Mechanics Of Solids

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Lecture - 03
Conditions of equilibrium in 2D and 3D

Welcome back. Welcome to the course mechanics of solids. So, in the last lecture we are discussing about the laws of mechanics and the steps involved to analyse any mechanical system, in the way whatever we generally see in the universe right. And then basically we talked about force, how to calculate force, how to quantify force and what are the consequences of the forces. So, those things we have seen. And then we discussed about moments moment of a force with respect to or a particular point in the space right. So, those things we have covered in the last lecture. And now we are in this particular lecture we are going to establish some laws whatever we have discussed in the last lecture that, every mechanical system will follow some law I mean it cannot be highlight the laws of mechanics.

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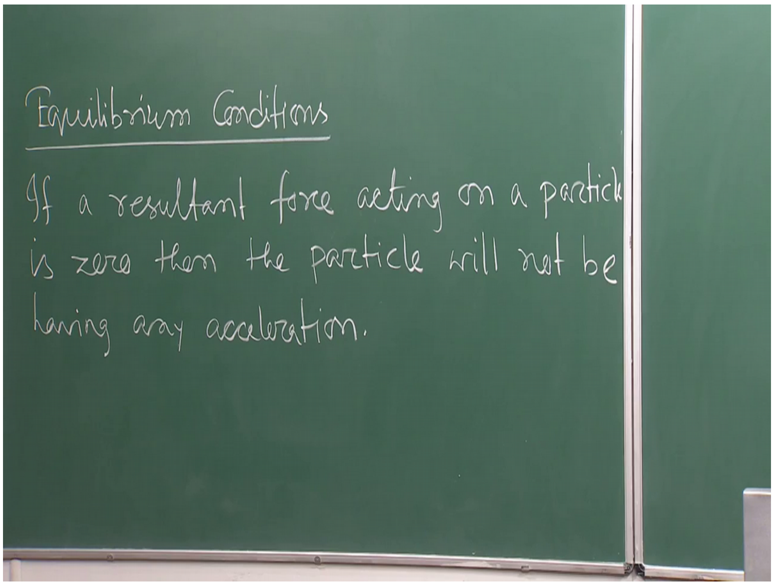

So, those laws we are going to explore in this particular lecture. So, basically the first thing is that, we will be talking about the equilibrium conditions. So, this is one law. So, if I say my mechanical system or any system is under equilibrium, then what do we understand by that. So, let us write down the statement, if resultant force acting on a particle is 0, then the particle will not be having any acceleration. And this condition is called equilibrium condition. So, what do we understand by that? If a resultant force, suppose, you are considering one system and that system is under the action of different forces. Not on a single force it can have a multiple number of forces and acting on that particular system. So, we will be having some resultant force right. So, as you know that because as I told you in the last lecture that force it is a vector interaction.

So, vectorially you can you can get the resultant, if a resultant force acting on a particle, not even in a whole system. Even if you consider a particle in that particular system then and so another force acting on a particle is 0 the resultant force is 0. Then the particle will not be having any acceleration. So, resultant force is 0. So, therefore, the particle will not be having any acceleration and these condition is known as equilibrium condition.

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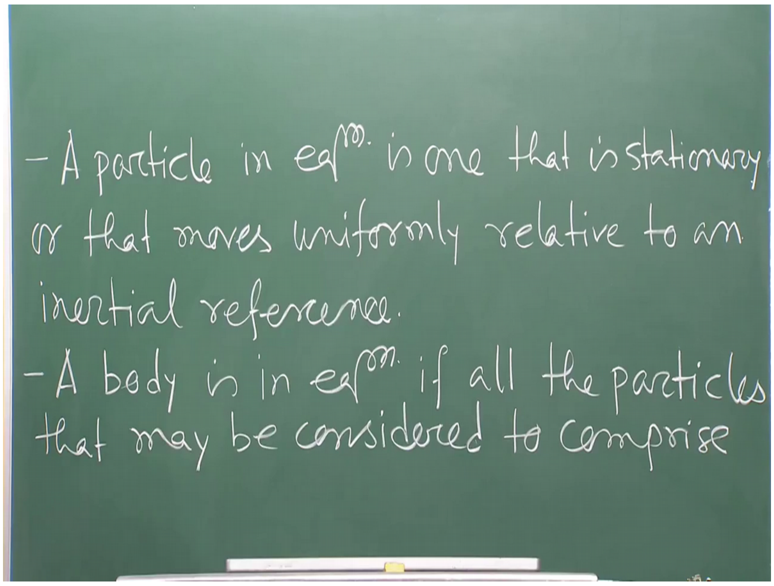

