Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 26
Strain Transformation

Welcome back to the course Mechanics of Solids. So, in the last lecture, if you recall we talked about the different strain components normal strain and shear strain, and how we can quantify the strain components that already we have said. And then we have established a relation between the strain and the displacement; and we have seen that for defining the state of strain in plane strain condition, you need three strain components that is epsilon x, epsilon y and gamma x y.

Now, pretty similar to your stress discussion if you recall that if you want to if you know the state of strain in x y coordinate system then can you define the state of strain in some arbitrary coordinate system like x prime y prime, so that we are going to do. And this is a little bit different then whatever we have whatever exercise we have done for our stress chapter.

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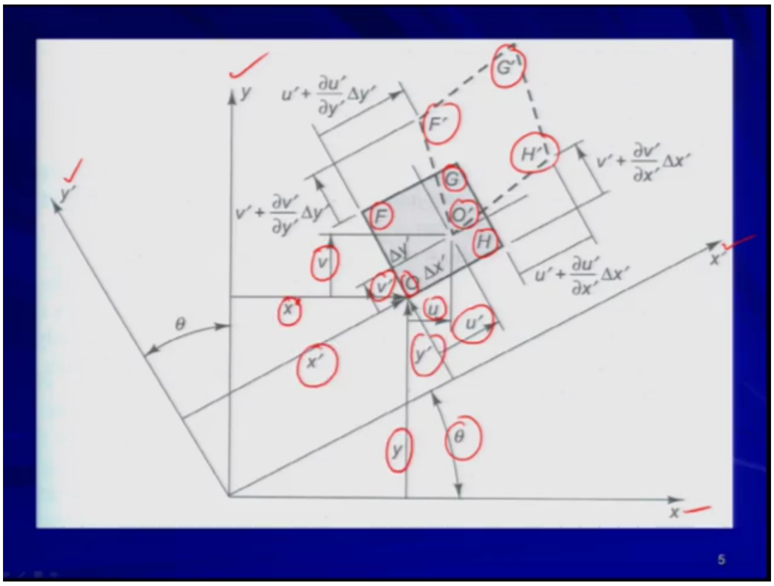

So, now if you look at this figure the basically this is your x y coordinate system. Now, we are going to define the state of strain in x prime y prime coordinate system which is oriented with respect to the angle theta and that means, the angle between the x prime axis and x axis is theta. Now, this is the body OHG F this is the body in say it is oriented in x prime y prime say coordinate system and everything is defined; that means, all the displacement vectors and your laying can all those things are defined in terms of x prime y prime coordinate system.

Though I mean you see this is this is the point O and this is the point O prime after deformation after deformation it takes the shape O prime H prime G prime F prime. So, u is the displacement vector along x-direction, whereas the same thing if you try to transform in x prime direction that will be u prime. Similarly v is the displacement vector along y-direction and that will be mapped that will be transformed to v prime in y prime-direction. So, this is basically the transformation I mean you need to know how to transform the axis. Well, so other things are defined. So, similarly I mean the coordinates of point O is nothing but x y. So, this is your x, this is your x and y; whereas, in x prime y prime coordinate system that is x prime y prime, so everything is ok.

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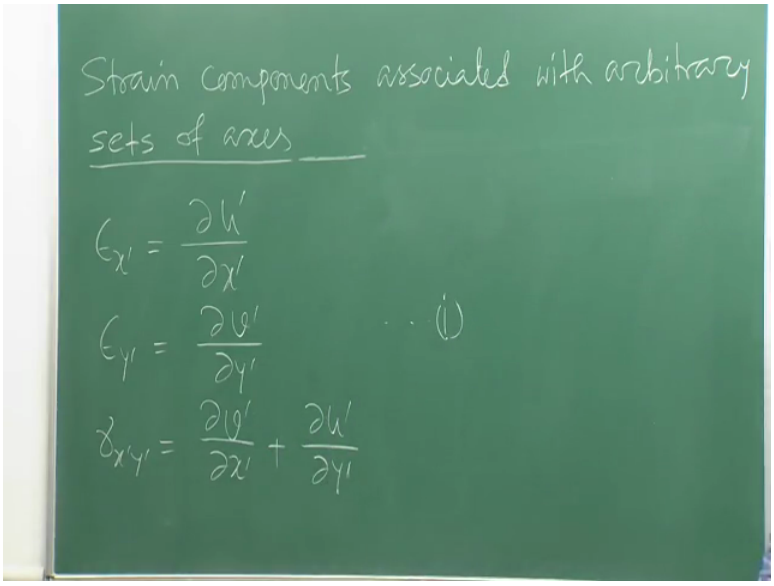

