Mechanics Of Solids
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Lecture - 25
Normal Strain and Shear Strain

Welcome back to the course Mechanics of Solids. So, in the last lecture we are discussing about strain right. So, they are we very similar to our plane stress thing we are having another term that is plane strain. So, what is plane strain we just talked about that thing in the last, we just started basically. So, a body whose particles all lie in the same plane and which deforms only in this plane, so you do not have the deformation suppose if you have x y and z, z is nothing, but the normal to the board if you consider.

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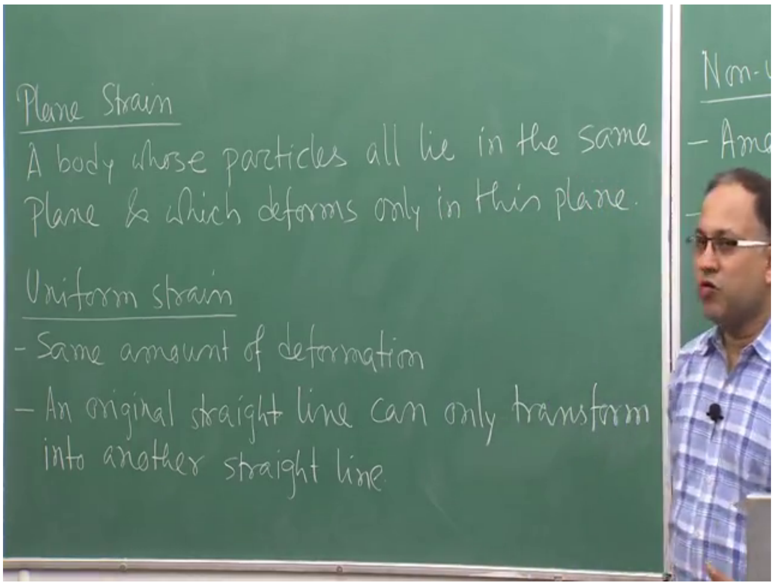

So, in that direction I mean z direction there is no deformation. So, whatever deformation you are observing, you are observing that deformation in the xy plane say and the classical example of this plane strain problem is something like a long wall or long say beam you would long say embankment retaining wall from civil engineering. So, these are the structures which will deform only in that plane. So, if you consider a continuous or long wall. So, wall I mean if you apply the load. So, you will be observing the deformation in the xy plane. So, z direction; that means, normal to that particular plane is not experiencing any deformation. So, that is that condition is known as plane strain condition and we will be sticking to this plane strain condition in this particular course and that is again 2D state of strain.

Now, we need to know few strain components that is uniform strain what do you mean by uniform strain in uniform strain basically you have same amount of deformation. So, same amount of deformation means you are applying the load and due to that you are observing the strain in the system and that strain is uniform that is same amount of deformation is happening everywhere and original straight line. So, if you have a original straight line which is which you are considering. So, suppose if I considered this chock which is originally straight or if you consider steel rod and you are just pulling it you are giving the externally applied tensile force then basically the elongation if we consider the uniform elongation then that will give you the uniform strain. So, the straight rod will be remaining straight even after deformation right.

So, an original straight line can only transform into another straight line. So, you will not be getting any bend in the straight line. So, straight line will remain straight.

