Mechanics Of Solids
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Lecture - 32
Tresca Criteria

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall, we talked about the complete equations of elasticity and then, we just started the condition for initial yielding and there we discussed about the first criteria that is nothing, but Von Mises Yield criteria, right and if you see the Von Mises Yield criteria is giving me the effect of all say principal stresses sigma 1 sigma 2 sigma 3 in such a way that a particular combination of sigma 1 sigma 2 sigma 3, that combination will satisfy the equation whatever was given earlier in the last lecture and that will give me the initiation of yield, right.

So, now in case for stress components with respect to non-prince principal axes, ok.

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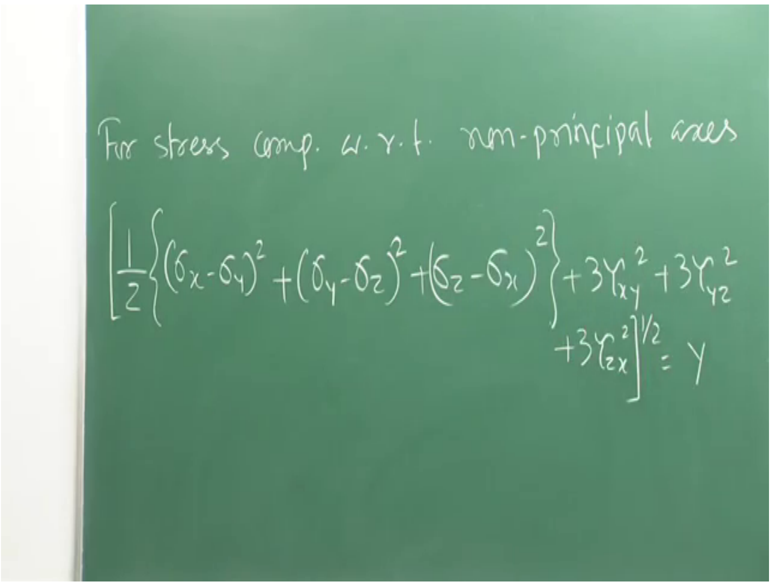

So, whatever we developed in the last lecture that is Von Mises Yield criteria based on the principal axes, right sigma 1, I mean 1 2 3 coordinate system, but if you say that your xyz coordinates system is not coinciding with 1 2 3 coordinate system that this 1 2 3 system or the principal axes are different than your xyz coordinate system. In that case, if you measure the stress components in xyz coordinate system, then your Von Mises Yield criteria will be modified, ok.

So, for stress components with respect to non-principal axes, that means xyz coordinate system is there which is not your principal for your system which does not coincide with 1 2 3 coordinate system. In that case, the Von Mises Yield criteria is something like this, half into sigma x minus sigma y whole square because now you will be getting the effect of shear stress as well because earlier you consider only the principal.

So, therefore, no effect of shear stress was there right, but in this particular case you will be having the effect of shear stress sigma x minus sigma y whole square plus sigma y minus sigma z whole square plus sigma z minus sigma x whole square plus 3 tau x y square plus 3 tau y z square plus 3 tau z x square to the power half equal to y. So, do not think that this is kind of empirical. It can be proved I am not showing the proof because I mean that proof will take a little bit longer. See derivations of those, but if you are interested you can refer any book to get the proof. However, I am not showing the complete proof for this particular thing, but it is not the empirical thing and it can proved, ok.

So, in xyz coordinates system your Von Mises riteria will be modified as per that. So, this is the effect of shear stress coming out of the picture. So, now whatever we are considering, this particular course as we have discussed that for I mean all the times you are considering the plane stress condition.

