Mechanics Of Solids

Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 29
Tutorial on Concept of Strain

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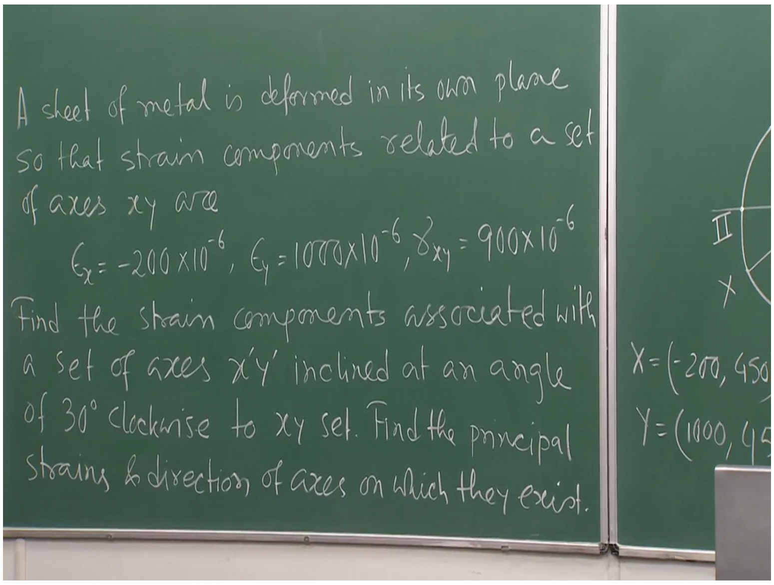

Welcome back to the course Mechanics of Solids. So, in the last lecture, we solved one numerical problem related to stress and their we have drawn the Mohr circle for stress now today another numerical problem will be taking related to the strain. So, the problem says a sheet of metal is deformed in its own plane of course, the plane strain condition so that the strain components related to a set of axes x y r epsilon x is given epsilon y is given and gamma x y is given. Now, find the strain components associated with a set of axes x prime, y prime inclined at an angle of 30 degree clockwise to x y set. Find the principal strains and direction of axes on which they exist.

So, we will solve we can solve this problem using our equations whatever we have derived in the previous discussion, but we will take this problem by drawing the Mohr circle. So, that you will be also I mean getting the idea that how we can draw the Mohr circle of strain and how we can solve this kind of problem using Mohr circle of strain well.

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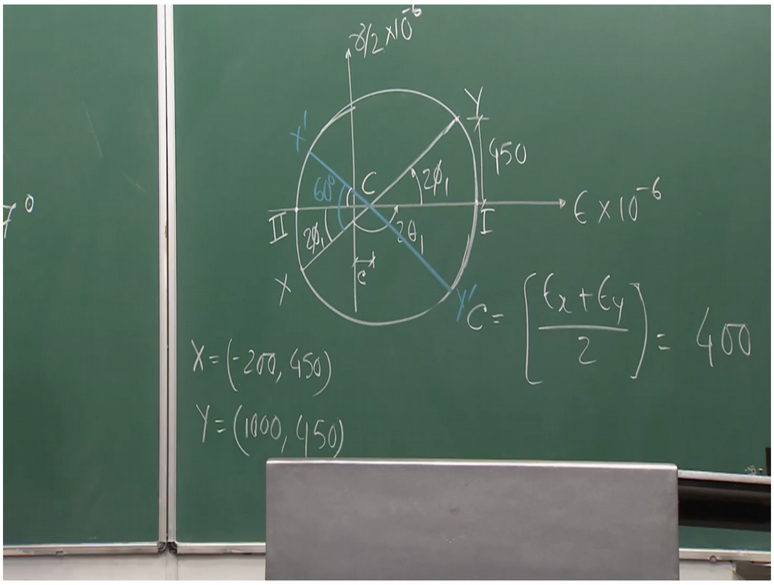

So, first we will try to draw the Mohr circle of strain for this problem. So, as you know we have the x-axis which is defined by normal strain and the y-axis is defined by half of shear strain. And we are taking the scale like this of 10 to the power 6. So, now, if you look at the problem, first what is your first job first job is to locate point x and y in this space. So, if you want to locate point x point x is nothing but coordinate of point x is if you look if you see coordinate of point x is minus 200 and half of gamma x y because we are floating everything in terms of half of shear strain. So, that will be 900 by 2, so 450, so this is the coordinate of x point. So, somewhere it will becoming like this say it is here.