Now, what exactly we understand from that. So, let us talk about or let us write down those statements and one by one we will discuss those things in the coming hour. A particle in equilibrium is one that is stationary or that moves uniformly relative to an inertial reference, a particle in equilibrium.

So, when I am talking about the equilibrium condition for a particle. So, a particle in equilibrium is one, that is stationary either stationary or that moves uniformly relative to an inertial reference. So, as I told you from this statement, it is clearly mentioned here that the particle will not be experience here or will not be having any acceleration. So, it can happen only in 2 situations. Either the particle in stationary or the particle moves with the uniform velocity or uniform relative say inertial uniformly with respect to some inertial reference right. So, under these 2 situations basically you will not be having any acceleration. So, a body now we are talking about a body is an equilibrium if all the particles that may be considered to comprise the body are in equilibrium.

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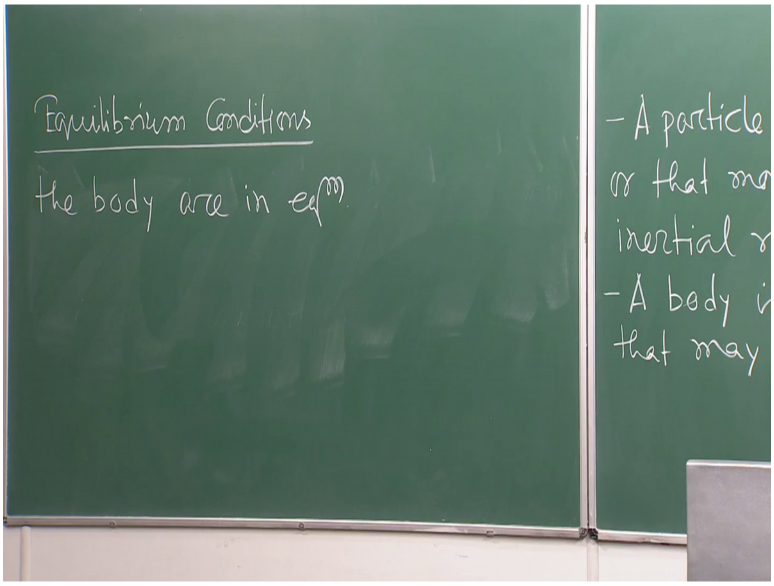

So, initially we talked about the particle. The particle is in equilibrium. So, particle is in equilibrium when either you have the particle stationary or you have the particle which is moving uniformly. Now if I say a body now we considered a teeny tiny I say a particle now we are talking about the body whole body. A body is in equilibrium if all the particles that may be considered to comprise the body. Which will form the body are in equilibrium. So, basically what I mean to say that if I say a body is in equilibrium; that means, each and every particle in that body will be in equilibrium. Now what are the laws of equilibrium. So, those things we are going to discuss now.

So, now basically if you look at the general equations of equilibrium how we can establish the equations of equilibrium, so the necessary and sufficient conditions of equilibrium is that the summish of all the forces; that means, the resultant force as well as the resultant moments, when I am talking about the force; that means, the force which are externally applied on the body.

