Mechanics of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 28
Tutorial

Welcome back to the course mechanics of solids. So, in the last lecture we were talking about the measurement of strain, and we just talked about or we were discussing about the construction of 45 more circle for the 45 degree strain rosette. So, mean if you come back to this figure again. So, basically we have got this construction of the more circle by following the 45 degree strain rosette ok.

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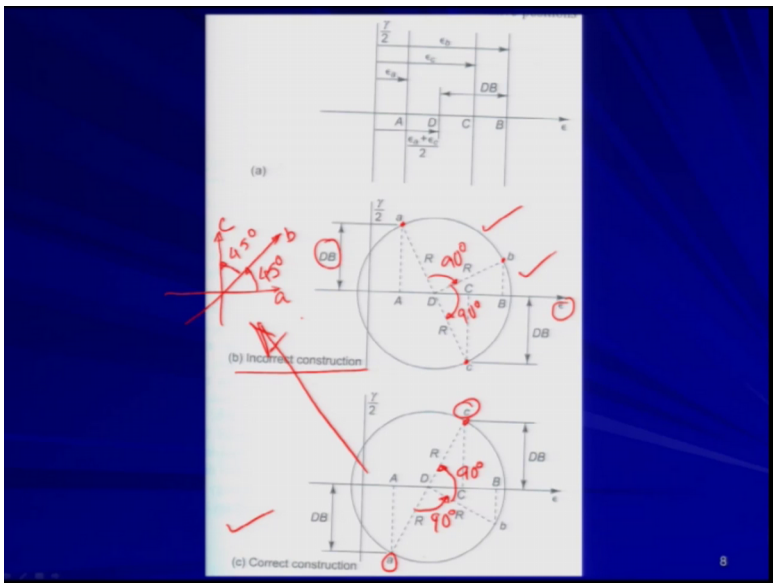

And we have got 3 points point A C and B. So, these three points basically are giving me the normal strain measurements for 45 degree strain rosette. So, that those normal strains are coming from the strain gage reading. So, now, once you construct this more circle basically you will be getting the information about other stress strains say in components epsilon x epsilon y gamma x y or any arbitrary coordinate system, the strain components are obtained from this more circle.

But in the last lecture we were talking about whether this construction whatever construction is done. So, that is correct or not and said this is incorrect construction now why it is incorrect. Let us discuss that thing about. So, now, if you see the 45 degree

strain rosette. So, the orientation of your 45 degree rosette is this is your a axis and this was b and this was c right. So, now, in this coordinate system if you go or if you rotate 45 degree in the anticlockwise direction, you should get b axis. From a axis if you rotate 45 degree in the anticlockwise direction then you should get b axis. Similarly from a or b whatever if you rotate another 45 degree then you will be getting c axis. Now you need to see or you need to check that whether you are getting the same sense of rotation in the more circle or not.

Now, let us see here now from A if we go to B in the more circle you need to go up to 90 degree right 90 degree anticlockwise. But here in this construction you have to come 90 degree in the clockwise direction similarly from b to c if you want to find out. So, that angle is again 90 degree in the clockwise direction. So, magnitude wise it is fine, but the thing is that the sense of rotation is completely reverses than the actual coordinate orientation right. The actual coordinate orientation says the angle between B and means angle between B and A is of course, 45 degree that is, but from A if you go the 45, degree rotation in the anti clockwise direction then you should get B.

Similarly, from B if you go 45 degree anti clockwise rotation you will get C, but in the more circle you are just getting reverse. So, therefore, this construction is incorrect now what should be the correct construction let us talk about that thing here. So, this is my correct construction. Now when we mean laid off point A or point C; we laid off point A in above the epsilon axis and point C below the epsilon axis right, but nobody told me that why point A will be should be laid off in the above of epsilon axis, who told me nobody told me just took it arbitrary right. So, laid off A which is equal to D B that is fine, but chopped the point A on that vertical in above the epsilon axis, but that may not be true. So, what we are doing here we are leaving of A in such a way, A point is coming below the epsilon axis, similarly the c point is going above the epsilon axis.