Similarly, we can plot y point whose coordinates are 1000 and 450. So, maybe it is coming say somewhere here. So, I am joining these two points. So, this is my x point this is my y point and I am joining with the Mohr circle. And your shear strain is positive therefore, x point is coming below the epsilon axes it has not gone above the epsilon axes because shear strain is completely positive so that is quite understood. So, this is your center. So center of the Mohr circle say c and this terms is a small c. So, therefore, small c can be written as epsilon x plus epsilon y by 2 so that gives me 400. So, now, if I say this angle, so this is your point one this is your point two, they are representing the major principal strain and minor principal strain. And we are considering this angle is say twice of phi 1 say and this angle say twice of theta 1. So, now, our job is to find out the magnitude of twice phi one and twice theta 1. So, because these things will be required when we will be talking about or when will be finding out the strain components on other plane.

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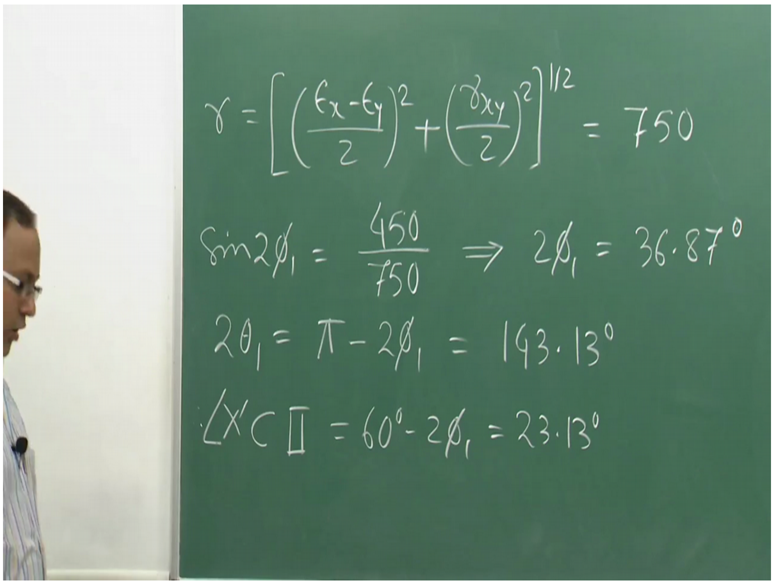

Now, next job is to find out the radius of the Mohr circle r and you know that is given by epsilon x minus epsilon y by 2 whole square plus gamma x y by 2 whole square to the power half. So, if you do that you will be getting that has 750 that is the radius of the Mohr circle. So, therefore, from the figure is twice of phi one is nothing but 450 by 750 twice of phi 1 is nothing but 450, what is 450 gamma x y by 2 that is the coordinate of say y axis I mean the coordinate of y axis that is the y component of y axis y point sorry. So, y point this is this is your 450 you know and 750 is a radius of the Mohr circle. So, that gives me twice of phi one is equal to 36.87 degree. Therefore, twice theta one can be obtained from this figure pi minus twice phi 1, which is nothing but 143.13 degree very simple.

And therefore, your now what will do we will try to look at point x prime and y prime on the Mohr circle. Now, in the problem it says that x prime y prime makes an angle 30 degree clockwise with x y set of axes. So, if that is true then my x y point are shown like that. So, x prime y prime point will be this is my x prime point and this is my y prime point and these angle is nothing but how much 60 degree double the angle actually made in the axes system so that is 60 degree. So, now, what I can write I can write x prime angle x prime c and two point that means, this angle this angle is equal to if this is two twice of phi 1, this angle is twice of phi 1, no doubt about that so that angle will be simply 60 degree minus twice of phi 1 that will come as 23.13 degree.