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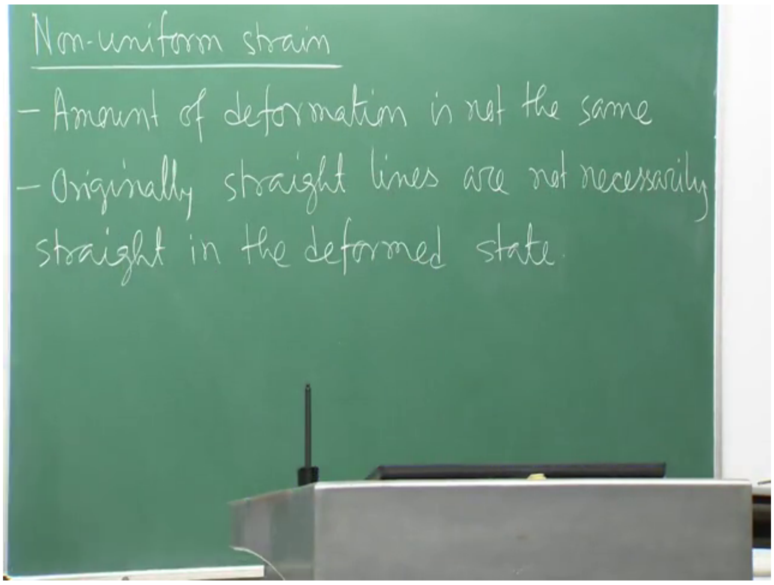

Whereas, if you come to the non uniform strain, so amount of deformation is not the same, so I mean in the particular system if it is under the externally applied force you are not observing the same amount of deformation everywhere and originally straight lines are not necessarily straight in the deformed state. So, that is very very important so; however, in this particular course we will be restricting ourselves to talk about uniform strain. So, we will not be talking about the non uniform strains. So, non uniform strain if you are interested. So, that will be coming in some advanced level course that is beyond the scope of this course. So, we will be sticking to this uniform strain so; that means, same amount of deformation and original straight line will remain or I mean straight even after deformation.

So, now we are trying to quantify the strain. So, how we can quantify the strain? So, before quantifying the strain components we should know that how many strain components you have I mean what are the different strain components you generally get from this study.

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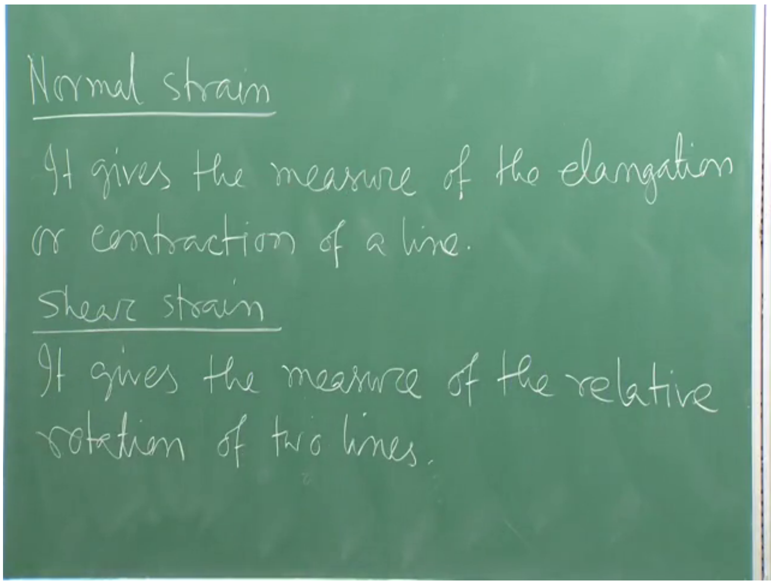

So, first one is normal strain. So, we are just basically getting the same thing whatever we have discussed for our stress. So, everything will be coming pretty same. So, there we got normal stress and shear stress here also we will be getting normal strain and shear strain. So, these are two components. So, what is normal strain right? It gives the measure of the elongation or contraction of a line. So, the whatever example we just talked about that is if you consider a steel rod I mean long steel rod straight steel rod and you are just pulling it if you are pulling it then you will be getting elongation or if you apply some compressive force in the steel rod you will be getting contraction; that means, the change in length either positive or negative right elongation or contraction and that kind of strain is known as normal strain.

So, basically it will not give you the twist in the system. So, you have the straight I mean system or the straight line and that straight line you are just pulling it or pushing it and due to that you will be getting elongation or contraction and that will give you the normal strain. Similarly you have shear strain, so how to define shear strain it gives the measure of the relative rotation of two lines. Now if you come back to this figure. So, then we can define what is normal strain and what is shear strain. So, what is the definition of shear strain? So, it gives the measure of the relative rotation of two lines. So, what does it mean? So, let us let us look at this figure.

