Mechanics Of Solids

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Lecture - 31
Von Mises Yield Criteria

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall we talked about the stress strain relation for the isotropic material, and there we concluded that the normal stress only develop normal strains and shear stresses only develop shear strain right. And based on that we established the generalized hooks law and based on that we have established the relation between the stress and strain right and then we talked about the thermal strain in case of elastic isotropy material and then we saw that if you have the mechanical strain as well as the temperature strain, then how we can combine these 2 effect; that means, mechanical strain and the combine strain together to get the total strain ok.

So, now today we are going to discuss about or going to write down the complete equations of elasticity. So, whatever we have learned so far. So, we are now writing one by one so that whenever you are dealing with some elastic say analysis and if you are we calling the theory of elasticity, then these equations must be satisfied to say that these body or the system is under equilibrium.

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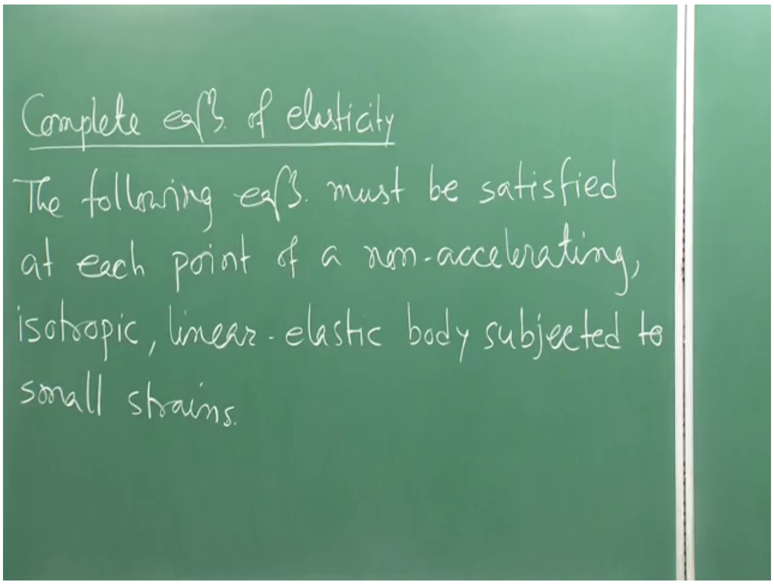

So, the following equation whatever we are going to write the following equations must be satisfied at each and every point of a non accelerating. So, there please mind it. So, that should be non accelerating isotropic linear elastic body subjected to small strain.

So, if you consider all these terms that is non accelerating isotropic linear elastic and small strain.

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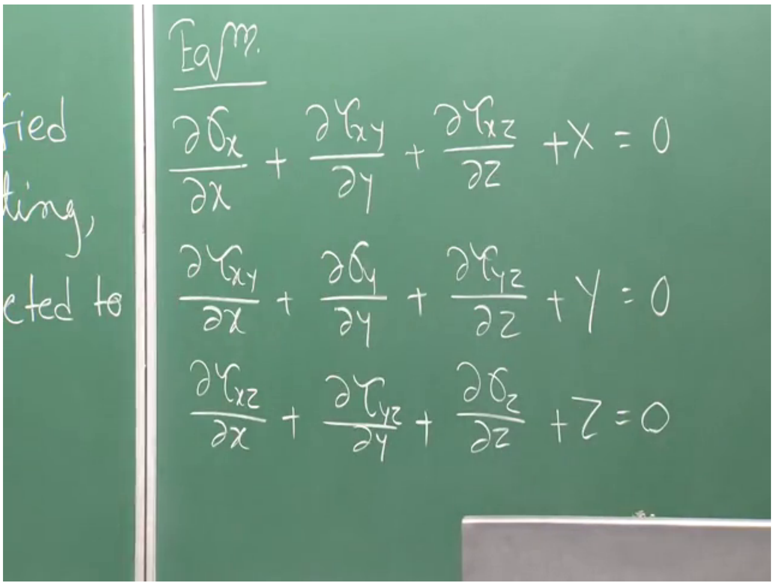

Then the complete set of equations of elasticity can be written as the first one is for equilibrium, that is for equilibrium condition already we have derived that thing for 2 d as well as we have extended the same thing for 3D. So, now, we can write down that thing for 3 D state of stress plus x equal to 0 del tau x y del y sorry del x plus del sigma y del y, plus del tau y z del z plus y equal to 0, and del tau x z del x plus del tau y z del y plus del sigma z del z plus z equal to 0.