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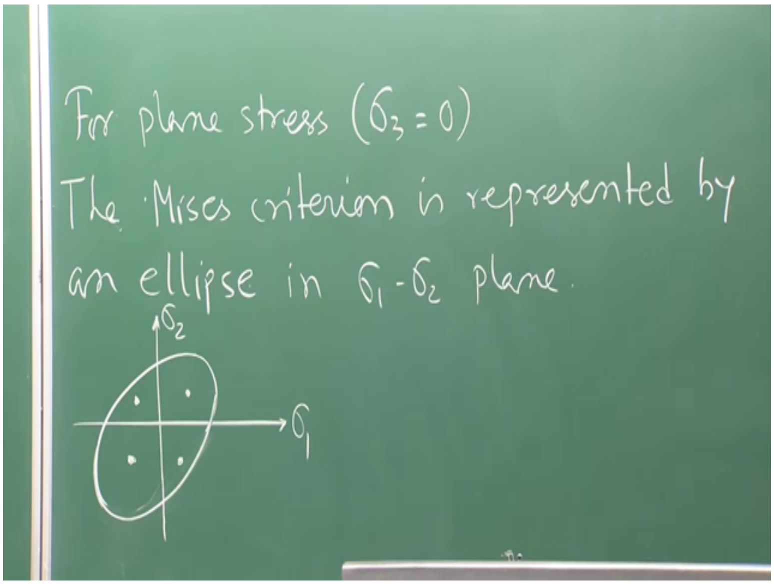

So, plane stress means your sigma 3 is 0 again. Again we are going back to our 1 2 3 coordinate system that is principal axes system. So, for plane stress condition sigma 3 is 0. So, the Mises criterion is represented by an ellipse in sigma 1 sigma 2 plane. Again it can be proved, however this proof is little bit say lengthy and you need little bit more background on that. So, that is why we are not talking about that thing.

So, anyway it can be proved, but you should know that what kind of shape it will take in sigma 1 sigma 2 plane, the mises criteria because the equation given in the mises criteria will define will represent some kind of say geometric figure and that is nothing, but one ellipse. So, if you look at the shape in sigma 1 sigma 2 space, this is your sigma 2, this is your sigma 1 say it will be an ellipse. So, it will be an ellipse. So, now what does it mean? If you look at this thing basically if you recall your Von Mises criteria whatever we have discussed in the last class, basically there you put sigma 3 equal to 0 and you will be getting the equation and that equation will be defining one ellipse in sigma 1 sigma 2 space in case of plane stress condition.

Now, this ellipse basically represent or it says what information you are getting from this ellipse. So, that is very important for the yield criteria, ok.

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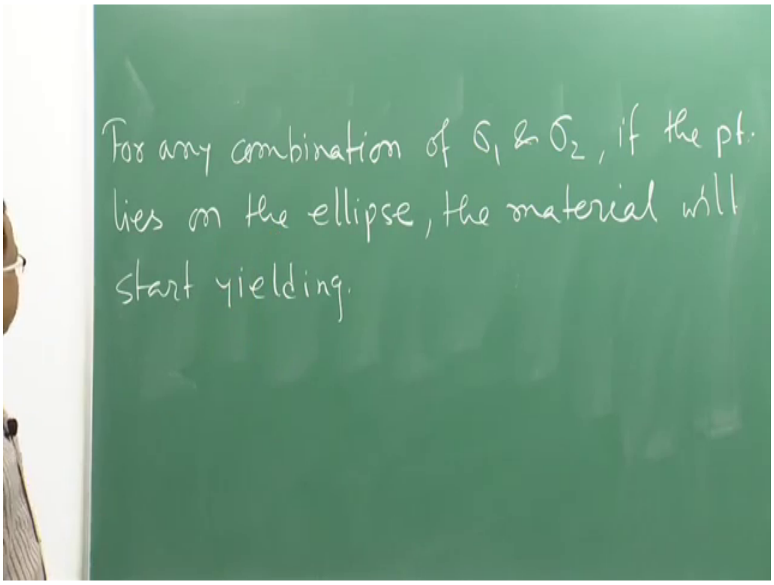

Now, for any combination as I told you earlier, combination of sigma 1 and sigma 2 if the point lies on the ellipse, the material will start yielding. So, what does it mean? It says that for any combination of sigma 1 and sigma 2 if the point say for any combination, so if I choose sigma 1 sigma 2 or if I get from the calculation if I get sigma 1 sigma 2 in such a way that the point is lying somewhere here or maybe somewhere here or maybe somewhere here or may be somewhere here. So, n number or infinite number of combinations you can have with sigma 1 sigma 2, right.