So, now, we are going to write epsilon x prime in terms of the displacement vector in x prime y prime coordinate system and we know that that should be del u prime del x prime. Similarly, epsilon y prime can be written as del v prime del y prime and we can write gamma x prime y prime is equal to del v prime del x prime plus del u prime del y prime. So, say this is my equation one. We can write that because of we know the displacement vectors in x prime y prime coordinate system, I can define the strain components normal strain and shear strain in x prime y prime coordinate system; only thing here we know epsilon x, epsilon y and gamma x y. So, we are going to express these three strain components in terms of epsilon x, epsilon y, gamma x y; that means, the strain components in x y coordinate system and at the same time the orientation of the transform or the I mean the new set of axis will be required. It is very similar to your stress; only things is that the process or the procedure of derivation is different.

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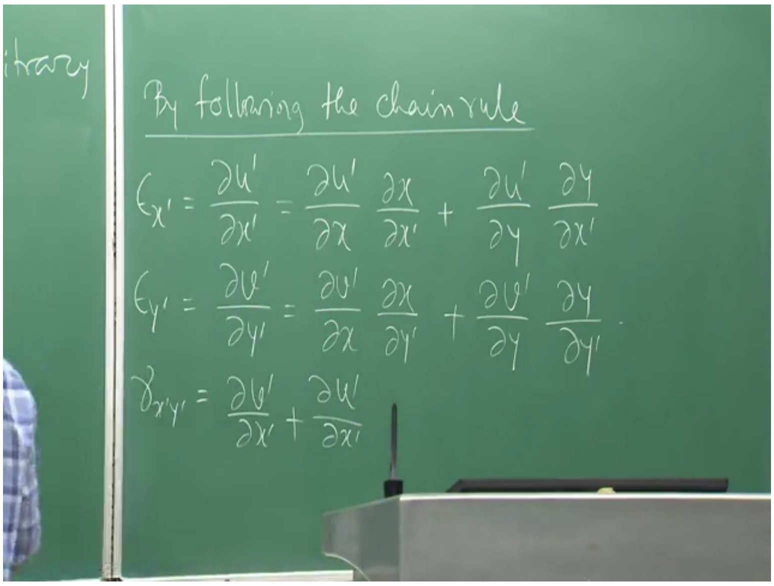

Now, by following the chain rule of mathematics, by following the chain rule, I can simply write epsilon x prime equal to del u prime del x prime that is given which is equal to del u prime del x del x del x prime plus del u prime del y del y del x prime. Now, write that yes I can write it so by following the chain rule. Similarly, I can write epsilon y prime is equal to which is nothing but del v prime del y prime that is nothing but del e prime del x del x del y prime plus del e prime del y del y del y. And finally, I can write gamma x prime y prime is equal to that is nothing but plus del u prime del x prime.