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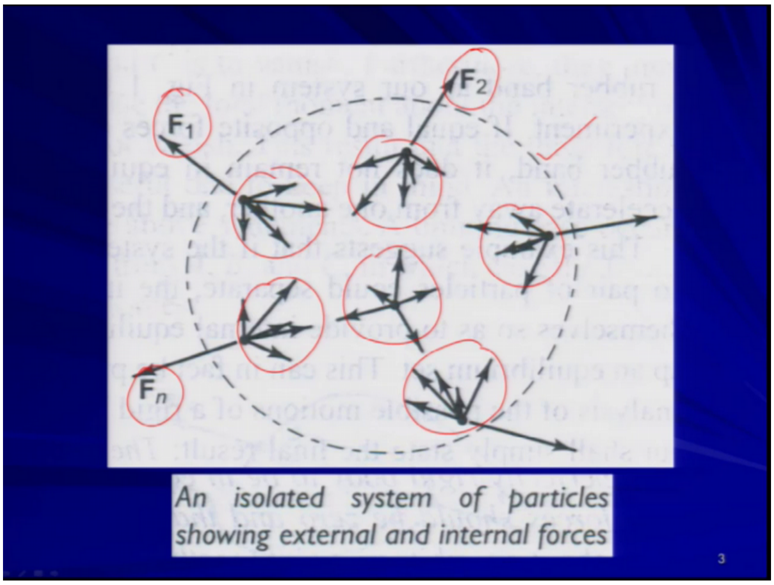

So, if you look at this figure basically in this figure you have the whole body. So, this is your whole body, this is your body. Now this body is under the action of different external forces F 1 F 2 up to Fn. And due to that external force application of the external force, you are getting the internal forces which are getting developed here right. So, these are all internal forces right. So, now, what I am saying when I am going to establish the equations of equilibrium, at the time we are talking that the resultant forces; that means, resultant forces of externally applied force. So, resultant forces and resultant moments that is becoming 0. Then only we will be getting the equilibrium condition.

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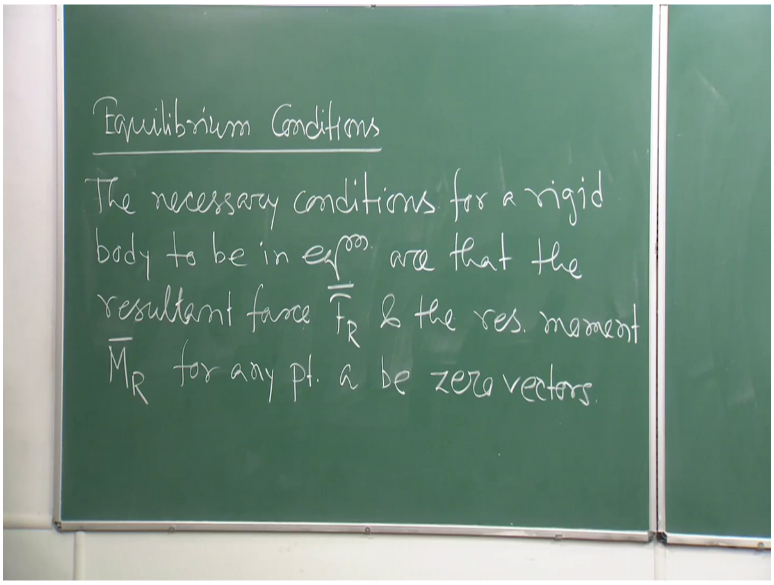

So, what I can write now. So, what I can write, now the necessary conditions for a rigid body to be in equilibrium are that the resultant force, force Fr and the resultant moment Mr for any point A be 0 vectors. So, this is the law. This is the law of your equilibrium condition that the necessary conditions for a rigid body; we are talking about the rigid body. So, for a rigid body to be in equilibrium are that the resultant force that is Fr and the resultant moment cause by that those externally applied forces for any point a. So, any point in that space and we have seen in the last lecture that how we can calculate the moment with respect to some fixed point. So, be 0 vectors. So, resultant force and resultant moment must be a 0 vector.

