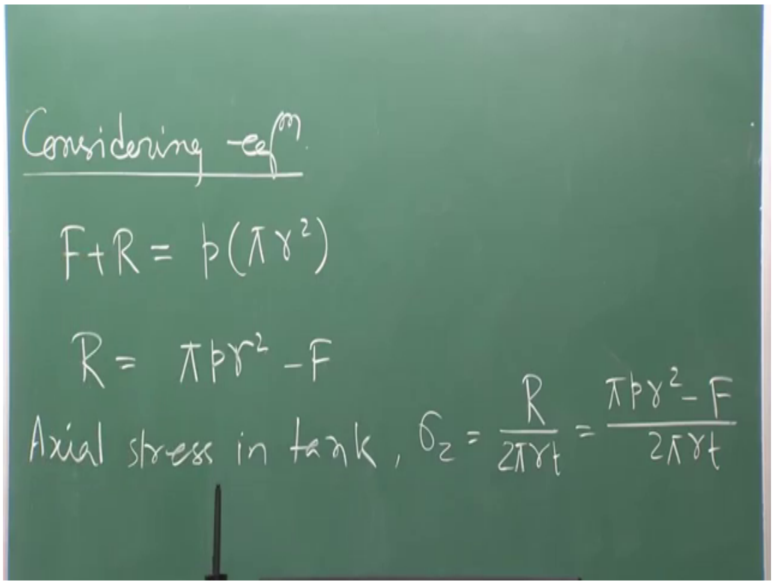
Now, I mean if you recall the discussion whatever we made for thin walled pressure vessel. So, that idea or that concept will be required here to find out the stresses I mean stresses, stress components there are in different directions. So, I am not going to drive that thing because if you apply the concept of thin walled pressure vessel then basically you will be getting the stress components in all directions there is sigma z sigma and sigma theta. So, basically you have three directions right in the polar coordinate system z r and theta. So, you will be getting the stress components you can calculate or you can determine the stress components in all 3 directions.

So, that thing I am not going to do because that you can perform or you can do by using the theory whatever we have developed for the thin walled pressure vessel earlier. So, we will be taking that thing directly, I mean that magnitude of the stress components will be taking in this problem directly fine. So, now, if you look at this problem it is very much clear form the figure itself because these 2 are the rigid wall. So, your epsilons z must be 0; that means the deformation or the strain in the z direction is simply zero right because the rigid walls will not allow any kind of deformation in the z direction. So, that is my say geometric fit you can say.

Now, considering the say free body diagram I am considering 1 say section here, section 1 1, in that section 1 1 if I draw the free body diagram. So, that will look like say, if you take if you consider one section 1 1 like this and you are taking you are drawing the flow diagram of both the parts. So, this is the left hand side part and this is the right hand side part. Now, at section 1 1 if you look at, if you consider 1 1 plane 1 z plane rather. So, on that z plane you are having this internal pressure is giving the force that small p will give you the force if it is a circular because this is a circular tank. So, on the circular cross section it will be applying the force and this will be the force because this, this tank will give the thrust on the wall. So, there for wall will be giving one opposite equal and opposite reaction on the tank. So, that is shown here that is F and that has to be calculated that is the objective and under this action of force and reaction and all those things external force you will be getting some force will be developed along the wall right, otherwise this and this R will be that is something like your tangential force.

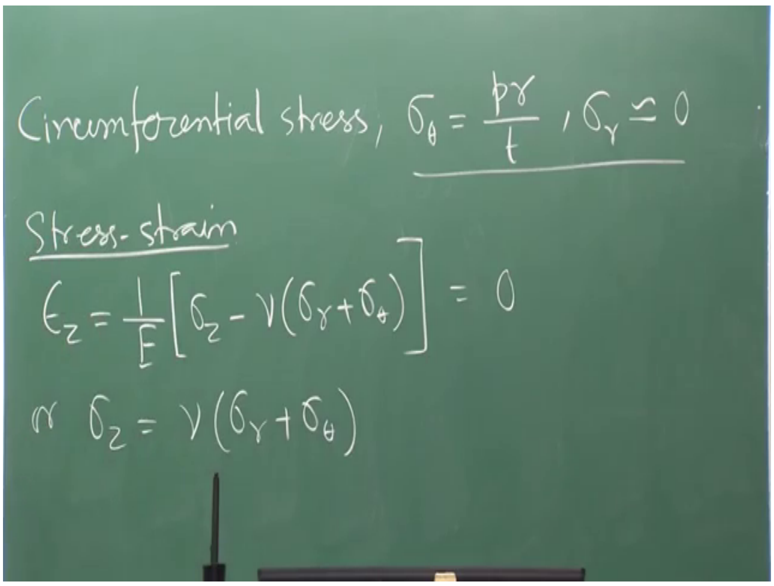
If you have the hoop like the, I mean tank like that so it will be tangential to the tank and if you. Similarly, if you consider the right hand side of the tank this R should be equal and opposite as you know from your discussion and of course, you will not be having any force in the radial direction by following the same argument whatever we did when we talked about the thin walled pressure vessel right. If you have the radial direction force then basically you are disturbing the symmetry right. So, by following the same argument we can consider we can, we can say that there is no radial direction force fine.