Now, you construct the more circle we are following the same process whatever we have discussed in the last lecture. So, if you construct the more circle, now you see from a you are getting 90 degree to reach B in the anti clockwise direction, and from B if you want to reach C you have to go to 90 degree rotation again anticlockwise direction and which is matching with this. So, therefore, this is your correct construction. So, this is all about your strain measurement and strain rosette that is arrangement and which is very very important if you are going to do some experiment and if you want to measure some

strain, and then from there if you want to get the value of stress and other things other information then this, discussion is will be very very required ok.

So, this concept will be very very essential. So, with this basically your stray concept of stress and strain chapter is over. So, now, we will take a couple of numerical examples to understand this whatever we are discussed in this particular chapter. So, let us talk about that. So, here is your first numerical problem.

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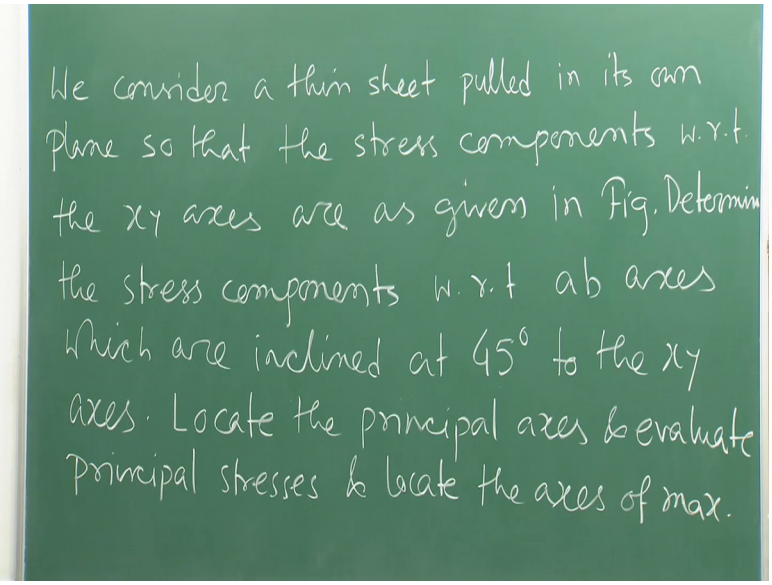

Let me write down we consider a thin sheet pulled in its own plane so that the stress components with respect to the x y axes are as given in figure. So, figure will be showing withdrawn determine the stress components with respect to a b axes which are inclined at 45 degree to the x y axes, and also find out locate the principal axes and evaluate principal stresses and locate the axes of maximum shear.

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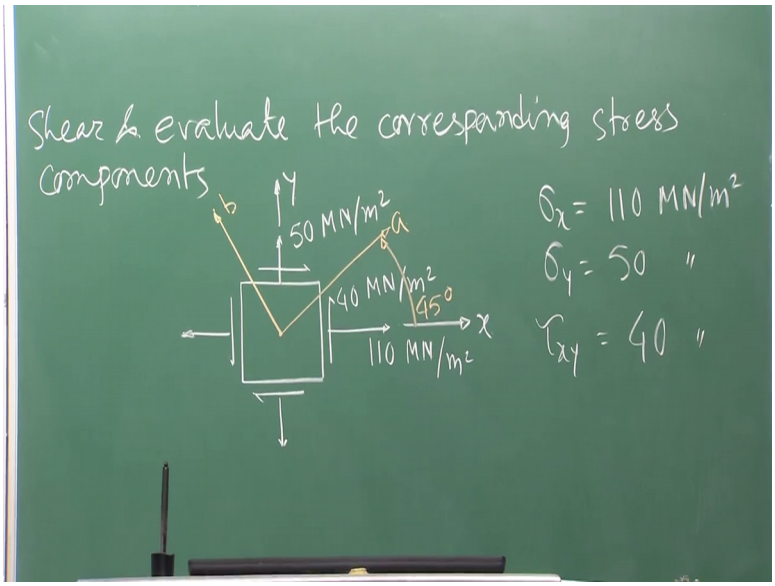

And evaluate the corresponding stress components. So, the figure is like this 110 mega newton per meter square, and this is your x axis this is your y axis your shear stress is 40 mega newton per meter square and tau x y is equal to tau y x. So, therefore, am not showing all mean everywhere 40 mega newton per meter square you gets quite understood and sigma y is mega newton 50 mega newton per meter square and you have the axes a b like this which is making an angle 45 degree with x axis . So, therefore, the problem says that we considered a thin sheet pulled in its own plane. So, that the stress components with respect to the x y axes are as given in figure. Determine the stress components with respect to a b axes that is your some arbitrary or set up axes, which are inclined at 45 degree to the x y axes locate at the principal axes and evaluate principal stresses and locate the axes of maximum shear and evaluate the corresponding stress components.