So, these are three equations of equilibrium which must be satisfied at each and every point for a non accelerating isotropic linear elastic material when you are talking about the I mean equilibrium conditions or the elasticity equations. So, now, what are these capital X, capital Y, and capital Z as we have discussed earlier these are nothing, but the body forces along x y and z directions right.

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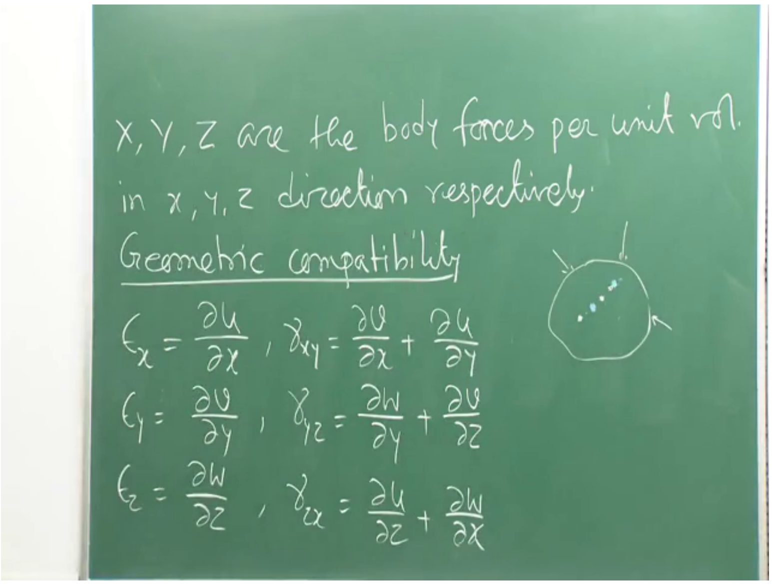

So, X,Y and Z are the body forces per unit volume these are the body forces per unit volume of course, in X,Y,Z directions so, that we can write in x y z direction respectively. So, this three equations are required for your equilibrium condition then you have geometric compatibility. Now what is that? Already from the name itself it is clear that you are going to satisfy some compatible condition right. So, you have seen that if you apply stress some strain or the deformation is happening, but this deformation cannot be arbitrary it should follow some rules, there is some rule of the game and that rule should be followed by this deformation.

That means, if I if I consider say a kind of say body and I will applying externally applied forces all round now if I consider 2 points here, now after deformation this point may move little bit say it is coming here and this point may move little bit and it is coming here say. So, I can use different colors that will be convenient for you to understand. So, after deformation this point is moving here and this point is moving here.

But no where this this this say point right point means some say particle. So, we are considering 2 particles and now after deformation this particle is going to this location and this particle is going to this location, now no where this particle should cross over right should overlap the previous particle. So, that is a kind of compatible condition and similarly you should not get the development of crack inside the system. Suppose if this the body you are applying some force, you are you your body is a continuum body right even after deformation that will be remain in continuum. So, you will not be getting any discreet gap in between the system or the body ok.

So, these are the things which can be satisfied by the geometric compatibility. So, the geometric compatibility is or can be written as in terms of the strain and the displacement as already you have seen already you have derived that epsilon x is del u by del x epsilon y is del v del y, and epsilon z is del w del z and similarly gamma x y equal to del v del x plus del u del y gamma y z equal to del w del y plus del v del z and gamma z x is equal to del u del z plus del w del x where your u v and w are the velocity or the displacement components right they are nothing, but the if it time dependent then it displacement (Refer Time: 08:46) velocity or otherwise displacement anyways.

