**Mechanics Of Solids**

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**Lecture – 24**
**Principal Stresses and Introduction to Concept of Strain**

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall we just talked about the construction of the Mohr circle and I hope that you have understood the process by which you can construct the Mohr circle because this is very very important this is not only for the stress even for the strain will gets similar kind of construction graphical construction that is Mohr cycle of strain. So, and sign convention all those things I hope that you have understood quite well.

So, now few things few things we can extract and few important information we can extract from this Mohr circle.

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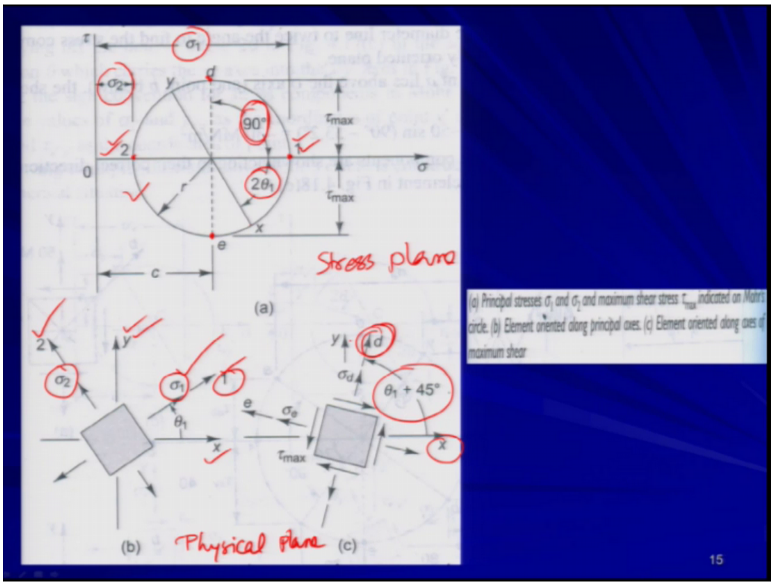


Now, if you come back to this figure in this figure. So, as I told you this is your physical plane and this is your stress plane as per our definition. So, now, in this figure few things. So, suppose I have constructed one more circle like this. So, this is my Mohr cycle. So, I have constructed by following the steps whatever we have discussed in the last lecture by following those steps we have constructed this Mohr cycle

Now, after constructing these Mohr cycle suppose if you look at these 2 points - point 1 and point 2 there is a specialty about these points. What is the specialty of these points? If you look at point 1 and point 2 point 1 is giving me the maximum possible normal stress that is nothing, but sigma 1 say and point 2 is giving me the minimum possible normal stress. So, in that particular say system, whatever orientation you consider. So, because you have xy coordinate system you can have infinite number of coordinate axis because if when you are defining the state of stress at a point you can have infinite number of planes at that particular point. So, in doing so actually when you are searching or when you are constructing infinite number of planes basically on some plane you will be getting maximum possible normal stress and another plane which is 90 degree away from that plane

So, that plane will give you minimum possible normal stress, but there will be no shear stress. So, if you look at point 1, point 1 is lying on the sigma axis similarly point 2 is also lying on the sigma axis. So, these 2 points will give you point 1 and point 2 will give you maximum possible normal stress and minimum possible normal stress respectively, but without any shear stress right. So, this is I mean in the mechanics this is known as principal stresses and the plane on which the principal stresses are acting those planes are known as principal planes this is very very important in mechanics.

So, several times and frequently will be using these terms principal stress or principal plane right, you must know what is principal stress. So, principal stress is nothing, but the maximum normal stress or minimum normal stress and the principal planes means on the plane only normal stress is acting no shear stress. So, now if you look at this points point 1 and point 2 we can simply write sigma 1 if you look at the figure sigma 1 is nothing, but the maximum possible normal stress and henceforth it will be defined as major principle stress.

