Mechanics Of Solids
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Lecture – 30
Elastic Stress-strain Relationship

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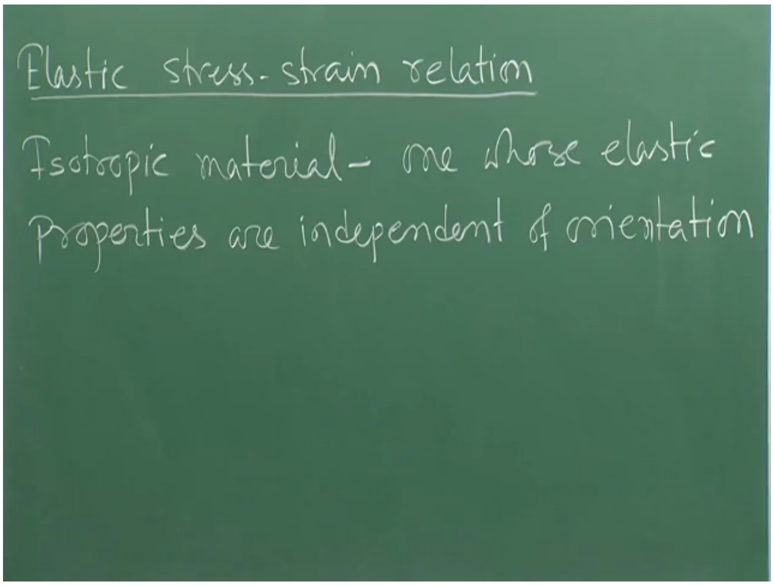

Welcome back to the course Mechanics of Solids. So, in the last lecture, we are discussing about the stress-strain relation. So, we have seen a few say definitions and few terms which will define the stress-strain behavior of a particular material. Now, we are going to discuss today this elastic stress-strain relation. So, now, we are going to deal only with because in this particular course we are not going to discuss any other type of material, we are going to discuss only about isotropic material.

Now, what is that the definition of isotropic material one whose elastic properties are independent of orientation. So, what is isotropic material isotropic material is one whose elastic properties are independent of orientation. Suppose, you are considering one body, so the body is having some say elastic properties in x-direction, y-direction and z-direction. Now, if you rotate the body, so that rotation will not cause any difference in the elastic properties.

Now, what are those elastic properties, we are coming to that point. So, mostly you know already we have talked about that thing modulus of elasticity E value, E earlier when we are discussing about the low deformation characteristics anyway. So, that is one of the elastic property. So, modulus of elasticity or the Young's modulus you know from your physics. Then another property is the Poisson ratio that will give you the relation between the lateral strain and the longitudinal strain. So, these are all elastic property. So, if you rotate the body in any direction that elastic properties will be remaining unchanged so that kind of material is known as isotropic material fine.

So, now, here at this point before establishing the relation between stress and strain. We should know that if I apply normal stress does it cause any shear strain or not, or if I apply shear stress does it cause any normal strain or not? What does it mean the suppose I am saying that if I apply normal stress only normal strain will happen and if I apply shear stress only shear strain will happen. So, that is the proposal I am making, but we can prove it that normal stress only causes normal strain, no shear strain will be caused by the normal stress. Similarly, shear stress only develop shear strain, not the normal strain.

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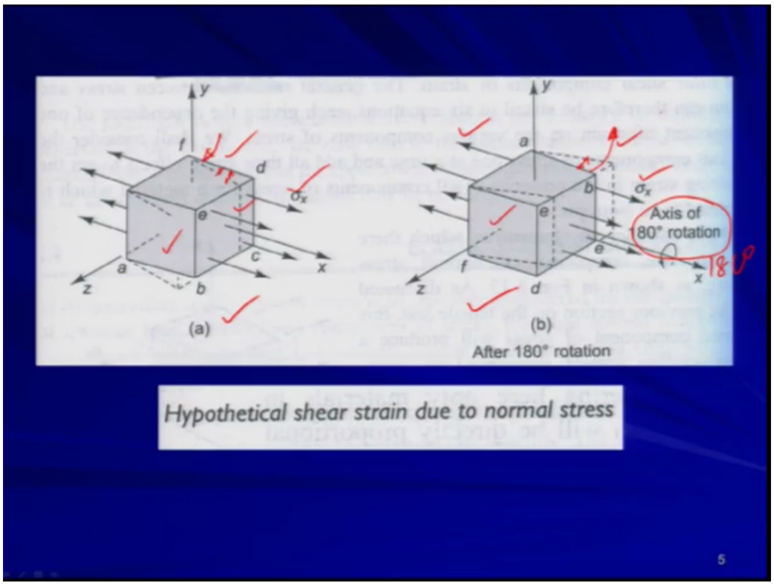