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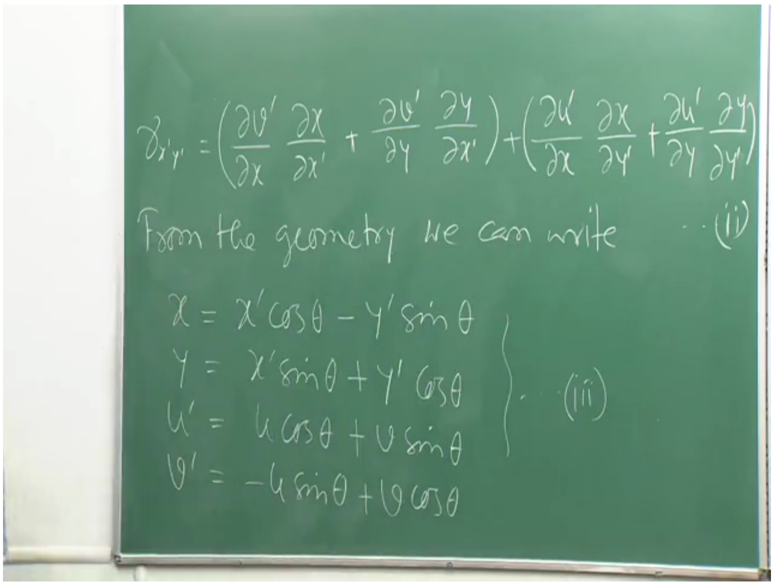

So, that I can write simply in the chain rule system we can write del e prime del x del x del x prime plus del e prime del y del y del x prime plus del u prime del x del x del y prime plus del u prime del y del y del y prime. So, all these equations I am calling equation 2. So, by following the chain rule, I am expressing everything or by I mean epsilon x prime epsilon y prime and gamma x x prime y prime by these expressions I can express that thing. So, now, form the geometry we can write so that means, we are just basically write the transformation equation if you look at the geometry; that means, we are going to transform I mean we are going to express x and y coordinate system in terms of x prime y prime coordinate system and similarly the displacement vectors. So, we are going to write down those transformation by following the geometry.

So, now if you look at the figure, it will very clear that x can be expressed as x prime cos theta minus y prime sin theta. This is the transformation rule from the geometry. Similarly, I can write y equal to x prime sin theta plus y prime cos theta. And similarly I can write the displacement term u prime is equal to u cos theta plus v sin theta and e prime equal to minus u sin theta plus v cos theta. So, this is a equation 3.

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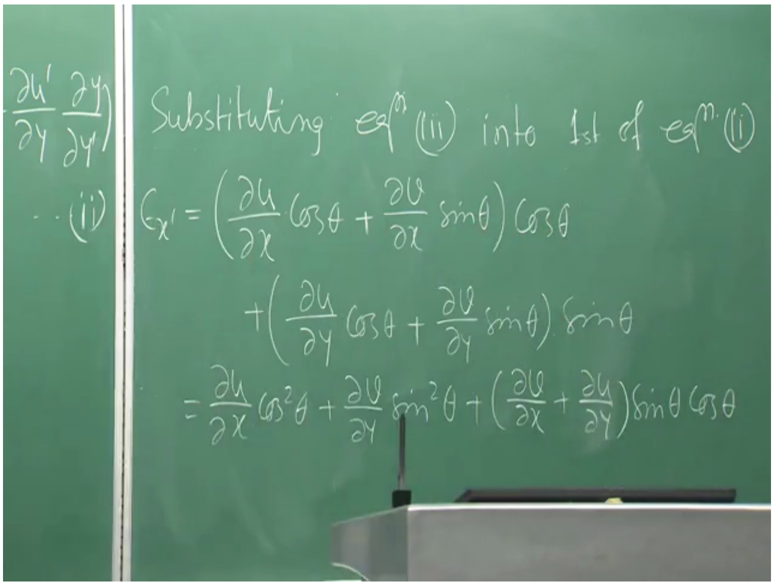

So, now substituting equation 3 into first of equation 2. So, if you substitute the equation 3, whatever expressions we are getting we are substituting that thing in first of equation 2; that means, the expression of epsilon x prime by following the chain rule. So, there if you look at we can simply write epsilon x prime equal to del u del x cos theta plus del v del x sin theta multiplied by cos theta plus del u del y cos theta plus del v del y sin theta into sin theta. Finally, if I can write del u del x cos square theta plus del v del y sin square theta plus del v del x plus del u del y sin theta cos theta.