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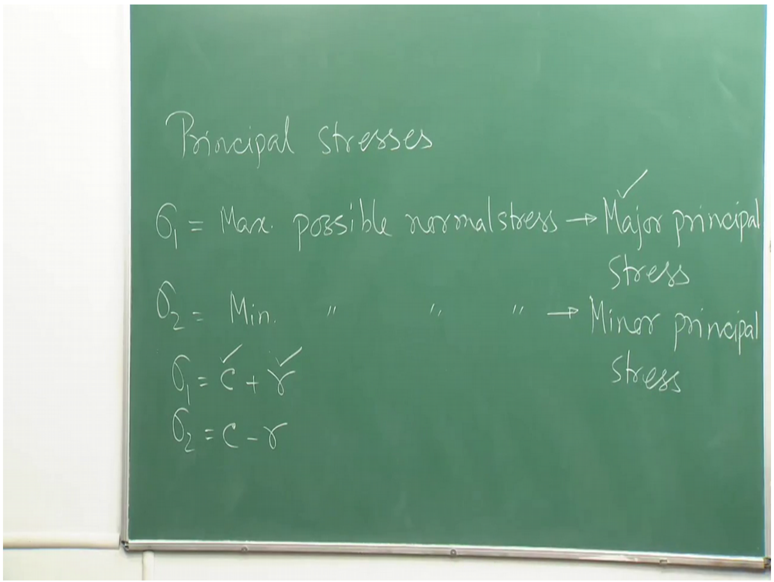


Similarly sigma 2 is the minimum possible normal stress and henceforth it would be defined as minor principal stress. So, you need to remember this thing. So, when I am talking about major principle stress what does it mean and when I am talking about minor principle stress what does it mean. So, major is the maximum possible normal stress and minor is the minimum possible normal stress.

So, as per our definition whatever we have defined earlier in terms of radius and axis of the center of the Mohr circle I can simply write sigma 1 is equal to abscissa of the center of the Mohr circle class radius and sigma 2 is c minus r. So, if you known the magnitude of the radius of the Mohr circle and if you know the abscissa of the Mohr circle or abscissa of the center of the Mohr circle then basically you can find out the major and minor principal stresses.

Now apart from that one more information we are getting and that is also very important because I mean why do we need all those things why do we need to find out the principal stress or other things. Similarly we will find out the maximum shear stress right because later on we will be discussing about the failure theories or failure criteria there you will be appreciating that these stresses if you do not know about these stresses then basically those failure theories will not be understood properly, because those failure theories are developed or based on these important stress, parameters principle stress, maximum shear stress and all those things.

So, for that you should know all these parameters now next important thing is that if you. So, now, we are coming to another 2 points this point d, another 2 point - point d and point e. Now before going to that now whatever we have developed for principle stress sigma 1 and sigma 2 in the stress plane now we are going to transfer that thing to the physical plane. So, this is your x y axis now what is the angle between because this is the angle twice of theta 1 is the angle between the x point and one point; that means, in the stress plane if it is making twice of theta one when you are transferring that thing in the physical plane. So, the x axis and major principle stress axis one will be making an angle theta one simply. So, that you are getting here.

So, this is my xy coordinate system this is the major principle stress axis and this is the minor principle axis and sigma 1 is acting normal to the major principle plane and sigma 2 is acting on the minor principle plane. There is no shear stress on this plane, so that is the specialty of this principle planes. Now as we are talking about another 2 points a point d and point e. Now what is the specialty of these 2 points let us talk about that. Now if you look at these 2 points point d and point e here actually your normal stress is same, what is the value of the normal stress? Abscissa of the center of the Mohr circle your normal stress is same on those 2 planes because d and e both will be defining 2 planes in the physical plane.

So, normal stress is same, but they are talking about or they are giving the information about the maximum shear stress. So, at point d or point e you will be getting the maximum shear stress.

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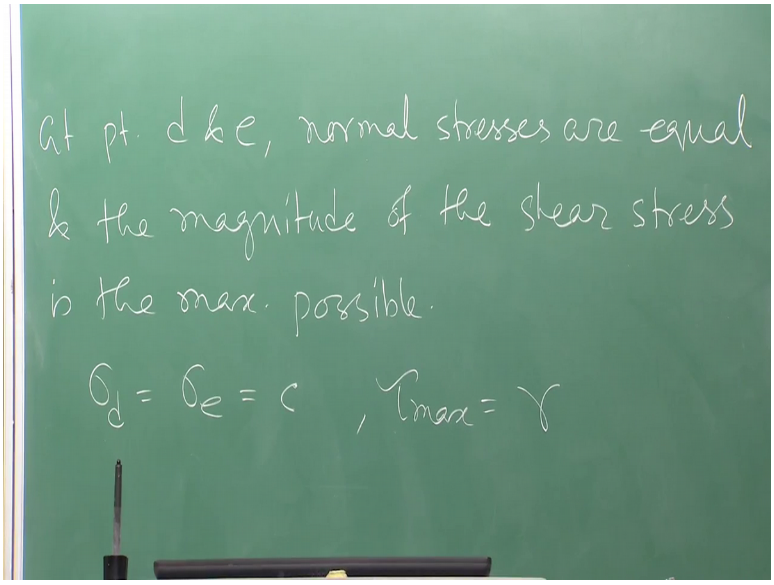