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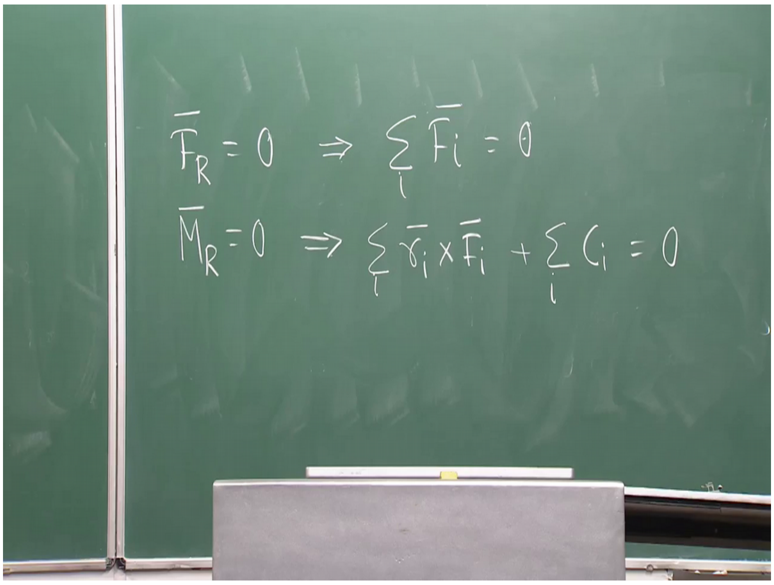

Now, mathematically how we can express that. So, mathematically I can express this. So, that gives me summation of all forces equal to 0 and resultant moment is 0. That implies the summation of the moments of externally applied forces in that way you can calculate plus you may have some concentrated moments acting on the body right. So, this is the moment due to the externally applied forces. And this is the moment which are concentrated moments acting on that particular body. So, summation of these things must be 0. So, mathematically I can express like this. Now if I consider a body let us let us talk about. So, I mean are these 2 equations add these 2 relations are sufficient.

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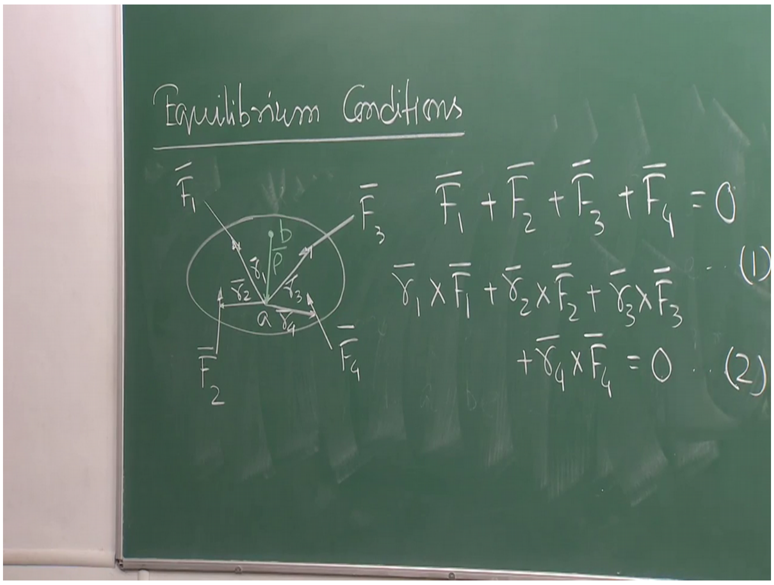

So, that is my question. So, let us see let us talk about that. Let me draw a figure small figure suppose this is one body. So, you have different externally applied forces, all the forces I am showing. So, if you if you understand this concept for the forces for the moments things will be pretty similar. So, these are few externally applied forces. And you are considering one point say A, and you want to find out the moment of these forces about point a. So, you need to know the displacement vectors. So, this is these are all your displacement vectors a r 2, r 1, r 3 and r 4. These are all your displacement vectors r 1, r 2, r 3 and r 4.

So, now if I want to establish these relations that is the equilibrium conditions, what I can write I can simply write F 1 because resultant force must be a 0 vector. So, summation of all the forces must be 0. So, F 1 plus F 2 plus F 3 plus F 4 equal to 0. So, that is my say equation 1. Now similarly my resultant moment is also 0 for the equilibrium condition.