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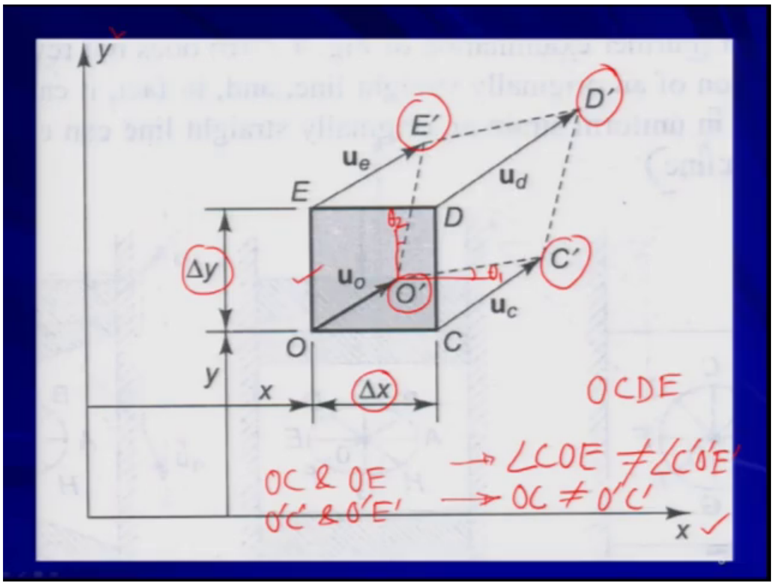

So, in this figure basically OCDE that is the original say body we are talking about the plane strain. So, we are just dealing with x and y plane. So, we are not talking about the z direction strain because as you know as we have discussed this is plane strain condition and the body is OCDE. So, OCDE this is your body right in xy plane. Now, that is before deformation and before deformation the OC length was delta x and OE length was delta y and this is a square I mean say a square body.

So, now, after deformation that O is moving to O prime, C is moving to C prime. D is moving to D prime and E is moving to E prime now OC and O prime C prime they are not same the lengthwise. So, the length of OC and O prime C prime is not same whereas, the angle E this angle COE is not equal to angle C prime O prime E prime right. So, two things we are getting OC is not equal to O prime C prime. Similarly other straight lines like I mean OC is one of the straight lines right OC remains straight because we are talking about the uniform strain. So, OC remains straight. So, OC remains straight like O prime C prime, but they are not the same magnitude the length of OC is not exactly same of length of O prime C prime, similarly OE the length of OE is not equal to O prime E prime. So, some strain has happened.

Now, in this condition I mean in this situation this OC is not equal to O prime C prime that gives you the normal strain whereas, this angle COE does not remain same as C prime O prime E prime and that is because of your shear strain and that is why I have written that it gives the measure of the relative rotation of two lines. Two lines means the COE angle was made by OC and OE that is the internal angle between these two lines now after deformation that C prime O prime E prime whatever angle you are getting that is because of O prime C prime and O prime E prime that is the internal angle between these two straight lines. So, these two angles are not the same. So, that is you how much relative rotation happened in that particular internal angle that will give you the measure of shear strain.

Now, let us let us define or let us quantify the amount of normal strain and shear strain happened in this figure.