Now, let us look at this figure. So, we can prove it actually. So, what is this in this figure? So, we are saying well. So, if I apply normal stress, so sigma x I am applying on this body that normal stress is causing some shear strain and that is shown by this dotted line so that means, you are getting some angular distortion. What I am saying that due to the application of sigma x, I am getting some amount of shear strain and that is shown by this dotted line fine. There is no point though no I mean no ambiguity in that. Now, what I am doing, I am just rotating this x-axis by an angle 180 degree I am giving some 180 degree rotation and with that 180 degree rotation this I mean body will take the configuration of this body.

So, sigma x direction of sigma x remains unchanged, there is no problem at all; only thing is that now you are getting the angular distortion in this direction. So, if I say this is the direction of your angular distortion that means, the shear strain is happening in that direction shear, strain is nothing but your angular distortion. Now, if you compare with this figure and this figure, what you are getting, you are just rotating the access nothing else. You are not changing the stress you are not changing the material no change anything I mean in anywhere I mean in this configuration only change your making that is you are giving some 180 degree rotation to the x-axis that is all.

So, with that 180 degree rotation you are getting the shear strain in opposite sends. So, in this figure, your shear strain was in this direction right, but in this figure you are getting shear stress in this direction that means, the angular distortion is happening in the reverse direction. So, you are getting just opposite sends of shear strain, but as long as you are dealing with isotropic material the rotation should not change the elastic properties. So, therefore, due to the application of sigma x, when your applying some rotation, you are getting the change in sends of shear strain. If you say this shear strain is positive then in the other figure, you are getting shear strain is as negative that is conflict, contradictory statement or the contrary findings.

So, that should not happen as long as you are dealing with the isotropic material therefore, only thing is possible I mean in at this stage only the possible thing is that this normal stress should not create any shear strain. If it clear shear strain then basically your getting this kind of ambiguity, but to avoid this ambiguity only you can say the normal stress does not I mean create or cause any shear strain that is the only possible thing to satisfy this isotropic material. Similarly, for other kind of shear strains, so if this shear strain whatever you are seeing other two components of shear strain can be nullified or can be said that they cannot be developed due to the application of this sigma x.

So, in that way we can say normal stress only produces normal strain that is the thing we can conclude that normal stress only produces normal strain. Similarly, we can prove that

that proof I am not showing; similarly, we can prove that shear stress only produces shear strain. So, shear stress does not produce a normal strain; similarly, a normal stress does not produce shear strain fair enough. So, with that proof or with the discussion, now we can discuss or we can establish the relation between the stress and strain.

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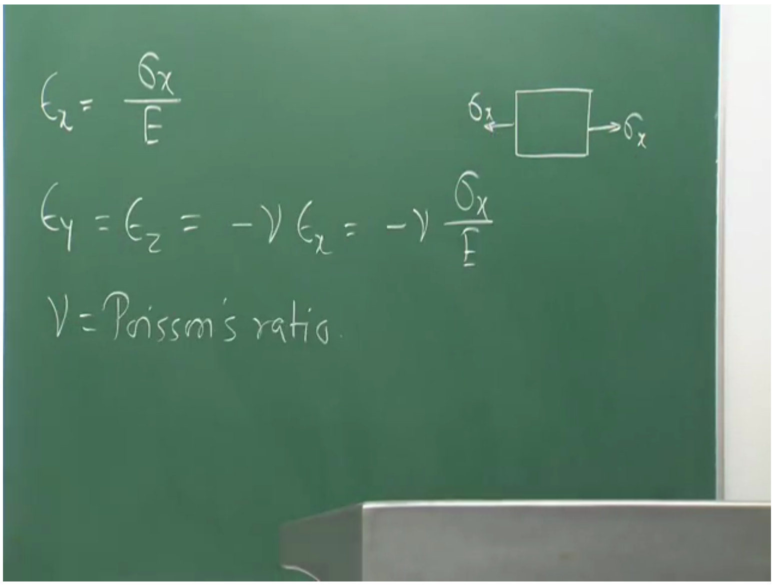

Now, let us talk about this epsilon x. So, epsilon x and the relation between epsilon x and sigma x as you know from your previous discussion in some physics book or physics, mechanics part of the physics that the normal string along x-direction is related to the normal stress along x-direction in this equation or in this fashion correct. Similarly, I can get epsilon y is equal to epsilon z is nu times epsilon x which is nothing but nu times sigma x by E. Where nu is nothing but your Poisson's ratio you know that. If you pull the body the other two directions will be contacted that is the common thing which generally observe and that has been observed by most of the materials and that can be written like that. This negative sign is because of that, because if you are getting elongation in the x-direction, in y and z-direction will getting the contraction and that contraction is defined by this Poisson ratio.