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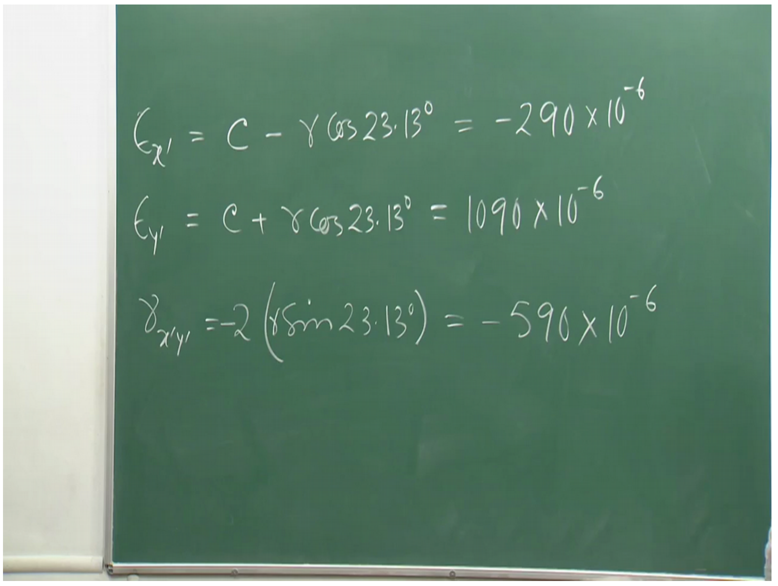

So, therefore, I can find out epsilon x prime that is the required thing by following this c minus r cos 23.13 degree. If you look at the Mohr circle, you will get it, so that gives me minus 290 into 10 to the power minus 6 epsilon y prime. Similarly, c plus r cos 23.13 degree that is coming as 1090 into 10 to the power minus 6. And what about gamma x prime y prime because that is the next thing what we are going to find out. Now, if you look at the figure the Mohr circle x point is getting map to the x prime point. So, x prime point is located above the epsilon axis, therefore your gamma x y should be negative because as per our sin convention gamma x prime y prime should be negative and that is given by twice of minus sin negative r sin 23.13 degree that comes as minus 590 into 10 to the power minus 6. So, these are the strain components in x prime y prime coordinate system.

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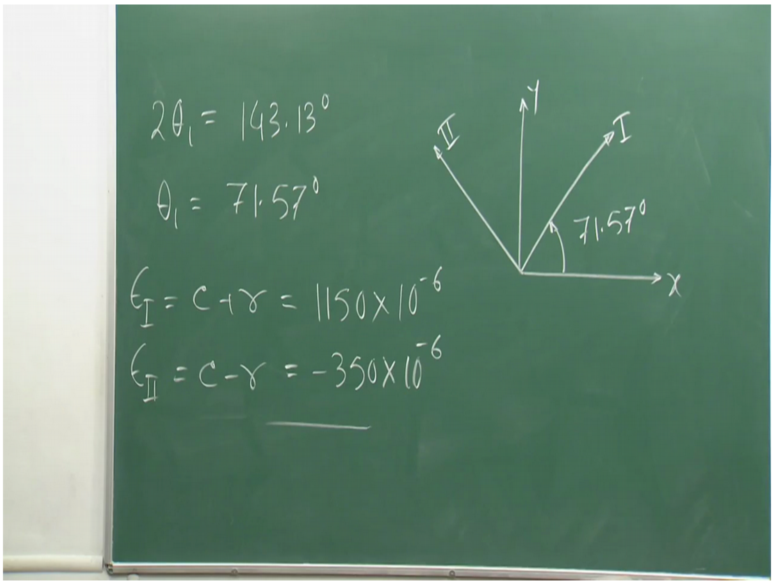

Now, the next objective is to find out the principal strains. So, the principal strains when we are going to find out. So, twice theta 1 value already we know 143.13 degree. So, therefore, theta one is nothing but 71.57 degree. So, the orientation of your principal axis this is x and y, this is your axis one major principal axis and this is your minor principle axis. And these angle because this is anticlockwise, so that should be anticlockwise of 71.57 half of theta 1 and epsilon I that is the major principal strain is nothing but c plus r that gives me 1150 into 10 to the power minus 6. And minor principal strain is c minus r that gives me minus 350 into 10 to the power minus six6, so done. So, whatever problem we have taken. So, very simply I hope you have understand the concept of Mohr circle of stress and Mohr circle of strain how we can find out all those required components strain components as well as stress component.