(Refer Slide Time: 23:42)



So, under now if I consider this part left hand side part then I can consider the equilibrium considering equilibrium, I can write F plus R both are in the same direction positive z direction is equal to p into pi r square pi r square is nothing, but the cross section area on which the internal pressure is acting. So, from this I can simply get R equal to pi p r square minus F. So, there for you are axial stress in cylinder axial stress in tank that is your sigma z is nothing but R that is the, that is the tangential force in the z direction force R divide by the area. Now, what is that area on which r is acting to pi r that is the circumference multiplied by thickness t as simple as that. So, this is nothing, but by putting this value pi p r square minus F divided by 2 pi r t.

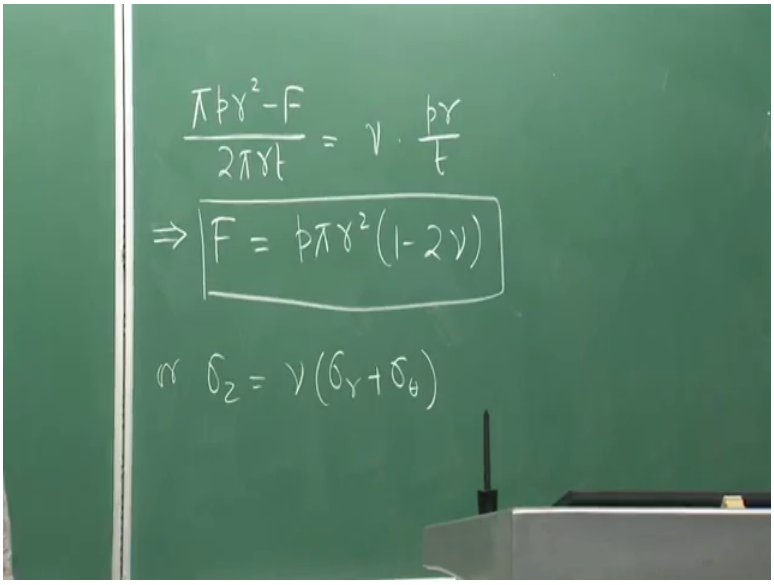
(Refer Slide Time: 25:17)



Now, circumferential stress, so this things we are taking from our previous discussion I mean even if you solve the thing of pressure vessel. So, these, these stresses stress components you will be getting. I am not roving or I am not determining that thing again in this particular in this particular problem; however, that you can you can get by following that saying theory whatever we have talked about for thin walled pressure vessel. So, the circumferential stress will be this you can calculate or you can determine for thin walled presser vessel that should come like that and your sigma sorry sigma r should be approximately 0.

So, this 2 components you will be getting from the theory of thin walled pressure vessel that you can determine by following the derivations whatever we did for our thin walled pressure vessel anyway that we are not repeating here. So, we have got 3 stress components sigma z sigma theta and sigma r. So, now, our stress strain relation if we invoke then we will be getting epsilon z is equal to by following because this material is following the hooks law sigma z minus mu into sigma r plus sigma theta which is nothing, but 0 already we have seen that because your wall is not allowing both the walls are not allowing the movement in the z direction. So, from this I can write sigma z equal to mu into sigma r plus sigma theta.

(Refer Slide Time: 27:33)



Now, by putting the values here now by putting the values of sigma z sigma r and sigma theta we are just, Getting like this pi p r square minus L by 2 pi r t that is your sigma z, I mean In sigma z into nu this poison ratio into p r over t, because sigma sigma r is 0. So, this is only the sigma theta. So, from this you will be getting F equal to p pi r square 1 minus 2 this is the force exerted by the tank on the wall. So, if you want to design your wall. So, what should be designed for that that much of force so, we have got that. So, I hope that you have understood the steps involved in this particular problem and in this therefore, I mean and I mean whatever, whatever process I mean by following the geometric feet and all those things you just, you just got this force because epsilon z is 0 and based on that you can find out this.

So, I will stop here today in the next class we will be continuing another couple of numerical problems.

Thank you very much.