So, that is why this at point d and e normal stresses are equal and the magnitude of the shear stress is the maximum possible. So, now, what is the angle between x point and d point? That is twice theta 1 plus 90 degree. So, that is happening in the stress plane. So, in the physical plane what should be the angle simply theta 1 plus half of that theta 1 plus 45 degree. So, this is the x axis and this is your d axis and that is making an angle theta 1 plus 45 degree. So, virtually if you look at. So, what we can conclude that whenever you are, you have defined the principal axis major principal axis or minor principal axis. So, your maximum shear stress direction or shear stress axis is making an angle 45 degree with the major principle stress axis.

So, because this is the angle 90 degree between point 1 and point d in the stress plane. So, in the physical plane the angle between one axis, one axis and d axis will be simply 45 degree. So, that is very very important. So, once you look at the principal stress axis or the major principle stress axis immediately you can define by drawing an axis at an angle 45 degree and that will be defining the maximum shear stress axis.

So, now with this what we can write for this sigma d is that is the normal stress on plane d that will be similar to sigma e and which is nothing, but the abscissa of the center of the Mohr circle and what is the value of tau max? That is the maximum shear stress that is simply the radius of the most circle if you look at the figure you will get it simply the radius of the Mohr cycle. So, if from the Mohr circle from the construction of the Mohr circle if you can find out these 2 parameters you can define all those important things. Now it can be proved though the proof I am not showing, it can be proved that I mean you can ask me that I am getting 2 principal stresses right if you consider three dimensional state of stress that is general state of stress three dimension if you consider x y and z. So, you will be getting three principal stresses one is major principle stress that is the maximum possible, another one is the minor that is a minimum possible and one is in between that is the intermediate principle stress.

So, you will be it can be proved that you will not be getting four principal stresses or five principal stress or something like that you will be getting three principal stresses. So, I am not I am not going to show the proof anyway. So, if you want to find out that proof then basically you have to refers some adverse solid mechanics book there you will be getting that proof anyway. So, in three dimensional or the general state of stress you will be getting three principal stresses major, minor and intermediate principal stress.

Now if you look at this figure I am considering say x y z coordinate system.

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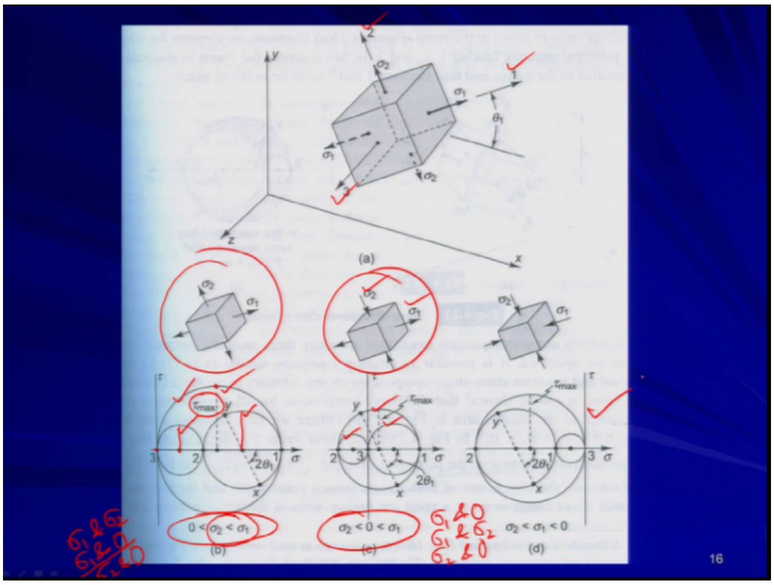

So, as I told you you will be getting three principal stresses; that means, three principal axis you will be getting 1 2 and 3 as shown in the figure this is the major principle stress axis, this is the axis 2 and this is axis 3.