So, for these combinations, you are not getting any yielding as per the Von Mises criteria because this is your boundary on which if the point lies, then you are getting yielding. That means, if you are getting for any combination if your point lies on the ellipse, you will get or the yielding will initiate, understood. So, that will give you basically this is the boundary of your yield condition or the yield criteria. So, this is the boundary within which you will not be getting anything, but if you are going on or if you are touching the surface that is the surface of the ellipse, you immediately will be getting yielding. That is the idea of your Von Mises criteria and that is very important actually. Isn't it? So, once you have established this ellipse from the criteria, then you just for any kind of combination of sigma 1 sigma 2, you just find out where is the point lies. Ig the point lies in this ellipse, there is no problem. So, you can go ahead, but if the point lies on the ellipse, then you must rethink because yielding will initiate, ok.

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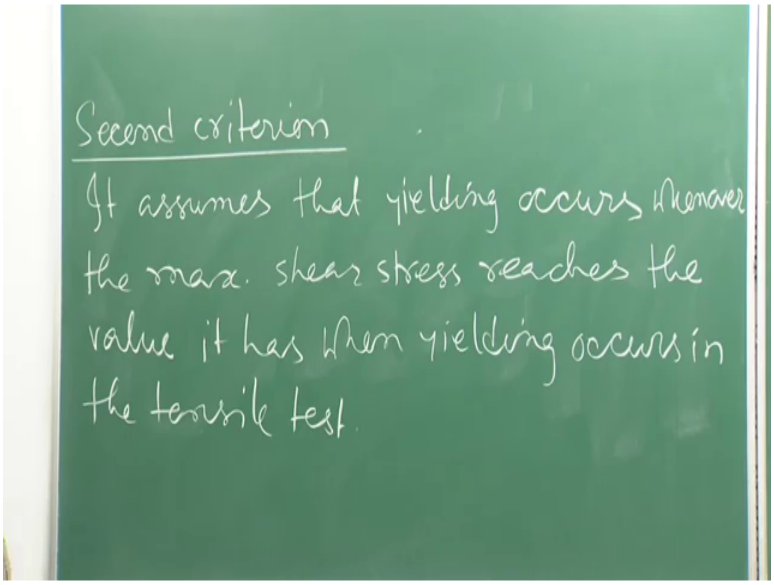

Similarly, the second criterion, it assumes that yielding occurs whenever the maximum shear stress reaches the value. It has when yielding occurs in the tensile test. The second criterion says that it assume that yielding occurs whenever the maximum shear stress by this time you know what your maximum shear stress is.

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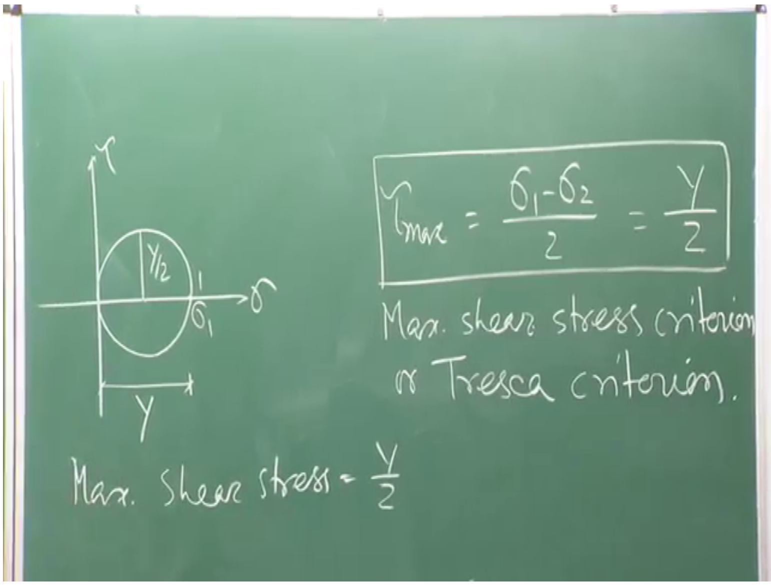

When you will get it, well the maximum shear stress reaches the value, it has when yielding occurs in the tensile test. So, if you draw the monocycle sigma tau space basically in case of say one directional, one-dimensional or 1 d say simple tensile test.