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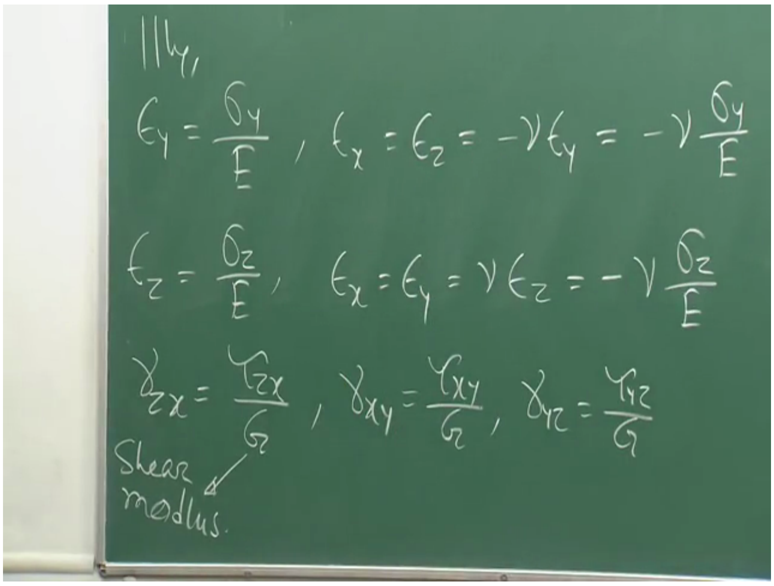

Similarly, I can write epsilon y is equal to sigma y by E, and epsilon x epsilon z equal to nu into epsilon y is equal to nu into sigma y by E, in this case only sigma x is applied. So, if you considered a body only sigma x is applied, and due to that your getting this relation. Similarly, in this case basically only sigma y is applied and because of that your getting the strain components. And likewise, you will be getting when only sigma z is applied, so we will be getting this you know that fair enough. So, normal stress only produces normal strain no shear strain, you see here the normal stress sigma x or sigma y or sigma z only produces normal strain, epsilon y and due to the Poisson's effect you are getting other to normal stress fine.

Now, for shear strain, you will be getting say gamma z x. So, this is the shear strain in z x plane that is caused by shear stress and one modulus and that modulus says known as shear modulus. So, G is known as shear modulus. Later on we will see that this shear modulus is not an independent say parameter, it is dependent on E and nu. So, the independent parameters are only two - E and nu; E is the elastic modulus and nu is the Poisson ratio. So, you can express G that is the shear modulus in terms of E and nu that we will see later on. Similarly, if you have gamma x y that is given by tau x y by G and gamma y z that is tau y z by G. So, now if you write down all the strain components, the relation between strain and stress then they will be looking like this.

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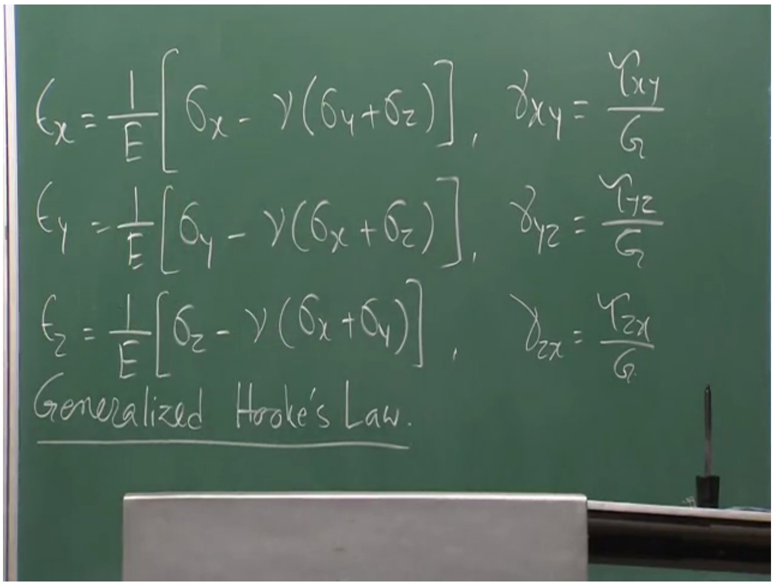