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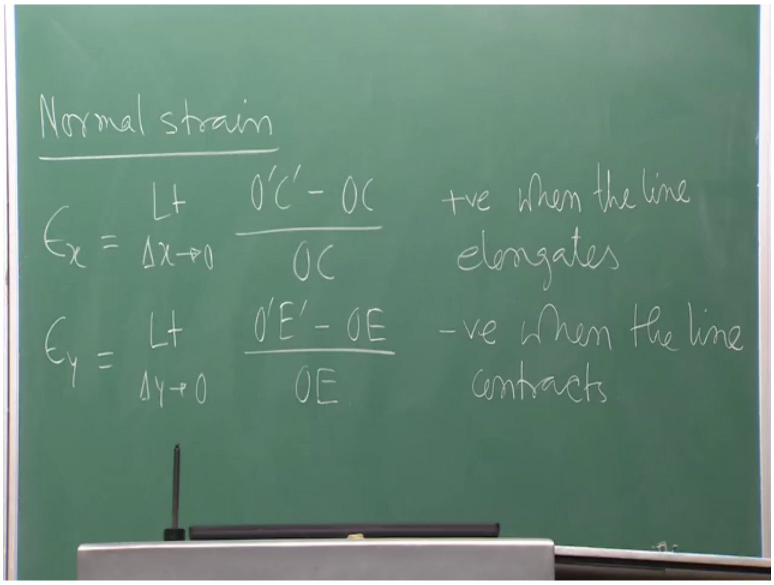

So, normal strain as you know from the definition that is change in length over the original length right. So, epsilon x as that means, the normal strain along x direction, epsilon x is given by limit del x tends to 0, O prime C prime minus OC over C clear. So, this is your normal strain. So, the change in length in the member OC right O prime C prime is the final length after deformation and OC is original length before deformation over the original length OC. Similarly you can define epsilon y which is nothing, but limit del y tends to 0, O prime E prime minus OE over OE any doubt right everything is pretty straight forward.

Now, we need to know the sign convention it would be positive normal strain will be positive when the line elongates. So, if the line elongates if you are getting the elongation in the line after deformation then that kind of normal strain will be considered as positive that is my sign, that is our sign convention in this particular course. And that is generally accepted in several I mean other books also. And it will be negative when the line contracts; that means, when you are getting the shortening in the line then your shear strain I mean normal strain will be negative. So, that is the, this is our sign convention.

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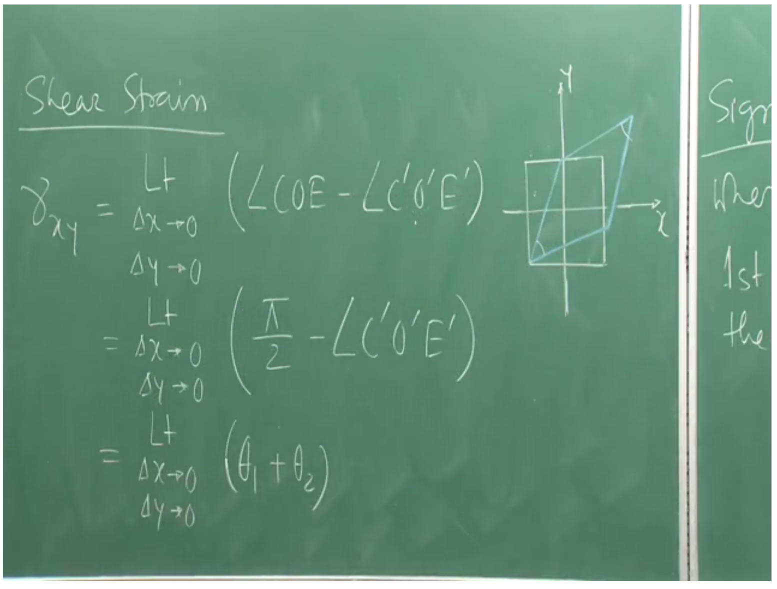

Now, let us talk about shear strain. So, how to quantify shear strain? Shear strain will be gamma xy right in xy plane we are considering that is a plane strain condition pretty similar to your plane stress situation planar the in that case you had planar stress, now you have planar strain. So, limit del x tends to 0 del y tends to 0. So, what is that? Change in angle internal angle that is angle COE minus angle C prime O prime E prime. So, this is your shear strain that is relative rotation. So, that can be further written as limit del x tends to 0, del y tends to 0, what was the value of angle COE in the figure it was ninety degree initially. So, I can simply write pi by 2 minus the angle C prime O prime E prime which can be further written as limit del x tends to 0 del y tends to 0 theta 1 plus theta 2.