So, basically your monocycle will look like this, right. This is your point 1 and this is nothing, but your sigma 1, right. So, that is nothing, but y right.

So, your maximum shear stress in the tensile test is nothing, but y by 2. So, this is your radius of the monocycle is your maximum shear stress. So, maximum shear stress is nothing, but y by 2. So, now if you consider only sigma 1, sigma 2 is acting, right. In case of plane stress condition, we are just considering plane stress condition or even if you consider a three-dimensional state of stress, you know the maximum shear stress is coming from the bigger circle because if you consider three dimensional state of stress, you will be getting three monocycles you remember. So, 2 will be smaller and that will be coming inside the bigger monocycle, right. So, the maximum shear stress will be dependent on the maximum or the bigger monocycle whatever.

So, if you have sigma 1 sigma 2, your tau max is nothing, but sigma 1 minus sigma 2 by 2. That is the radius of the monocycle. Agreed or not? Tau max will be always sigma 1 minus sigma 2 by 2 and that is equal to y by 2. So, this is your criteria and this criteria is known as maximum because it is based on maximum shear stress. So, maximum shear stress criterion or popularly known as tresca criterion. So, we have learned two different criteria. Von is one, Mises another, one is tresca and they are very much used in mechanics. Apart from these, this failure criterion or the yield criteria you will be getting several yield criteria which will capture the behavior of a particular or the specific material. So, that I am not going to discuss. So, if you do some advanced level course, you will be getting those criteria later on. So, for the time being we are just talking about only these two criteria i.e. one is Von Mises and another one is tresca, ok.

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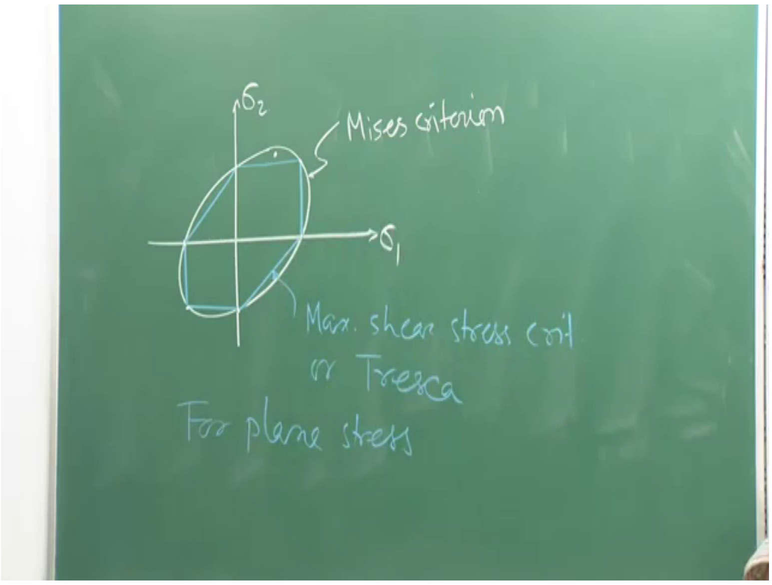

So, if you try to plot this tresca criteria in sigma 1 sigma 2 space or the plane like whatever you have done for Von Mises, then this is sigma 1 and this is sigma 2. So, your Von Mises criteria was an ellipse. Am I right? So, that was your Mises criterion whereas, if you see this equation will develop this kind of say hexagon. So, this is nothing, but maximum shear stress criterion or tresca. This is for plane stress for three-dimension state of stress. How you will get? So, those things we are not going to discuss of course. If in case of three dimensional stress, these Mises criterion will be looking like a circle on the pipe plane that is some special plane which can be defined by knowing something else, so that we are not going to discuss. Those are little bit in the advanced level and then, that cylindrical surface.