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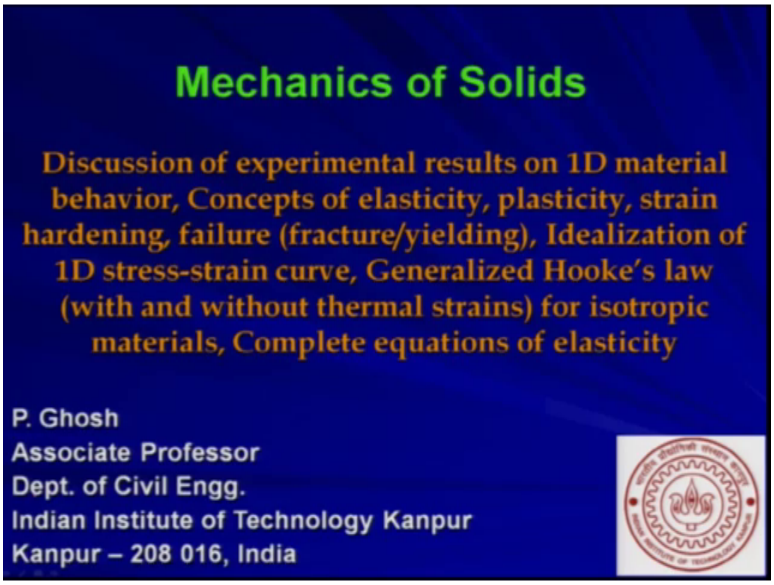

So, with this I will conclude this chapter that is stress strain consider of stress strain. Now, we are moving to the next chapter that is the establishment of relation between stress and strain. So, in this chapter, basically we are going to discuss this thing discussion of experimental results on 1D material behavior, concept of elasticity, plasticity, strain hardening, failure, idealization of 1D stress-strain curve, generalized Hooke's law with and without thermal strain for isotropic material of course, complete equations of elasticity. So, these are the things we are going to discuss in this particular chapter particular topic. So, before moving to that we need to know few definitions. So, what are those?

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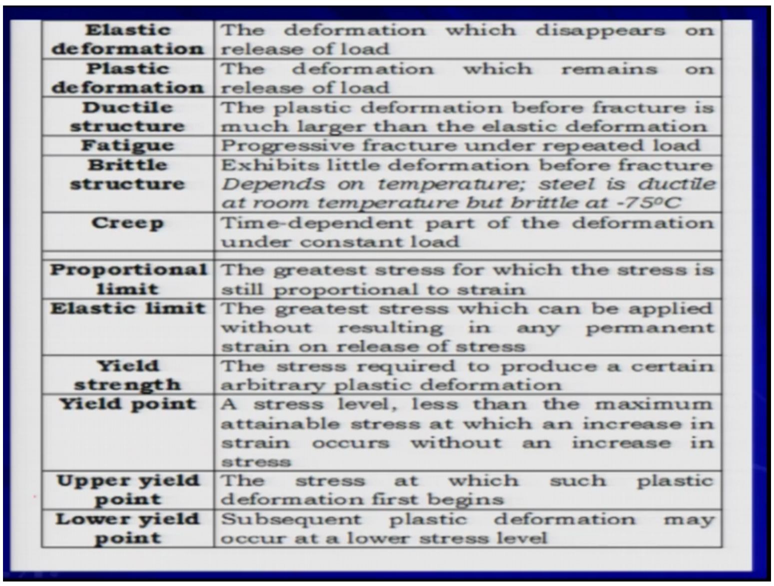

Elastic deformation	The deformation which disappears on release of load
Plastic deformation	The deformation which remains on release of load
Ductile structure	The plastic deformation before fracture is much larger than the elastic deformation
Fatigue	Progressive fracture under repeated load
Brittle structure	Exhibits little deformation before fracture *Depends on temperature; steel is ductile at room temperature but brittle at -75°C*
Creep	Time-dependent part of the deformation under constant load
Proportional limit	The greatest stress for which the stress is still proportional to strain
Elastic limit	The greatest stress which can be applied without resulting in any permanent strain on release of stress
Yield strength	The stress required to produce a certain arbitrary plastic deformation
Yield point	A stress level, less than the maximum attainable stress at which an increase in strain occurs without an increase in stress
Upper yield point	The stress at which such plastic deformation first begins
Lower yield point	Subsequent plastic deformation may occur at a lower stress level

So, these are few definitions if you look at the point wise, look at the slide. So, point wise it is given. First one is elastic deformation. So, do mean by elastic deformation, the deformation, which disappears on release of load as you know. So, your loading, so the material is showing some deformation you release the load the deformation will disappear so that kind of deformation is known as elastic deformation. Similarly, you will be having plastic deformation the deformation which remains on release of load; that means, even after releasing of load the deformation still exist so that kind of deformation is known as plastic deformation.