So, now this problem can be solved by using the equations whatever we have derived, but; however, to understand the more circle in a better way we are going to going to solve this problem using more circle construction ok.

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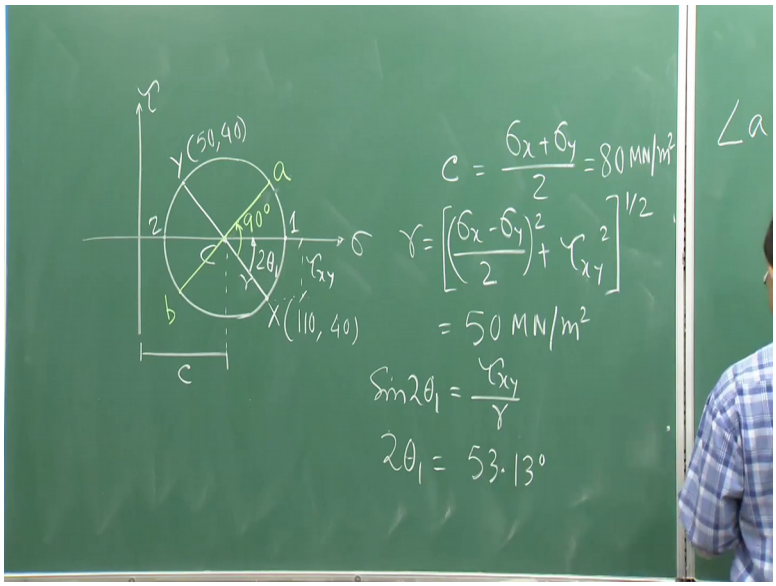

So, let us do that. So, let us construct the more circle first. So, this is your sigma tau space this is the more circle of stress we are going to draw, now what are the things know. I know sigma x equal to 110 mega newton per meter square, sigma y equal to 50 mega newton per meter square and tau x y is positive or negative that will be positive because on positive plane along positive direction the shear stress is shown. So, that is also positive 40 mega newton per meter square.

So, all stress components are positive, sigma x sigma y and tau x y all are positive. So, therefore, the more circle let me draw it. So, this is your more circle this is the center of the more circle, and these are two points basically this points you have to locate first have drawn the more circle first that does not mean that you will be get in the more circle. First you will locate as per our construction procedure; first you locate point x point y as per that combination. So, point x coordinate is 110, 40 and point y coordinate is 50, 40. Now you may ask me why have shown x point below the sigma axis and that is known to you, because we have following the sign convention where if you have positive shear stress then it will be the x will be plotted below the sigma axis ok.

So, here your stress shear stress is positive. So, therefore, x is coming below the sigma axis. So, now, from this we can calculate C. C is nothing, but the distance this is your distance of the center of the more circle from the tau axis. So, c can be obtained as sigma x plus sigma y by 2 right this is your sigma x 110 this is your sigma y. So, this will be

coming at the midway. So, sigma x plus sigma y by 2 and that comes as 80 mega newton per meter square. Similarly the radius of the more circle that you know already we have put the expression sigma x minus sigma y by 2 whole square plus tau x y square to the power half, if you put you will be getting the value 50 mega newton per meter square ok.

So, these are the values we will be required mean these are the parameters we will be required later on for calculation of remaining a stress components well. So, now, your a b if you look at this is your physical plane, this is your stress plane as per our definition. So, in the physical plane the angle between x axis and a axis is 45 degree in the anticlockwise rotation sense therefore, can locate here the angle the axes the plane will basically a plane and b plane like this. So, from x point have to rotate 90 degree in the anticlockwise direction to get plane a.