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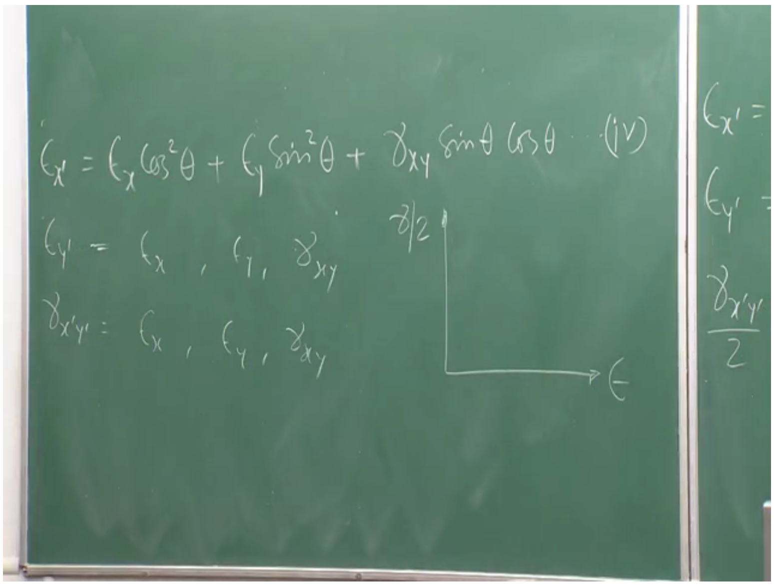

And finally, we can write finally, we can write epsilon x prime. Now, what is del u del x that is epsilon x. So, epsilon x cos square theta plus what is del v del y epsilon y. So, epsilon y sin square theta plus del v del x plus del u del y is nothing but your gamma x y sin theta cos theta, so equation 4. In similar way, if you substitute equation 3 in the second and third term of equation 2 then we will be getting the expression for epsilon y prime and gamma x prime y prime in terms of epsilon x, epsilon y, gamma x y that you can do it by following the similar thing I mean very pretty straightforward expression. Now, if I do the trigonometric calculations and where the relations are developed based on the double angles, trigonometric relations based on double angles if we impose here whatever we did for our stress then we can get the expression of epsilon x prime, epsilon y prime and gamma x prime y prime in this fashion.

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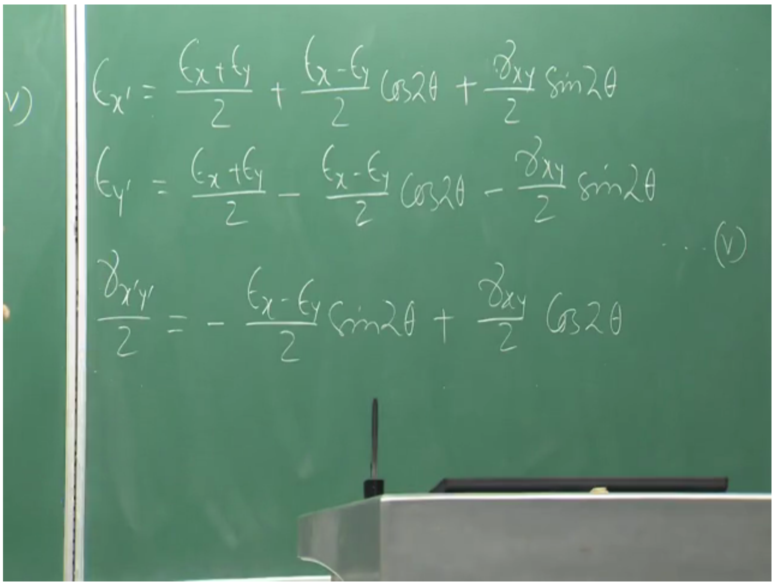

So, we will be simply getting epsilon x prime is equal to epsilon x plus epsilon y by 2 plus epsilon x minus epsilon y by 2 cos 2 theta plus gamma x y by 2 sin 2 theta. Similarly, I can write epsilon y prime is equal to epsilon x plus epsilon y by 2 plus epsilon x minus epsilon y by 2. This is minus cos 2 theta plus gamma x y by 2 sin 2 theta and gamma x prime y prime by 2 is equal to minus epsilon x minus epsilon y by 2 sin 2 theta plus gamma x y by 2 cos theta - the equation 5. So, these are the equations by which you can get epsilon x prime, epsilon y prime, gamma x prime y prime in terms of epsilon x, epsilon y and gamma x y. So, that means, if you know the straight of strain in x y coordinate system, you can find out the state of strain in any arbitrary set of coordinate system. If you know epsilon x, epsilon y, gamma x y and theta if you know then you can find out. So, that you have got from this expressions. So, we have derived that thing.