So, I can write r 1 cross F 1, plus r 2 cross F 2, plus r 3 cross F 3 plus r 4 cross F 4 must be 0. So, equation 2 is any doubt I hope there is not. So, we are following these equations, nothing else for this, for this body. Now I am considering another point here. So, this point is say b and the displacement vector between b and A is say rho. So, I am considering another point B, and let us let see that if I chose another point whether my equilibrium condition will be different or not. So, then we can say this is these are 2 my basic equations for your equilibrium condition right, that in that way we can say that.

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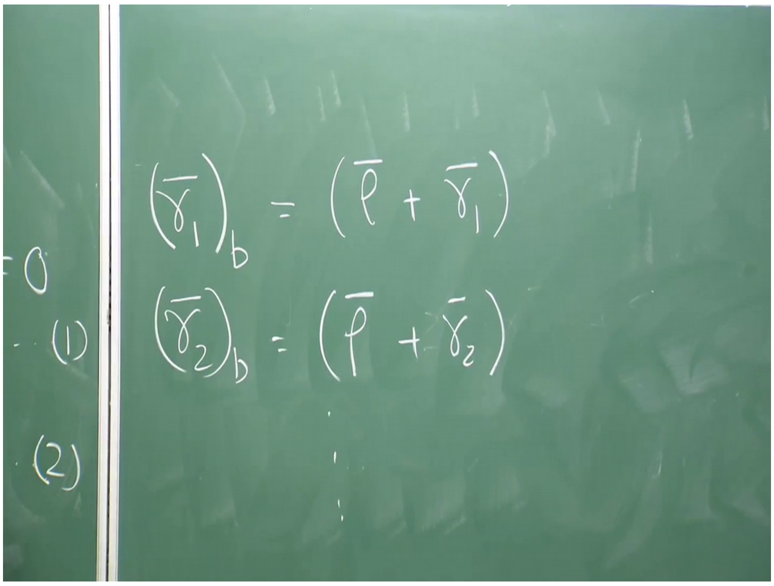

So, if I consider another point say B, in this body and the displacement vector between b and A is A rho. Then I can simply write for the new point b I can simply write down that r 1 b. What is the r 1 B, r 1 b is the displacement vector for force F 1 from point B; that means, that is a displacement vector to the point where F 1 is acting from point b.

So, that is that is defined by r 1 b, so r 1 b from your vectorial. So, equation or the vectorial say rules you can write down r 1 b is simply equal to displacement vector rho plus r 1. Is not it? What is r 1, r 1 is the displacement vector between the point where F 1 is acting and point a. So, this is a displacement vector between A and b. So, in that way I can get the displacement vector from point b the newly chosen point b. Similarly, I can write r 2 b is equal to rho plus r 2 and so on. So, now, what I can write. So, therefore, if I if I try to exploit the equilibrium condition that is a moment equilibrium like the resultant moment with respect to b is 0. As I told you the moment resultant moment must be 0 moment in at what point we are choosing, now we are going to prove that if we chose different points it would be remaining the same that is my actually the objective. So, now, if I choose point b, and if I take the moment equilibrium that is that is your summation of Mr with respect to point b is 0.

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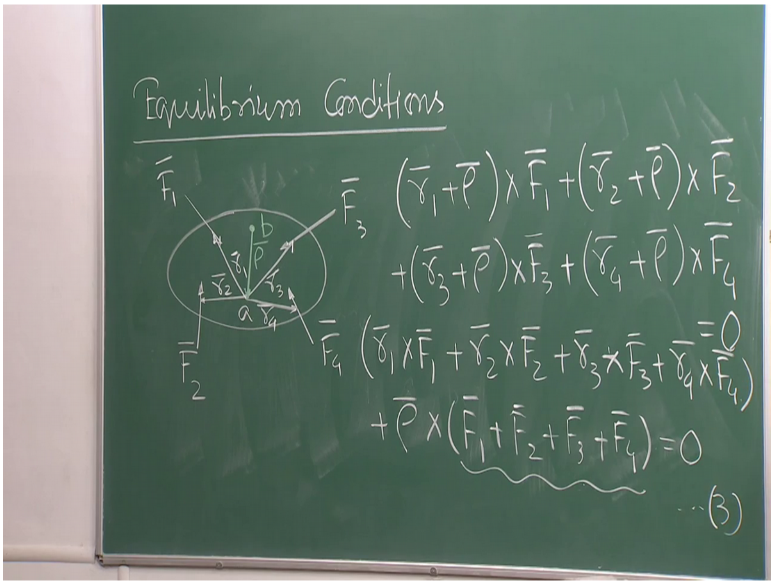