Then ductile structure what do mean by that the plastic deformation before fracture is much larger than the elastic deformation. The plastic deformation as we have define just now the plastic deformation before fracture before fracture means you are getting the failure. So, before facture whatever plastic to information you are getting that is much larger than the elastic deformation, so that what the material which is showing this kind of behavior that means, high amount of plastic strain plastic deformation it shows so that kind of material is known as ductile structure say something like your steel you might be knowing.

Then because these terms will be frequently used in this particular chapter, so you should know or you should I get a custom with this. Then fatigue progressive fracture under repeated load. So, you apply the load repeatedly and suddenly I mean at certain point you will see that some fracture has happened so that kind of behavior is known as fatigue. Then brittle structure it exhibits little deformation before fracture. So, something like your glass. So, if you apply load on the glass, so you will not observe any deformation at all and all of sudden it will collapse, the facture will happen, so that kind of material is known as brittle material, but however, it depends on temperature. Now, if you consider steel, steel is ductile at room temperature, but if you increase or if you decrease the temperature say for minus 75 degree centigrade then the steel will behave as a brittle material. So, it depends on temperature.

The next term is creep. Creep is nothing but there are time dependent part of the deformation under constant load that means, it is a time dividend phenomena. So, you have the material and on that material you are having now sustained load and that load is existing over a period of time and then due to that kind of sustained load you will be getting some deformation and that deformation is known as creep.

Then we are defining few limits in the stress-strain a property because we can plots stress-strain curve we will see that later on that proportional limit the greatest that is the greatest stress for which the stress is still proportional to strain, so that means, what you get you try to plot stress strain. So, at a very initial stress you will be getting stress and strain both are proportional. So, if we increase the stress, you will get the increase strain and that will show some linear behavior, so that kind of limit is known as proportional limit. And it is very difficult to find out exactly the professional limit anywhere you that is different issue.

Then elastic limit the greatest stress, which can be applied without resulting in any permanent strain on release of stress because you know versus elastic deformation. So, therefore, elastic limit is nothing but the maximum stress at which I mean even after that if you I mean release the load or the release the stress the material will come back to its original position and that is known as elastic limit. Now, coming to the yield strength, so this is very, very important because you should design I mean whatever material you are choosing for design any structure, so this yield strength is very, very important. Because you should know when it will behave as I mean when the threshold I mean the strength of the material will go to the threshold value of the elastic limit anyway. So, this yield strength is defined as the stress required to produce a certain arbitrary plastic deformation. Some amount of plastic deformation if it shows that string that at that point whatever strength is there so that strength is known as yield strength.

Then coming to the yield point. So, there are two types of yield point that I will be coming after word, but what is yield point. So, yield point is nothing but a stress level less than the maximum attainable stress at which an increase in strain occurs without an increase in stress so that is the stress level. So, you are increasing the stress you are getting the increase in strain, but at certain point you will see that even if you increase the I mean without any increasing stress your strain is continuously increasing that means, your deformation is going on increasing without an increase in stress, so that point is known as yield point. And you generally get for different yield points most of the materials one is upper yield point another one is the lower yield point. So, upper yield point is the stress at which such plastic deformation first begins. So, whatever plastic deformation I am saying you so that means, without I mean that is the end point of the elastic deformation and plastic deformation initiates so that is the point where you are plastic deformation first begins.

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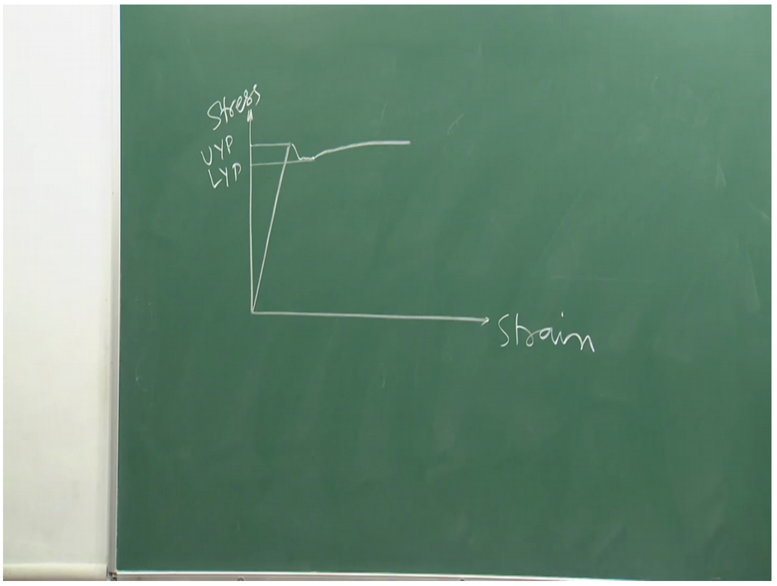