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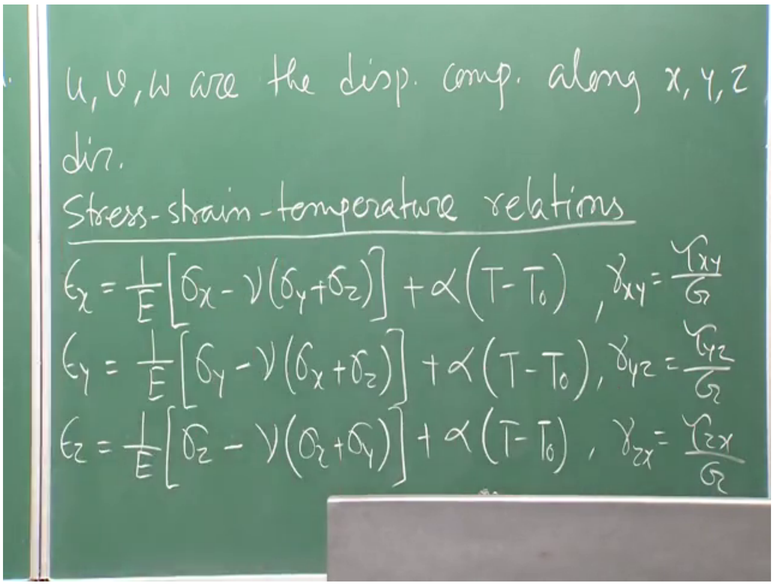

So, that is u v and w are the displacement components along x y and z directions.

Already we have derived that see we are not now writing down all those things systematically, and at the same time we are defining those equations as per their function right. Previously we have defined the equilibrium equations so that is required for satisfying the equilibrium condition and this set of equation is required to get or to satisfy the geometric compatibility and next set is nothing, but your stress strain temperature relations.

So, all those equations we have derived somehow we are writing down one by one systematically, epsilon x can be written as in the previous class we have discussed that sigma x minus nu sigma y plus sigma z plus alpha coefficient of thermal expansion into T minus T naught that is the difference in temperature. Similarly epsilon y is equal to 1 by E sigma y minus nu is nothing, but that is poisson ratio sigma x plus sigma z plus alpha T minus T naught similarly epsilon z is equal to 1 by E sigma z minus nu into sigma x plus sigma y plus alpha into T minus T naught and your gamma x y similarly gamma x y can be written as tau x y by G that is G is nothing, but the shear modulus and gamma y z can be written as tau y z by G and gamma z x is equal to tau z x by G right. So, this three are there are six equations are nothing, but the stress strain temperature relations. So, this is basically the complete equations of elasticity.

So, if you are dealing with theory of elasticity or whatever type of system I told that in non accelerating isotropy, linear elastic and of course, small strain if you are considering then these equations must be satisfied ok.

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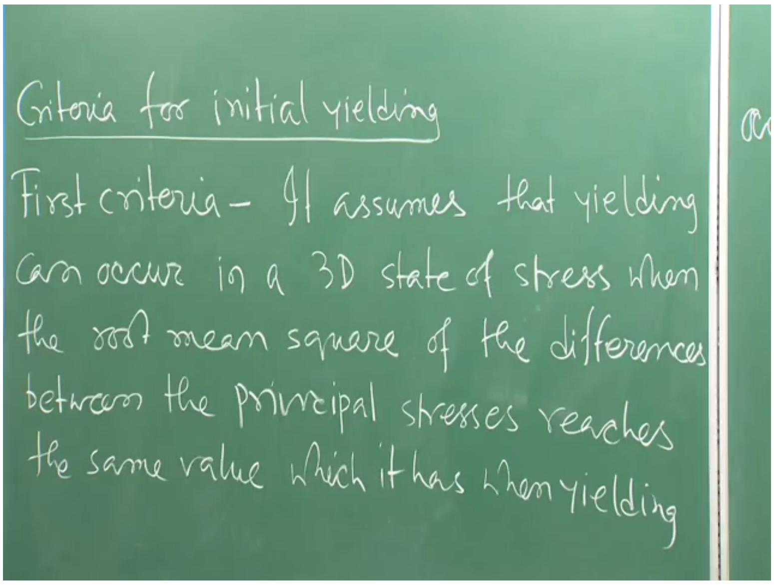