So, Mises criteria will be defining a cylindrical surface on that pipe line whereas, this tresca criterion will be defining regular hexagonal say cylinder which will be inscribed in the Von Mises criterion, so that you can see that tresca criterion or this hexagon, that blue hexagon is basically getting inside the Von Mises criteria. What does it mean? That means, it says that at these points tresca at this this point basically, tresca and Von Mises both are defining the same, I mean yield point. I mean the same yield condition actually, ok.

So, if you are combination of sigma 1 sigma 2 is such that you are getting these points. So, these I mean whether you are going by tresca or Von Mises, I mean no matter whatever method or whatever failure criteria you choose, you will be getting the yielding, but if your combination of sigma 1 and sigma 2 are such that that you are here, right. Suppose I am not here, suppose you are here. So, as per tresca you are getting yielding, but as per Von Mises you are not getting yielding, right. So, when this point will move to this ellipse, then only you will be getting yielding as per Von Mises, but as per tresca you might have got the yielding when you reach an this surface. So, this blue hexagon will give you the boundary for the tresca criteria, ok.

So, with this I mean whatever we have discussed about the failure criteria and initial yielding and all those things that is over, right. So, now we are going to talk about the very important, another important say concept there is engineering strain and engineering stress strain and true stress strain, right. So, that thing will be talking about in the next lecture, however what basically engineering stress is and true stress. Now, engineering stress is nothing, but the stress which is calculated based on the initial configuration because if you see you might have seen this thing when you have done the uniaxial test, uniaxial tensile test or whatever you have done in your strength of materials or mechanics of solids lab. There you might have seen this kind of situation.

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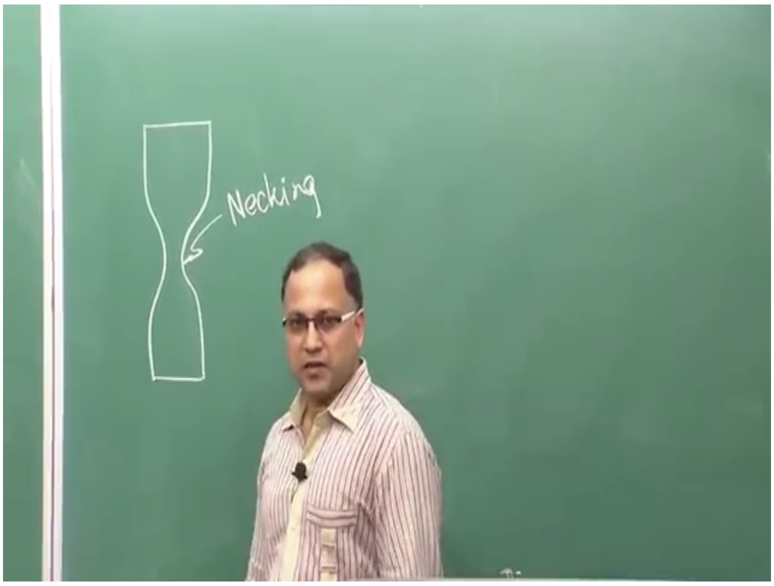

So, this is your original say diameter original dimension if you are considering the steel rod. So, at certain point, you will be getting this kind of reduction because when you are pulling it at the failure, you will be getting this kind of reduction in the material, right in the rod or the one direction, one-dimensional say bar whatever you considered and this is known as necking, right. You know that happens generally just before the failure. So, what I mean to say that means, when you are pulling something or when you are pushing something, when you are considering compression or attention.

So, in the process of your application of load or the stress, your cross-sectional area will not remain same, right; so if you define the stress based on your original dimension that may not be the true situation or the true stress. So, whatever stress is defined based on the original dimension, that is known as engineering stress and the stress which is defined based on the change or the altered configuration of the dimension that will be defined as the true stress and similarly, the strain components.

So, I will stop here today. In the next lecture, we will be discussing this Engineering Stress Strain and True Stress Strain and then, we will solve some numerical problems from this chapter and will conclude this chapter.

Thank you very much.