So, epsilon x is given by 1 by E sigma x minus nu into sigma y plus sigma z; epsilon y is equal to 1 by E sigma y minus nu into sigma x plus sigma z; epsilon z equal to 1 by E sigma z minus nu into sigma x plus sigma y. And gamma x y equal to tau x y by G; gamma y z equal to tau y z by G; and gamma z x equal to tau z x by G. So, this is nothing but you are as you know generalized Hooke's law. This is very, very important as long as we are discussing about the elastic relation elastic stress-strain relation though the plastic stress-strain relation is not included in the syllabus. So, this is your elastic stress-strain relation or now of course, the linear here we are considering isotropic material we are considering. So, you if you know the stress components sigma x, sigma y ,sigma z, and tau x y, tau y z, tau z x you can find out the strain components or other if you measure the strain components because you cannot measure the stress. So, if you measure the strain components then from this equations you can find out the stress components.

Now, as I was saying that this G value is not an independent parameter this is G is not an independent parameter. So, G is dependent on E and it can be prove that. However, this proof I am not showing, it can be proved that G equal to for the proof any classical say mechanics book you can refer. However, I am not showing that proof figure here. Thus it can be proved that is G equal to E by twice of 1 plus nu. So, G is dependent on E and nu.

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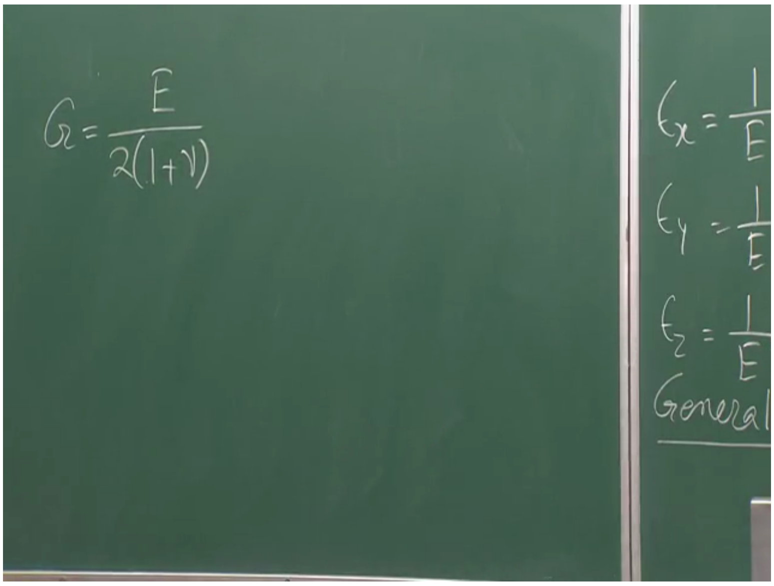

So, the basic parameters are two that is E and nu. So, once you know E and nu, you can find G. So, once you know G, you can establish this relation shear stress versus shear strain fine.

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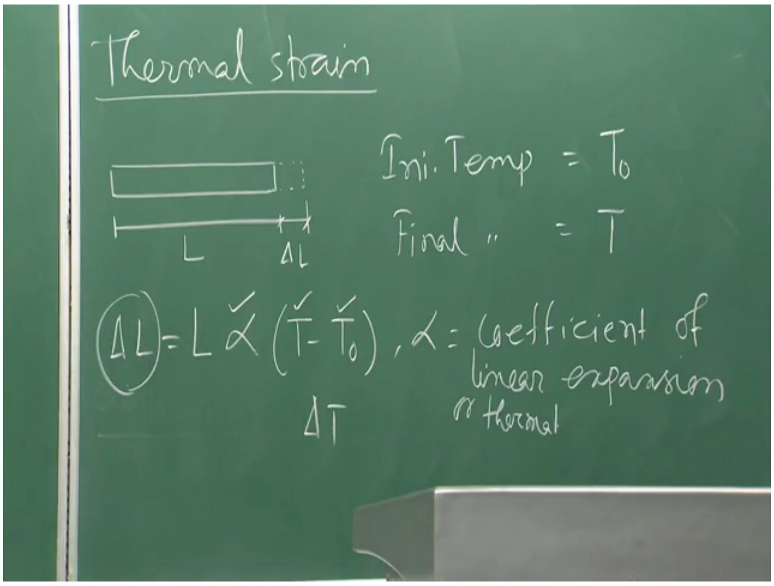

Now, we will come to thermal strain. Now, we will come to thermal strain. Now, what is that mean there suppose if you consider anybody any material any system, so if that material is under some temperature, if you raise the temperature to certain extent, so due to that increase in temperature you will be getting some deformation developed in the material that you know from your thermal expansion coefficient, and all those discussion. And as long as your material is behaving as elastic that means, if you release or if you decrease the temperature from that point to the original temperature then again that much of stain will be disappeared.