Now, if you look at this I mean this equation, so equation five. So, that is pretty similar to the equation whatever we have derived for our stress only exception is that here you have half. So, there you had tau x y only that is the shear stress, but no half was there right, here you are getting one half by definition I mean when we derived that I thing we got this thing. So, now if we look at this expression then basically one thing is coming out from this discussion that we can again construct the graphical representation very similar to the Mohr cycle of stress, we can have Mohr circle of strain if we plot the coordinate system of gamma by 2 versus epsilon; that means, normal strain versus half of shear strain. Now, if we plot or if we construct the Mohr circle in this space epsilon versus gamma by 2 space then basically these three equations will give you or will form you find a Mohr circle, is that clear?

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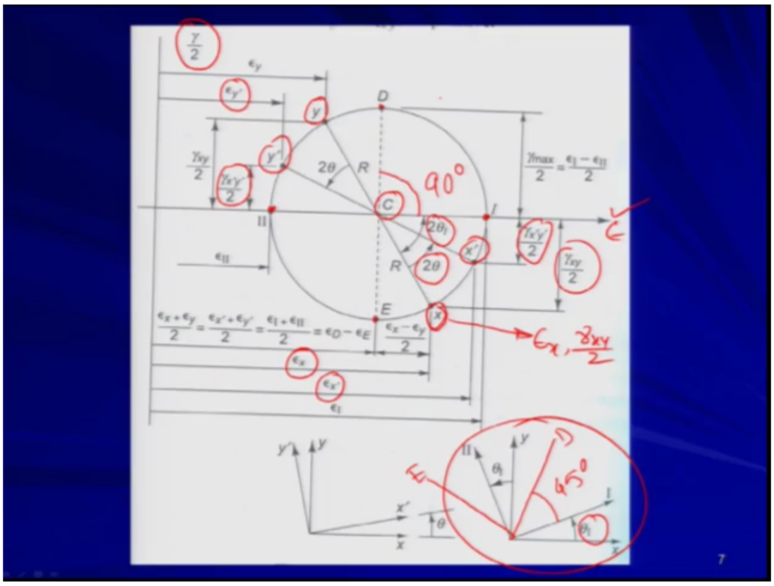

So, that more cycle if we look at, so this is the Mohr circle we look like. So, as I told you so it is basically epsilon versus gamma by 2 space. And the construction of this Mohr circle is pretty similar whatever we did for stress. So, therefore, I am not going to repeat again I mean the whole process only thing is that this is the point x whose coordinates are epsilon x gamma x y by 2. So, this is given by this. Similarly, you can get point y which is diametrically opposite. So, once you know these two points then basically you can locate the center of the Mohr circle by connecting these two points x and y with the straight line. Then once C is known you can find out the radius of the Mohr circle and other necessary parameters. And then you can find out the state of strain on x prime y prime coordinate system by just following the same sense of rotation, but twice.

So, the angle between x and x prime is theta, but here in the Mohr cycle you will be getting twice of theta that is pretty similar to your stress. So, you are just pointing out the points x prime y prime. So, the coordinates of x prime y prime epsilon x prime and gamma x prime y prime by 2. Similarly, the coordinates of y prime point is epsilon y prime and gamma x prime y prime by 2.

Now, there are few things, which are pretty similar to your stress. Here also you are getting two points point one and point two, where you have maximum possible normal strain and minimum possible normal strain without any shear strain. So, these normal strains are known as principal strains and the plane on which they are acting those planes are known as principal planes I mean and that also we can prove I mean though I am not going to prove that thing again. So, that principle I mean principle stress is associated with principle strain. So, anyway, so this point one will give you the normal, I mean principle measured principle strength and point two will give you the minor principle strength and that is shown here. So, that rotation I mean that orientation of the principle axis is shown in this figure.