Then I can simply write r 1 b into cross F 1. So, that directly I can write r 1 plus rho cross F 1 plus r 2, rho plus F 2, plus r 3 cross F 3, plus r 4 cross F 4 must be 0. That is also equally valid. Because instead of choosing a because nobody is telling me that which point has to be chosen for taking the moment equilibrium. So, I am choosing some other points a b. So, then the equation will be transformed like that right equations will be getting changed and that will take form like this. So, now, if I simplify this I can get simply r 1 cross F 1 I mean by following the distributed law of your a vector, I can simply write r 2 cross F 2, plus r 3 cross F 3, plus r 4 cross F 4 plus rho cross F 1 plus F 2, plus F 3, plus F 4 equal to 0 correct yes of course, we can write right.

Now, if you look at this equation this part is already 0 because we are talking about the equilibrium condition and that is already established that is the. So, resultant force must be a 0 vector. So, this is already 0. So, what is left out? This is left out. Now if I say this is my equation 3, then equation 3 is nothing, but your equation 2 if you if you go back to your lecture. So, equation 3 ultimately or eventually becomes equation 2, because this part is of course, 0.

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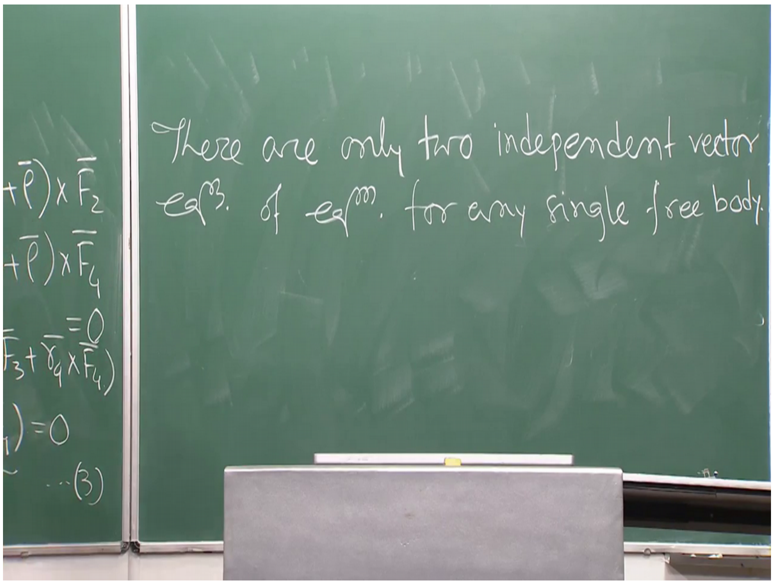

So, now I hope, I can write from this discussion that that. So, I can write there are only 2 independent vector equations of equilibrium for any single free body. This is very important. So, there are only 2 independent vector equations what are those summation of Fr equal to 0 summation of Mr equal to 0 right; that means, Fr there is a resultant force is 0 resultant moment is 0. These are 2 basic equations of equilibrium for any single free body. And rest if the equations whatever you want to develop all those things ultimately you will follow these 2 equations. And that already we have seen see if it is not following if this rule is not followed, then what we will we should get we should get different equations. So, equation 3 should be different than equation 2 that you are not getting.

So, that I mean insists that you are you are only getting 2 independent equations of equilibrium. So, now, few consequences few say I mean discussion may come out from this say the topic that.