And these equations will be required to get the complete information about the system. Now we are going to discuss about the criteria for initial yielding. So, criteria for initial yielding. So, there some times this is also known as your phyllio criteria or whatever I in different books you will see phyllio criteria or criteria for initial yielding or say yield criteria ok.

So, different names have been proposed in different books however. So, this is the criteria by which you can say that if you apply I mean say the if any system is under different externally applied loads or the forces, then basically at which point the material start yielding that kind of information is very much required for the design engineer. So, for that particular purpose you should know for any particular material or say if it is one D or 2 D or 3 D it does not matter, but the thing is that whatever material you are considering or whatever system you are considering you are applying some externally applied loads and under that circumstances whether the material will yield or not.

And you will know by this time that yield point is a very very important say point. So, this point is basically I mean making the demarcation between the elastic limit or the elastic say behaviour and the plastic behavior right. From the yield point basically your plastic deformation initiates. So, that point is very very important to be determined otherwise what will happen say suppose you are going on increasing the load on the material or the body at some point you will see that the plastic deformation starts and without taking any external load the whole material starts collapsing suddenly right. So, because as you have seen that when if you are talking about the perfect plastic condition then you have seen that after reaching the yield point there is no increase in the stress rather the strain will be go on increasing. So, to avoid those kind of situations in the real life, we should know that at which point or at what kind of combination of loads or the stress the material will start yielding right.

So, if you consider 1 D condition. So, yield criteria will be dependent only one directional or 1 D a set of stress. If you consider 2 dimensional set of stress then basically 2 stress I mean directions will be I mean making this yield criteria if you consider the three dimension set of stress then all the stress components. So, it is a what I mean to say that, it is a combination of stress which will govern the yield initiation of yielding right.

So, the first theory the first criteria says what does it say because we are now we are defining some rules and the material I mean it is not required that all the materials will be following this criteria. So, we will be defining 2 criteria basically this is first criteria and we will be defining the second criteria that is not necessary that all the material. So, whatever you are seeing in the universe that all the materials will be will be say following this first criteria right some material will follow the first criteria some material will follow the second criteria and depending on that basically phyllio theory have been developed in the plasticity ok.

So, now we are defining the rules or the criteria now we are not talking about the which material is following this. So, that depends on the material behaviour that we are not talking about, but we are defining the rules or the guidelines. So, first criteria says it assumes it assumes that yielding can occur in a 3 D state of stress, when the root mean square of the differences between the principle stresses reaches the same value which it has when yielding occurs in the tensile test.

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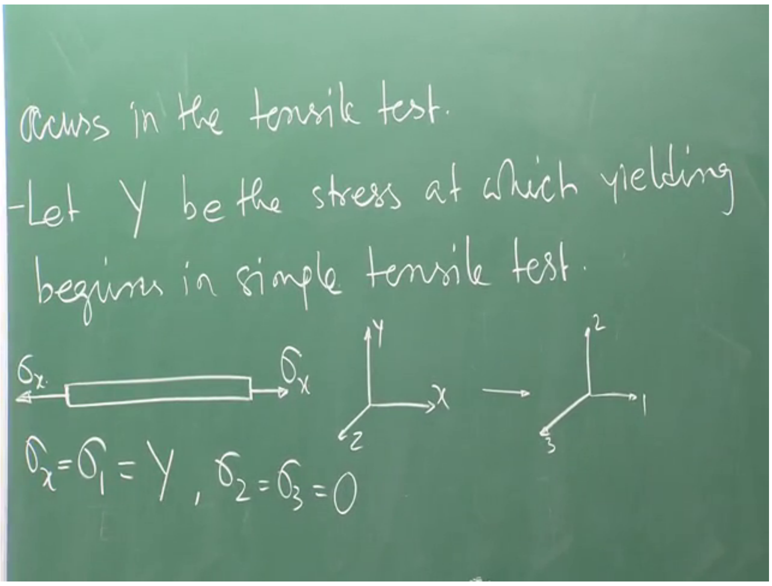