So, the angle between x point and one point is nothing but twice theta one in this Mohr circle, so that will be taking theta one in the actual coordinate system. And now very similar to your stress you are getting two points D and E where your normal stress is same. But you are getting maximum possible shear strain that is your gamma x y is becoming or gamma value on those planes will be maximum, not gamma x y gamma value that is the shear strain will be becoming maximum on this planes. So, and this plane is basically making an angle 90 degree with the principle normal principle strain axis. So, therefore, if you want to draw the axis on which your maximum shear strain is acting say D and E, so this angle will be 45 degree all right pretty similar to your stress.

Now, one thing you need to remember. Here also we are following the same sin convention to draw the Mohr circle whatever we considered in case of stress do you remember that. So, we considered that if the normal stress is positive if the normal stress is tensile in nature then that will be positive; if it is compressive in nature that will be negative. Here also if your epsilon x is a elongation in nature then that will be positive, epsilon I mean epsilon is contraction in nature then that will be negative. But in case of shear strain, so if your shear strain is positive then x point will be coming below the epsilon axis; if shear strain is coming negative then x point will go above epsilon axis, so that is also pretty similar or in line with whatever sin formation we considered to draw the Mohr circle for stress. The same thing we will be continued here also.

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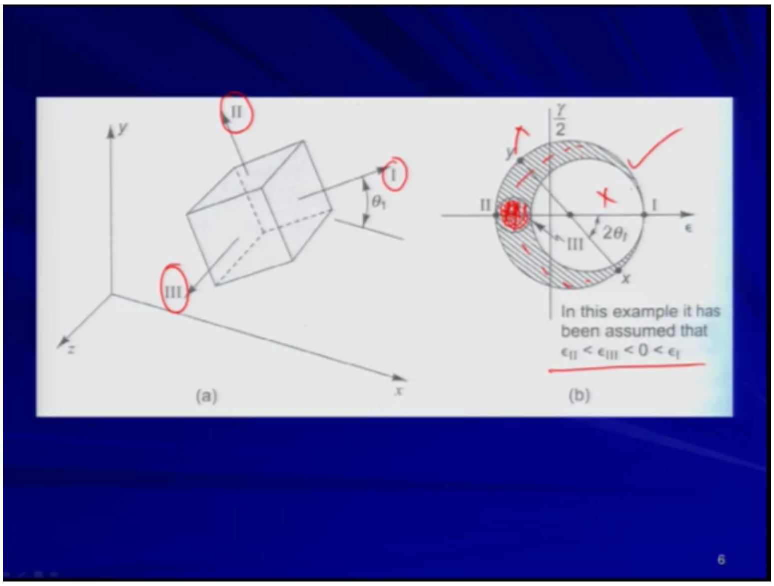

Now, once you have got the Mohr circle for strain in case of plane strain condition, we can draw the Mohr circle of strain in case of three-dimensional or the general state of strain that is pretty similar. Again you will be getting three different normal strain, three different principals strain you will be getting epsilon 1, epsilon 2 and epsilon 3. So, I mean in case of 3D state of strain you will be getting three different principle strains. And similarly very similar to your stress, you will be getting the Mohr circle will be like that. So, three different Mohr circles will be getting and with this combination I can get this kind of Mohr circle.

But in all the cases, I mean this figured I am here it is wrong. So, basically your state of strain will be always lying in this shaded part, not inside this circle, not inside this circle, not outside this circle. So, it will be always within this shaded part. So, with this basically your stress and strain concept is over. So, now you are familiar to the stress as well as strain and how to quantify those things, how to find out those things in some different arbitrary coordinate system all those things are covered.

Now, in the next lecture, we will be talking about the measurement of strain using strain rosette. So, I mean there we will see that this Mohr circle this representation of Mohr circle in the strain I mean plane strain condition or in the three-dimensional state of strain will be very helpful, very handy to find out the strain in some arbitrary coordinate system by getting the response from the strain rosettes. So, I will stop here today.

Thank you very much.