So, 1 2 3 these are all principal stress axis and this is still under I mean says your sigma three is always say 0 in this case say for the example now if I consider this configuration where sigma 1 and sigma 2 both are acting and both are intension in nature and we are satisfying this condition where sigma 2 is greater than 0 and sigma 1 is greater than sigma 2. So, in that situation, if I consider this combination sigma 1 and sigma 2 I will be getting. So, if I consider this combination sigma 1 and sigma 2 if I consider this combination I will be getting this Mohr circle. If I consider sigma 1 and 0, 0 means I do not have any stress any normal stress on that particular plane; that means, on plane three we do not have any normal stress so, but it is also free from shear stress so; that means, it is it is virtually one of your principal plane.

So, if I consider this combination sigma 1 and 0 then basically I will be getting this Mohr circle the bigger one because there this is the point I mean which will give me the sigma three and in this case particularly sigma 3 is 0. Now if I consider sigma 2 and 0 then I will be getting this circle right, but whatever is the case my tau max is this that is the maximum possible shear stress do not get confused that this should be your maximum tau max I mean this would be or this should be your max I mean maximum shear stress know the maximum shear stress will be always associated with the outer circle. Similarly if I consider this combination where sigma 1 is tension and sigma 2 is under compression and this is the criteria which we are going to satisfy that is I mean sigma 2 is lesser than 0 and sigma 1 is greater than 0; that means, your sigma three is 0 in this case also. So, when we are considering this combination sigma 1 and 0 then basically I am getting this circle, when I am considering sigma 1 and sigma 2 I am getting the bigger circle and when I am considering sigma 2 and 0 I am getting this circle. So, likewise I am getting this construction.

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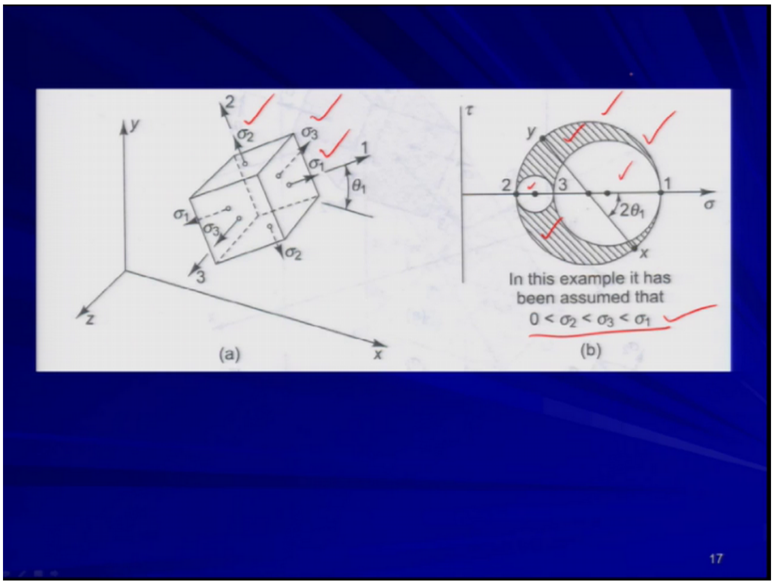


Now if you get all the principle stresses or if you have all the principle stresses present in the system sigma 1 sigma three sigma 2 all principle stresses major minor as well as intermediate principle stresses they are present then basically you will see and if you suppose for example, for a particular or special construction we are making or we are considering this condition that is sigma 1 is greater than sigma 3 is greater than sigma 2 is greater than 0. If this condition is satisfied then I will be getting the Mohr cycle construction like this. If you consider sigma 1 sigma 3 sigma 2 sigma 3 and sigma 1 sigma 2 something like that and then you will be getting three Mohr circles like that.

So, this will give you the general state of stress and it can be proved again I am not showing that proof it can be proved that whatever possible stress I mean situation or state of stress is possible in this particular say combination they will be lying only within this shaded zone. You cannot get inside this or inside this or outside this bigger circle it will be always lying on the shaded zone. So, in the 3D state of stress it can be proved that the state of stress will be always lying in the shaded zone.