Now what is theta 1 plus theta 2? If you look at this figure, so this angle say this is theta 1 and say this is theta 2. So, change or the rotation of the line from its original position, so theta 1 is the angle between say O prime C prime with the respective say OC, OC line similarly the theta 2 is the angle between O prime E prime and the respective line OE. So, this will give you the measurement or the magnitude of the shear strain. So, that will be I mean we are dealing with a small deformation problem. So, this will be pretty small right this squeezing of the angle.

Now, you may have this theta 1 plus theta 2 whatever you are observing; that means, the angle C prime O prime E prime may be greater than COE or less than COE; that means, before deformation you had 90 degree angle pi by 2, after deformation you may get less than 90 degree angle; that means, it is squeezing the angle is squeezing internal angle or it may expand; that means, it may go beyond pi by 2 depending on that you have to define a sign convention. So, let us define the sign convention for shear strain likewise whatever we have defined for the normal strain. So, the sign convention for shear strain is little bit tricky.

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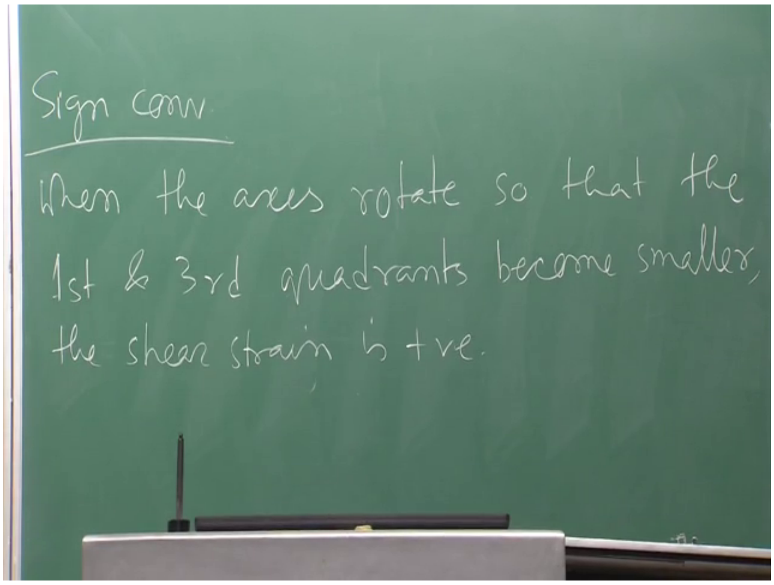

So, the sign convention for shear strain when the axes rotate so that the first and third quadrants become smaller the shear strain is positive. So, when the axes rotate so that the first and third quadrants become smaller then the shear strain is positive now what does it mean say if this is my say axes; if I have. So, this white red square is your say originally original body whatever you have before deformation now after deformation it is taking the shape say the blue one. Now you if you look at this figure actually the first and first and third quadrant is becoming smaller right first and third quadrant both are becoming smaller. So, if they are becoming smaller then this whatever shear strain you are getting that is positive. Now if the first and third quadrant become larger than the original one then your shear strain will be negative. So, that is our sign convention will be following in this particular course. I hope you have agreed or you have you understood the thing properly right.

So, this is a sign convention, so if your first and third quadrant becomes smaller than your shear strain is positive otherwise it will be negative. So, now, let us define the shear strain or let us define the strain displacement relationship because whatever you are getting whatever strain as we discussed or as we have agreed earlier that the strain is due to your displacement or the movement right movement of particles. So, we need to relate between or you need to relate the strain and the displacement. So, let us talk about that.