For that is so that kind of strain will be participating or that kind of strain will be the extra strain on top of this mechanical strain. So, these are all mechanical strain epsilon x, epsilon y, and gamma x y due to the application of stress. So, these on top of this mechanical strain due to the temperature rise or temperature fall, you will be getting the thermal strain and that thermal strain will be additive or subtractive due to I mean if you increase the temperature or if you decrease the temperature. So, now, we should know how we can calculate this thermal strain, and how we can couple the thermal strain and the mechanical strain.

So, suppose as we know from our discussion from the physics. So, this is say any material this is a bar of length L, and due to the temperature rise this much of length is getting increased say delta L. So, initial temperature say T naught and final temperature say T, then I can write delta L from the physics L into alpha into T minus T naught where alpha is nothing but coefficient of linear expansion, linear or thermal; sometimes we say thermal expansion, coefficient of thermal expansion fine.

So, if you know I mean it depends on the material if you use steel, if you use copper, if you use aluminium depending on that alpha value will be differing and that you will be getting from you from any standard book. So, if you know the initial temperature and final temperatures that means, a delta T basically the temperature difference if you know you can find out how much difference in length or how much elongation is happening. It could be elongation, it could be contraction depending on the temperature rise or fall.

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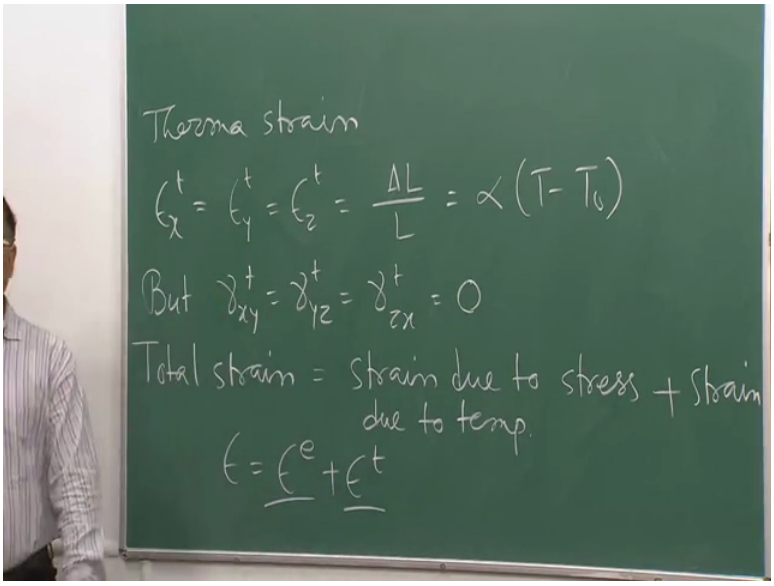

Now, I mean based on that your thermal strain can be calculated as your thermal strain can be calculated as epsilon x due to thermal T. And they will be same epsilon y, epsilon z is equal to delta L by L, where delta L is here. So, it is simply alpha into T minus T naught. But you should remember because the temperature is uniformed increasing in the system you are not increasing the temperature for one part of the system, another part is not expresses the temperature rise or fall. So, you are global your over overall temperature raise is happening. So, you are shear strain must be 0; only you will be getting normal strain due to the temperature rise.

So, therefore, the total strain equal to strain due to stress that is a mechanical strain you can say plus strain due to temperature that is your total strain if I say, what is the total strain. So, total strain will be the mechanical strain plus temperature strain so that is expressed as epsilon that is a total strain is equal to epsilon e plus epsilon t. So, that is coming from your stress part and that is coming from your temperature part. So, whenever I mean this will give you so unless until it is mentioned I mean if it is mentioned then fine; otherwise you can considered only the mechanical strength. But the idea is that for a system the total strength depends on of course, the material is behaving still in the elastic limit the total strain is dependent on the strain due to stress that is a mechanical strain and strain due to temperature that is thermal strain.

So, I will stop here today. So, we will continue in the next class with general state of I mean general I mean equations of general state of stress I mean the equilibrium equations for general state of stress so that thing or the general equations of the elasticity that will be discussed in the next class. And then we will be discussing about the failure criteria.

So, thank you very much, I will stop here today.