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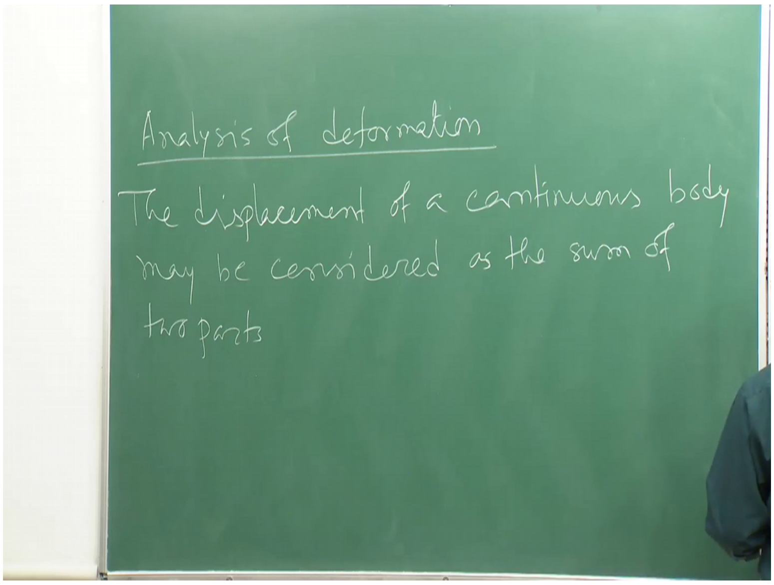


So, with this I can say that we have we have discussed enough about the state of stress and stress and plane stress condition now we will move to the strain that is another important thing we should know and let us talk about strain or the deformation of a model. Analysis of deformation - The displacement of a continuous body may be considered as the sum of 2 parts which are those first one is a translation and or rotation of the body as a whole and that is generally called as rigid body motion.

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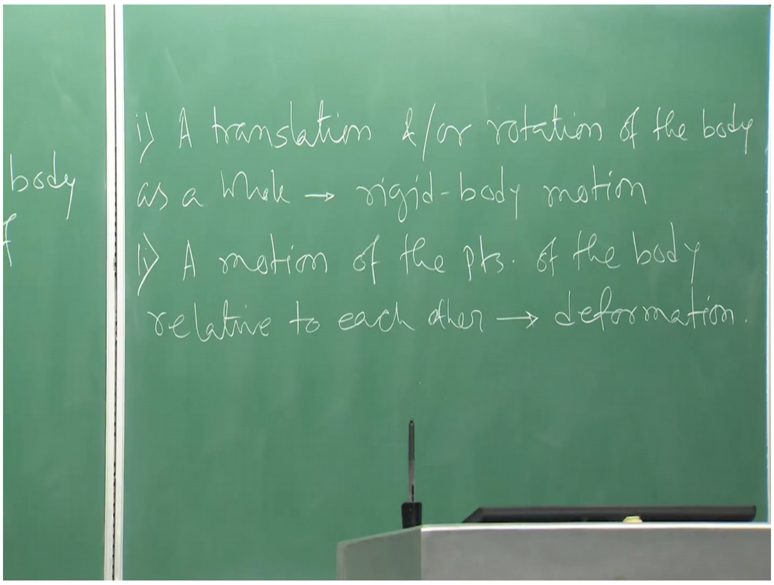

And the second one is a motion of the points of the body relative to each other that is called deformation. That means, when we are talking about the displacement of a continuous body. So, if I consider a continuous body there is no gap, there is no crack, there is no fracture. So, we are considering one continuous body.

So, displacement of a continuous body may be considered as the sum of these 2 parts the first part is a translation and or rotation, translation as well as rotation or translation or rotation whatever of the body as a whole; that means, if this is the body I am just taking here form I am just translating this body from this point to this point. So, this is known as rigid body notion motion. So, we are not going to discuss this thing because we are interested to find out the deformation.

The second part is a motion of the points of the body, motion of the points of the body related to each other; that means, if you consider a body and then you apply the external, apply external forces and then you just I mean see a look at the how the relative movements are happening inside the body particles. So, that is known as deformation.