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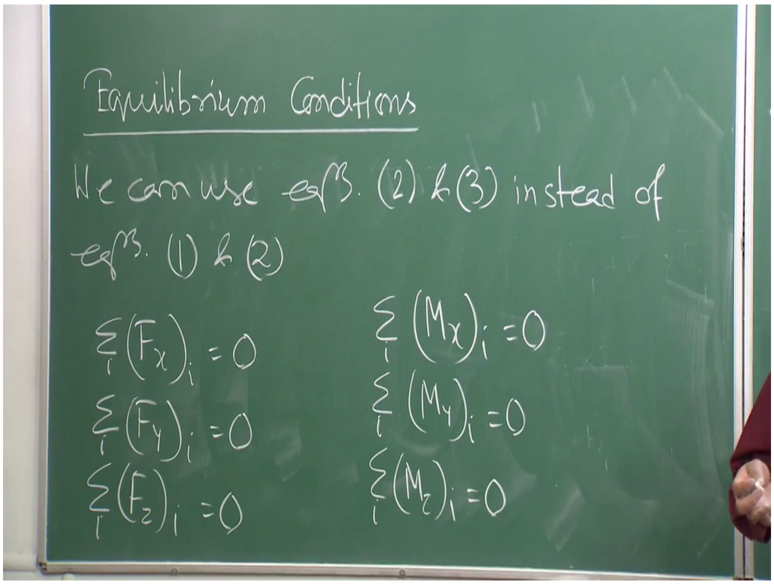

So, few discussions make out from this topic that we can use equations 2 and 3 instead of equations 1 and 2 right. So, what was there in equation 1? There is a resultant force is 0. What was there in equation 2? That summation of moment with respect to point a was 0 right. What was there in equation 3? Summation of moments with respect to point b was 0. Now these as I told you these are 2 your basic equations independent equations equation 1 and equation 2, one is force balance another one is moment balance with respect to point a. Now instead of that instead of taking the force balance that is the resultant force is 0 instead of taking that I can choose these 2 equations; that means, both equations are talking about the moment balance; that means, I am talking about the summation of moment with respect to point a is 0, and summation of moments with respect to b is 0. I can use those 2 equations to solve our equilibrium problem.

That we can do there is no problem in that. So, I mean in the subsequent lectures. So, when we will be talking about different structures and different kind of configurations we will see that as for your convenience basically sometime you use this this set; that means, all the moment balance you are taking moments at different locations and you are finding out the moment balance or you can get the combination of moment balance as well as the force balance right. So, there is no problem in that now as you have seen that what we are talking about we are talking about the resultant force must be 0 and resultant moments must be 0. Now for any force or any moment if you consider in the space and if you consider a mutually perpendicular coordinate system x y z right, mutually perpendicular coordinate system x y z, then any force can be resolved in the in the space if you consider any force that force can be resolved in this x y z coordinate direction. Similarly, the moment can be resolved in x y z coordinate direction right. Like if you have the force F can be decomposed or resolved along x axis, say Fx along y axis say Fy and along z axis say Fz similarly the moment mx my mz.

So, most of the times we are not talking or we are not taking this thing in calculation that is the force, whatever force is there we are not taking the summation of that particular force or for our convenience. Sometime what we do that we reserve all the forces along the mutually perpendicular coordinate system x y z and similarly the moment we also resolve and then, we take or we exploit the equilibrium condition.

So, for that what we can write. So, when you resolve the forces in x y z coordinate system then you can simply write summation of Fx equal to 0 summation of Fy equal to 0 and summation of Fz equal to 0. So, instead of taking the F in the space you are just resolving this force in x y z coordinate system, and then you are taking the equilibrium along individual coordinate direction Fx Fy Fz. Similarly, I can write summation of mx equal to 0, summation of my equal to 0, and summation of mz equal to 0. So, that will that will be convenient to use for any kind of system, and later all you will appreciate that you will be frequently using these relations to solve our system which is under equilibrium.

So, I will stop here today. So, in the next lecture, we will be talking about the free body diagrams and the different force configurations.

Thank you very much.