And the lower yield point is that subsequent plastic deformation may occur at a lower stress level that means, I mean if you look at this a typical stress-strain curve lets draw a typical stress-strain curve here. Stress generally stress is plotted along y-axis and along x-axis strain is plotted. So, suppose this is your upper yield point and this is your lower yield point. So, this is your stress strain behavior initially it will be proportional and then suddenly I mean at certain point you will be getting the termination of your elastic deformation and then plastic deformation initiates. So, this is your completely the plastic deformation. So, this is at this point your plastic deformation initiates first begin. So, this is your so upper yield point and then subsequently your getting the yielding at cert at lower level lower level of stress, so that is known as lower yield point. So, these are few definitions.

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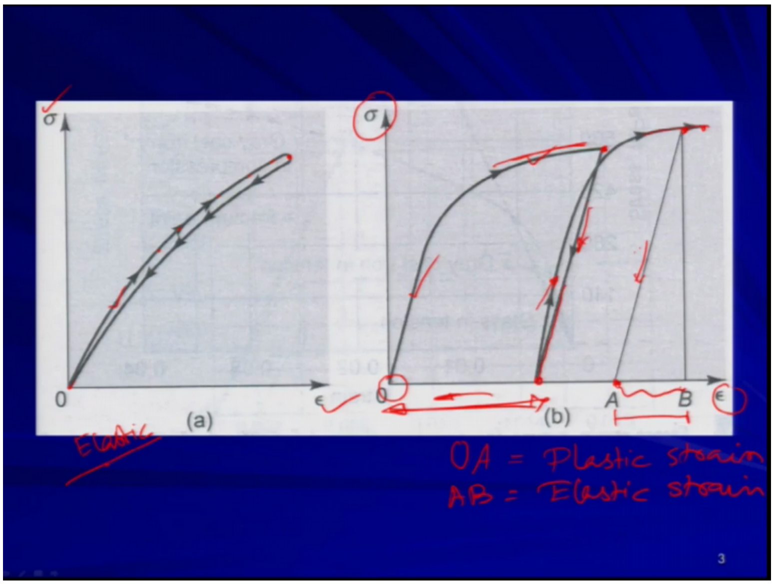

Now, we are going to discuss the effect of loading, unloading and reloading. So, what happens generally if you load a material, so if you look at this plot, so sigma versus stress versus strain plot. So, this line is typically this is a non-linear behavior; this is not a linear behavior of the material. So, this line is showing the loading. And if the material is elastic, this material is elastic, so after reaching this point, you are unloading the material. So, if you unload it, it will come back to its original position. So, this kind of behavior is known as elastic behavior as I was talking about so elastic deformation. So, whatever deformation you observe, so at any point I mean within this loading path at any point if you unload the material the material will come back to its original position. So, that is typically elastic deformation behavior.

Now, in case of this material say what you are doing this curve, this line is showing the loading after reaching this point you are unloading. And then basically I mean if you look at this curve. So, this part is experiencing some plastic deformation. Now, after reaching here, you are unloading it through this path and you are reaching here; if this point is within the elastic limit then basically due to the unloading I mean activity you must come back to this point the original position, but you are not getting it back. So, therefore, some plastic deformation is happening. So, this much of plastic deformation is happening. So, this much of permanent deformation you are observing due to the unloading; though the load is not there low load is 0, the stress is 0, but still you are not able to come back to this original position.