Now, what does it mean? So, this is the criteria this is the definition of the first criteria. First criteria says that it assumes that yielding of a material can occur in a 3 D state of stress when the material or when the system is under 3 D state of stress when the root mean square of the differences between the principle stresses root mean square of the differences between the principle stresses reaches the same value which it has when yielding occurs in the tensile test; now what does it mean physically ok.

So, let us do let us see that thing physically now let Y be the stress at which yielding begins in simple begins in simple tensile test. So, we are just defining let us let y I mean I do not know that that will be different for different material if use steel y it be different if you use copper it will be different for different material different say metal or different material whatever you are considering. So, for that y is a I mean yield stress for simple tensile test.

So, what does it mean what is simple tensile test as I mean I think you might have done that thing in your undergraduate some simple tensile test of the steel rod or the aluminum rod or whatever. So, y is that yield stress now simple tensile test what we do generally in the lab we are taking a rod or one 1 D one dimensional say member of a particular material and we are applying tensile stress. So, if my coordinate system is like this and say that is we are just mapping that thing 1 2 3 coordinate system 1 2 3 coordinate system means principle stress I we are just saying that may be x y z coordinate system actually it is a x y z coordinate that is a general state of a coordinate system, but; however, I we are just mapping that thing in 1 2 3 coordinate system which are the principle axis, and where one axis is matching with x axis 2 axis is matching with y axis and so on or the timing we are just assuming that thing it is not necessary that all the times one axis will be matching or following the same direction of x axis not necessary already we have seen that thing in different numerical problems as well as whatever we have discussed.

Anyway, so this is the 1 D say 1 D tensile test we are doing we are performing on some material. So, we are applying only one direction stress that is sigma x along x direction. So, in this case if we will say and we are what we are observing on this plane this is my x plane this is my positive x plane this is my negative x plane whatever from your definition you know now on the x plane basically you have only the normal stress acting. So, therefore, it is eventually the one of the principle planes yes or no right because there is no shear stress acting on that particular plane. So, x plane is virtually becoming one of the principle planes and that is nothing, but your major principle plane yes or no that will be a major principle strain and x axis is virtually becoming major principle axis that is shown here as one right.

Similarly, if you see the y direction, so this plane is say y plane this plane is y positive y plane this plane is positive negative y plane and the y plane basically you do not have any stresses. So, if they if you do not have any stresses and therefore, of course, you do not have the shear stress. So, if shear stress is 0 on a particular plane so that plane must be one of the principle planes is not it. So, that you have got. So, y plane is also another principle plane and that is defined by say axis and the y direction is becoming one of the principle axis and that is say axis 2 1 so on and z plane is also because we are only applying unidirectional say tensile force right. So, z plane and y plane are completely free from any stress. So, they have to be the principle planes and y and z axis have to be principle axis ok.

So, sigma x is nothing, but your major principle stress from the discussion we are getting it is nothing, but say y because we are continuously applying the tensile force that is sigma x we are going on increasing and we are observing the strain happening and at certain point you will be observing the if you if you plot the stress in graph at certain point you will be observing the yielding initiates right. At that point that is the maximum say sigma x is possible which is I mean possible at the point where yielding starts is not it. So, that is nothing, but by our definition that is nothing, but y. Now what about other principle stresses sigma 2 and sigma 3 because sigma x is nothing, but sigma 1, but sigma y and sigma z are 0. So, therefore, they must be 0 understood the logic and discussion whatever has happened here. So, we have got sigma 1 sigma 2 and sigma 3 for a uni axial tensile test right ok.