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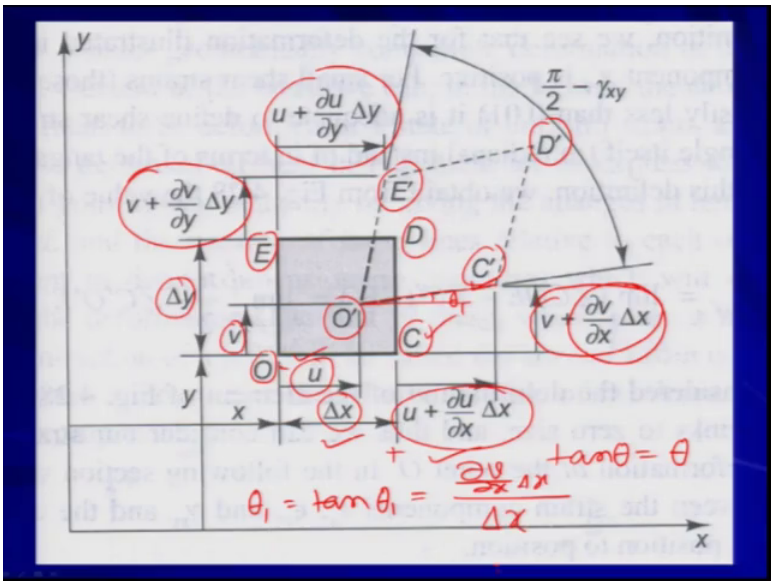

So, in this figure if you see OCDE is your original body, after deformation it takes the position O prime C prime D prime E prime. So, originally OC was equal to delta x and the length of OE was equal to delta y. Now after deformation actually when O is moving to O prime. So, this is basically defined by this displacement vector u. So, u is the displacement vector along x direction whereas, v is the displacement vector along y direction. So, O to O prime if you move. So, you are getting u displacement along x direction and v displacement along y direction.

Similarly, so I mean as per our differential calculus this C prime point if we compare C prime point with respect to C point. So, we will be getting this is my displacement that is the variation in v is happening right v plus del v del x delta x, differential variation. Similarly this from C to C prime if you move in the along x direction you are getting this much of displacement - u plus the directional derivative. Similarly this is coming and this is also coming.

Now, we are going to establish the relation between the displacement and the strain. Let us see how we can define that.

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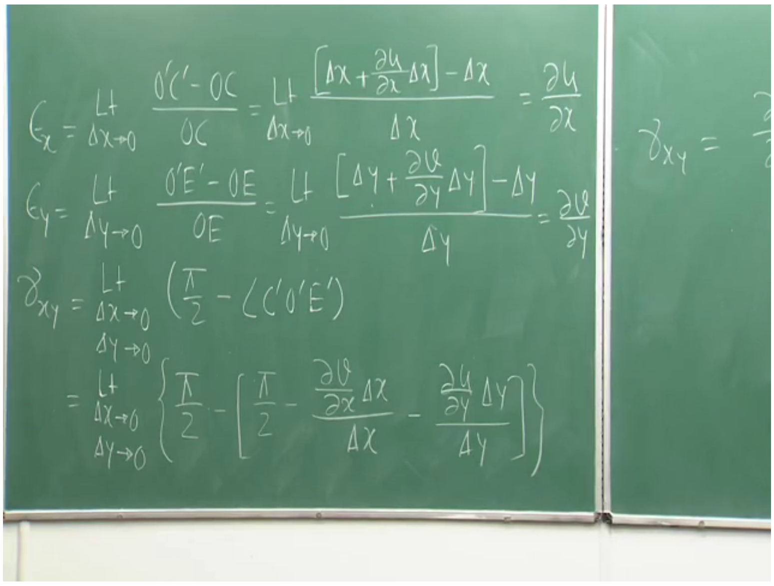

So, your epsilon x already we have seen that is given by limit del x tends to 0, O prime C prime minus OC by OC which can be written as limit del x tends to 0. So, O prime C prime can I write delta x plus del u del x delta x that is my O prime C figure. So, delta x plus del u del x delta x, why it is. So, by following the derivatives right directional derivative whatever you are getting here right. So, O prime C prime means this is the length we are going to find out right. So, I mean the length along x direction. So, length along x direction if you consider, that will be delta x plus this minus this right. So, delta x plus this minus this, that will give you the length of O prime C prime.