Then again you are reloading. So, this is your reloading curve. So, your reloading it and you are reaching here and then again you are unloading. So, again you are unloading or you are say at this point you are unloading. So, you are come back, you are coming back. So, you are coming back to this point A. Now, this OA, if you look at this OA is your plastic strain that is your permanent deformation or the permanent strain happened due to this loading, reloading and unloading the activity. But this AB part is your elastic strain. This AB part is your elastic strain because you reached up to this point right then due to the unloading you came back to this point A. So, you this much of this much of deformation you would recover, so that is strain is known as the elastic strain.

So, for a material if you go up to the plastic deformation basically that plastic deformation will be having some part of some component of elastic deformation plus plastic deformation something like your OA and AB. So, now, the most of the materials it is not possible to describe the entire stress-strain curve with a simple mathematical expression try to understand. So, sometimes this behavior is very, very complicated. So, it is I mean this most of the times a stress strain curve cannot be expressed in a simple mathematical expression. So, therefore, it is required to get some kind of say idealized stress-strain curve. So, those idealized stress strain-curve we are now going to discuss and you will appreciate that these idealized stress-strain curve I mean capture the behavior of stress strain of a material very, very simple way.

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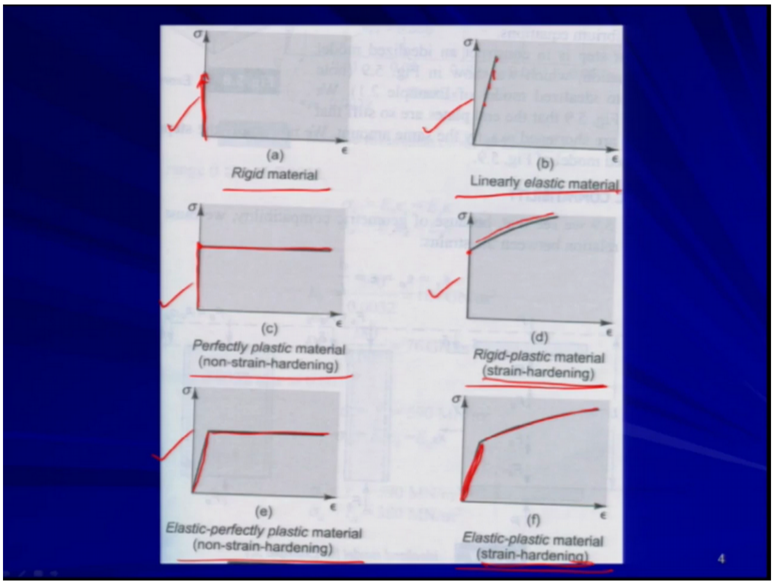

So, these are the idealized stress-strain curve. This stress-strain curve is for rigid material. Now, rigid material if I say rigid material, you will not be observing any deformation as if you go on increase the stress there will be no deformation. So, you are just following this line. You are increasing the stress sigma is increasing, but without increasing epsilon so that is the idealized behavior of rigid material. Then this plot is for linear elastic material, you see the stress-strain curve is completely linear. So, this is linear elastic material. And any point on this line, at any point, if you release the stress, it will come back to its original position.

Then you come to this, this is known as rigid perfectly plastic material initially it is rigid after reaching here it is showing perfectly plastic that means, without any increase in stress this strain is going on increasing so that is known as perfectly plastic material. And of course, it is non-strain hardening. Now, what is strain hardening, we are coming to that point right now. So, if you see this figure this plot basically this is giving me rigid plastic material with strain hardening. So, what is happening here? So, it is behaving rigid up to this point, after that you are getting slight increase in stress, but strain is increasing enormous. So, this is your plastic behavior plastic deformation is happening but it is not exactly the linear I mean the parallel to the epsilon axis, it is not like that, it is not like your perfectly plastic material, but it shows some strain hardening happening. So, some of amount of increase in the stress is also happening, so that is known as strain hardening behavior.

So, now, if you come back to this plot this sigma epsilon plot. So, this is known as elastic perfectly plastic material. So, initially it is elastic linearly elastic and perfectly plastic. Similarly, this plot if you look at this is elastic in linearly elastic initial stage linearly elastic and then plastic, but strain hardening, so that is happening with increase in stress. So, I hope that you have understood the idealized stress-strain curve generally we follow over we considered these idealized stress-strain curve to define the stress-strain behavior of most of the materials well.

So, I will stop here today. So, in the next lecture, we will be continuing this discussion and we will be trying to establish the relation between stress and strain.

Thank you very much.