So, that is nothing, but. So, this is your a and this is you b, and this angle is 90 degree. Same sense of rotation from the physical plane we are coming to the stress plane only twice right. That was 45 degree now it will be coming as 90 degree. Now if define this angle as twice of theta 1 then can calculate that angle as sin twice of theta 1 is equal to tau x y tau x y you can see tau x y by r. So, psi this is your tau x y and this is your r. So, sin twice theta 1 is equal to tau x y by r from this can get twice theta 1 is equal to by putting the values numerical values will be getting that equal to 53.13 degree ok.

Now, angle a c 1 what is one this point is one and this point is say 2.

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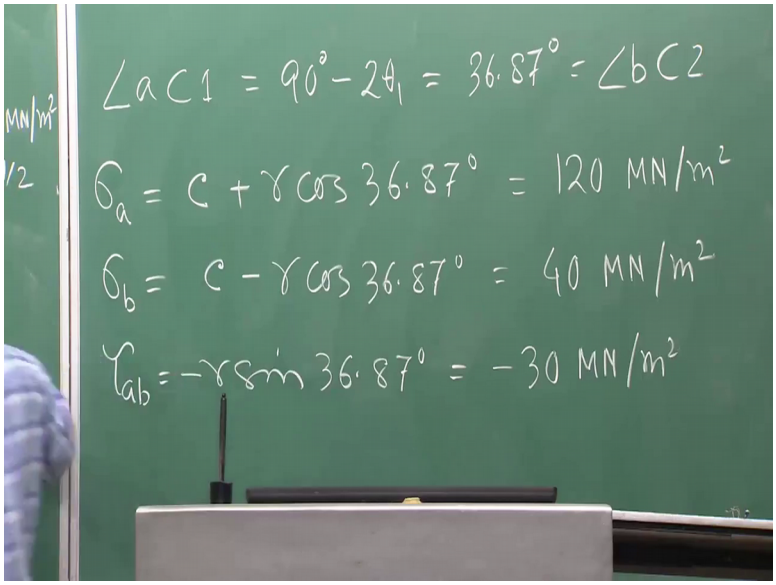

So, angle a c 1 is therefore, your 90 degree minus twice theta 1, which gives me 36.87 degree which is equal to b c 2 which is equal to; obviously, which is equal to b c 2these angle. So, from this can get the stress components in a b coordinate system; that means, the arbitrary set of coordinate system sigma a is nothing, but c plus r cos 36.87 degree. Just look at the figure sigma a that is the c plus this much from c to this part that will be obtained as r cos 36.87 degree.

So, if you put the values it will be coming 120 mega newton per meter square. Similarly sigma b can get, c what is sigma b? Sigma b is here right. So, this is your sigma b. So, sigma b is nothing c minus r cos 36.87 degree that comes us 40 parameter per meter square. Now what about tau a b? Now tau a b is nothing, but this is your tau a b, this is your tau a b or this is your tau a b whatever you say this is your tau a b. So, tau a b is r sin 36.87 degree now what is the sin. So, sigma a is positive fine because that is in the positive sigma direction sigma b is also positive. Now what about tau a b is it positive or negative? You just look at this, this x point is getting map to a point; that means, if a point is coming above sigma axis then that shear stress is negative as power sin combination because x is getting mapped two a or x is getting rotated to point a.

So, a is lying now above sigma axis. So, instead of a you can considered x prime. So, x is mapped to x prime and x prime is lying above the sigma axes. So, therefore, your tau a b must be negative as per our sin convention. So, that is coming 30 mega newton per meter square. Now you quickly we can say sigma one that is principle stress and principle stress axis and maximum shear stress and maximum shear stress axis. So, this two things we are going find out.