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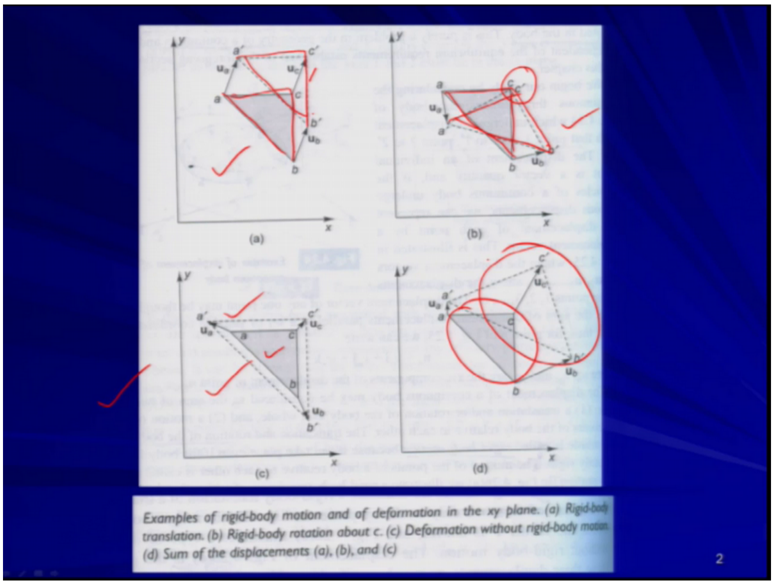


So, we are interested to talk about this. Now before that if you look at this figure then basically this first one is giving me the rigid body translation. So, this is the actual body, this is the actual body now after translation this has taken this position this is rigid body translation no deformation is happening no relative movement in the body parts. If you look at this, this is giving you the rigid body rotation with respect to point c. So, initially it was like this and now it has taken this position. Now this part is giving me deformation without any rigid body motion. So, this was the body initially and after deformation. So, that is getting enlarged. So, this is the bigger shape of the body. So, body has got enlarges after deformation. So, this is called deformation without any rigid body motion.

Now if you have all these things together; that means, rigid body translation rotation as well as this deformation together then basically your initial configuration of this body will be taking the final configuration like this. So, that is actually happening if you consider both rigid body motion as well as deformation, but for the time being we are just concentrating on the deformation part and let us talk about the deform how a body will behave under deformation and if there is any deformation particularly under the action of the externally applied forces then how to quantify those deformations. So, here also in this particular thing or particular chapter we are not going to discuss the 3D state of strain.

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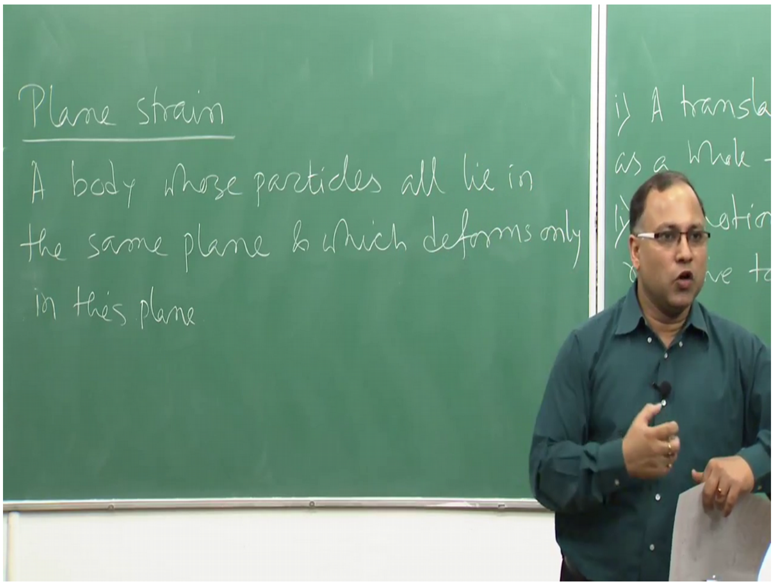


So, deformation means basically deformation can be quantified by the strain right. So, we are going to define all the plane strain like whatever we have discussed as plane stress, we are going to discuss the plane strain. What is plane strain? A body whose particles all lie in the same plane and which deforms only in this plane right. Suppose if you consider a long wall. So, you will be getting the deformation only if you consider the long direction is your z direction or the say y direction, you will be getting the deformation only in the x z plane.

So, y direction along the y direction the strain or the deformation is negligible. So, that kind of thing long cylinder or long say shaft or long member like beam of the building or the or the long say dam kind of thing. So, these all these structures are generally showing will be showing the deformation only on that particular plane. So, out of the plane deformation is not there. So, that kind of situation is known as plane strain situation of the plane strain problem.

So, I will stop here today. So, the next class will be continuing with the deformation chapter.

Thank you very much.