Similarly, what is the length of OC? That is simply delta x over delta x now this will give me del u del x that is very very important, this is very very important relation between the normal stress along x direction and the displacement vector along x direction. Similarly epsilon y can be defined as limit del y tends to 0, O prime E prime minus OE by OE that can be further written as very similar to the previous expression I can write delta y plus del v del y delta y and originally OE was delta y over delta y that gives me del v del y where v is the displacement vector along y direction. Now similarly I can define gamma xy where gamma xy is nothing, but the shear strain that is defined by limit del x tends to 0 del y tends to 0 and we define that thing by pi by 2 minus angle C prime O prime E prime. So, this can be further written as limit del x tends to 0 del y tends to 0 pi by 2 this pi by 2 remains minus.

Now, C prime O prime E prime we can write what we can write C prime O prime E prime we can write pi by 2 minus the angle say theta 1, right. So, that is nothing, but del v del x delta x by delta x. So, what is that angle? So, we are just measuring this angle. So, here we are writing because theta is very small. So, we are writing tan theta equal to theta. So, this angle if I say theta 1, so theta 1 is nothing but tan theta 1 is nothing but del v del x delta x right this one by over the length delta x right. So, this I can write like this, minus theta 2 I can write del u del y delta y by delta y. So, this gives me this gives me gamma xy equal to if you simplify this thing you will be simply getting del v del x plus del u del y.

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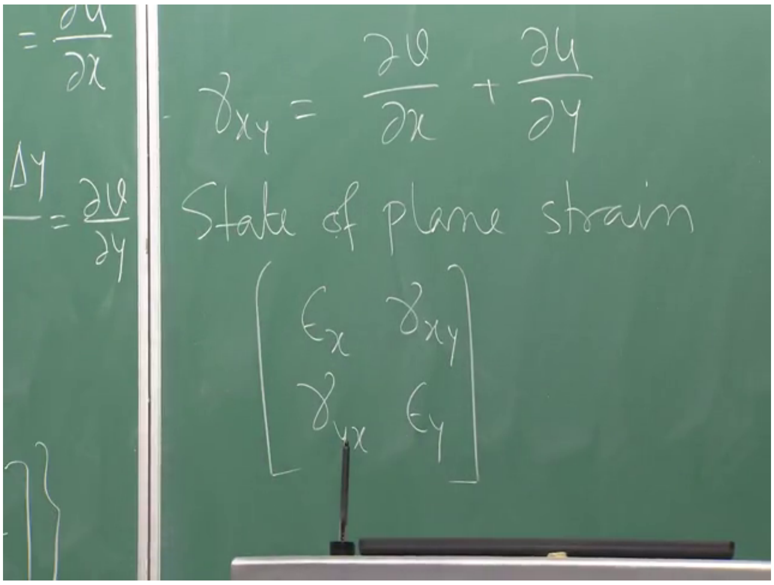

So, all strain components depend linearly on the derivatives of the displacement components if you look at epsilon x is equal to del u del x linear I mean linear derivation derivative of u along x direction epsilon y also is equal to del v del y that is a linear derivative and gamma xy is given by this. So, that is so strain components are dependent on the linear derivative of the displacement components. So, therefore, your state of plane strain, if you want to define the state of plane strain which is pretty similar to your state of plane stress state of stress we define right around at a point we define the state of stress in case of plane stress right.

So, the state of plane strain can be defined as epsilon x gamma xy, gamma yx and epsilon y. So, this is your state of strain in case of plane strain in xy plane. So, you need three I mean I mean again you can prove the gamma xy is equal to gamma yx and ultimately it will be coming down to three strain components epsilon x, epsilon y and gamma xy.

So, I will stop here today in the next class or the next lecture we will be trying to find out the transformation of strain; that means, if you know the state of strain in x y plane can you know or can you find out the state of strain in some arbitrary coordinate systems like x prime y prime whatever we did for our stress.

Thank you very much.