And the in that case basically sigma x sigma y sigma z both all are principle stresses for surely because there is no shear stress on the planes. So, now,. So, as per the of criteria what does it say that it says that the state of stress the three dimensional state of stress when the root mean square of the differences between the principle stresses reaches the same value.

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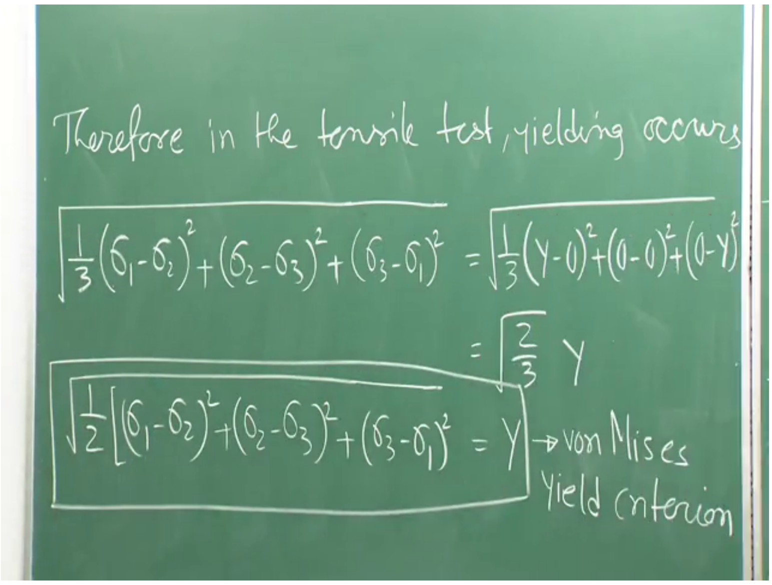

Which it has when yielding occurs in the tensile test that is a first that is a criteria actually. So, we are now defining the. So, therefore, in the tensile test yielding occurs root mean square of the differences of the principle stresses; one third this is nothing, but your root mean square of the differences of the principle stresses right sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma1 whole square.

This is your root mean square of the differences of the principle stresses for your uniaxial or the simple tensile test that will be becoming as one third sigma 1 what is the value of sigma 1 simply y. So, y minus 0 whole square, plus 0 minus 0 whole square, plus 0 minus y whole square. So, from there we can get root over 2 by 3 y. So, we can simply write root over half sigma 1 minus sigma 2 whole square, plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square equal to y.

So, this we are getting from our first criteria; that means, for initial yielding or the first this is my first yield criteria. So, what you are getting here. So, what basically the interpretation coming from this equation; that means, if you have the body or if you of you consider any body under the action of sigma 1 sigma 2 sigma there these are the principle stresses say. So, three dimensional state of stress. So, under the action of sigma 1 sigma 2 sigma 3, now for the combination of sigma 1 sigma 2 sigma 3 which will satisfy these condition so that condition or that situation will cause the yield. For the combination of sigma 1 sigma 2 sigma 3 if you arrive to this situation where this left hand side is becoming equal to y what is y? Y is nothing, but your I mean your that is a stress at which your yielding starts in case of simple tensile test. So, you perform. So, whatever material you are considering in three dimensional set of stress, you just consider the or you just perform the simple tensile test on the same material you find out y from the laboratory and then that y should be equal to the combination of sigma 1 sigma 2 sigma 3 in such a way that it should satisfy this equation.

So, up to that point basically I mean once you reach that condition or reach that situation then yielding will initiate for that particular material. Because that is coming from the first criteria and this is known as in mechanics this is known as mises simply mises or say von mises von mises yield criteria. So, this is popularly known in mechanics as von mises yield criteria. So, you have got now what is von mises yield criteria right this you will be getting from laboratory experiment, once you get that then for any combination of sigma 1 sigma 2 sigma 3 if it is satisfying this these equation then that will cause yielding in that particular material ok.

So, I will stop here today in the next lecture we will be continuing with the second yield criteria and then we will be discussing something about engineering strain and true strain engineering stress true stress engineering strain true strain and that is all ok.

Thank you very much. I will stop right today.