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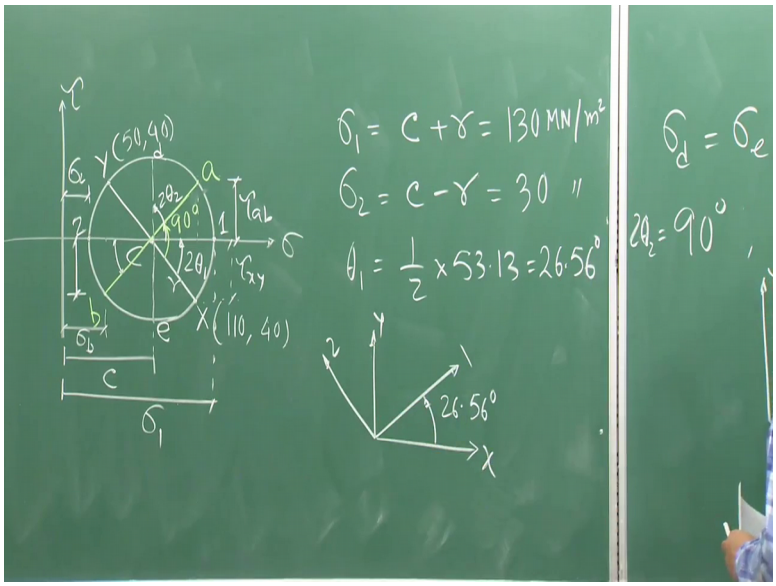

So, sigma 1 by definition, sigma 1 means this one is sigma one right this is your sigma 1 by definition c plus r. So, that will be coming as 130 mega newton per meter square, and sigma two is c minus r this is your sigma 2. So, that is coming as 30 mega newton per meter square and know this twice theta 1 is known to me right.

Twice theta what is twice theta 1? These angle is nothing, but the angle made between x axis and major principle stress axis; that means, axis one. So, in the stress plane this is twice theta 1. So, in the physical plane it will be theta 1. So, theta 1 value is nothing, but your half of 53.13 is nothing, but 26.56 degree. So, therefore, your, this is your x and y axis and this is your one and two axis which is making and angle in the anticlockwise sense of rotation 26.56 degree. So, this is done principle stresses and orientation of principle axis ok.

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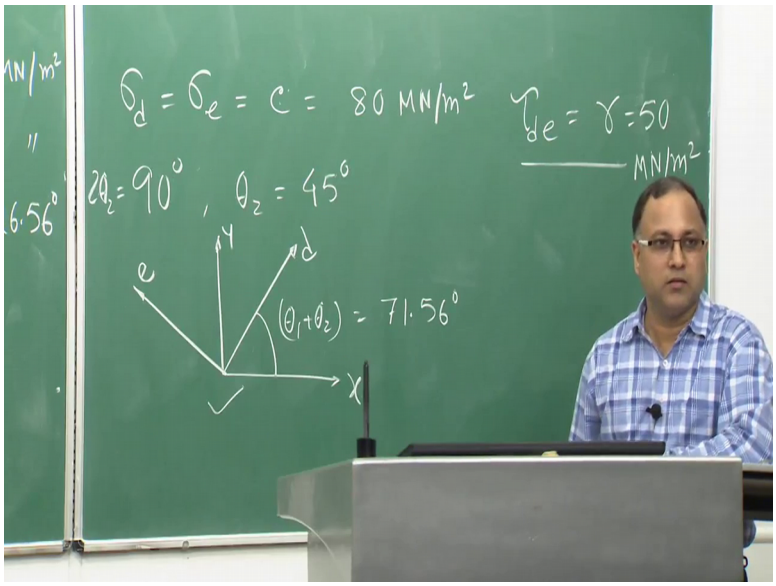

Now coming to the sigma d this point this point is say d this point is e. So, sigma d is nothing, but equal to sigma e is equal to c right.

So, that is 80 mega newton per meter square that is the normal stress on the plane on which maximum shear stress is the acting that will be same and twice theta two if we considered this angle as theta two twice theta two; that means, that angle between the measured principle stress axis and the axis on which the maximum shear stress is acting that is nothing, but your 90 degree. So, twice theta two is equal to 90 degrees therefore, theta two equal to 45 degree right. So, therefore, if this is your x y plane, from one axis it will be further 45 degree right. So, this angle is your d and e, and these angle is your theta 1 plus theta 2 that is nothing, but 71.56 degree.

So, you have got the and of course, the maximum shear stress that is also important to be determined that is nothing, but the radius of the more circle and that is 50 mega newton per meter square. So, you have got maximum shear stress, you have got measured principal stress, minor principal stress and the orientation of the principal axis and orientation of the axis where maximum shear stress is occur. So, will stop here today will take one more numerical example in the next class and then we will try to establish the relation between stress and strain ok.

Thank you very much.