Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 23
Construction of Mohr's Circle

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall, we have derived these three equations.

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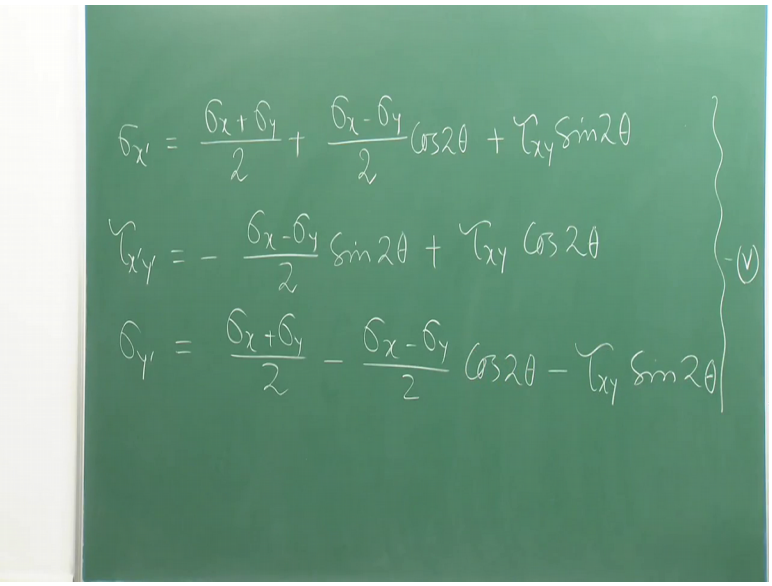

Sigma x prime tau x prime y prime and sigma y prime; that means, we are going to find out the state of stress in some rotated coordinate system x prime y prime with respect to xy coordinate system. So, if you know the state of stress in xy coordinate system that is sigma x sigma y and tau xy if they are known to you and if you know that what is you rotated coordinate or the transformed coordinate system x prime y prime; that means, if you know the magnitude of theta. So, from these three equation basically you can find out the unknown stress component sigma x prime, sigma y prime and tau x prime y prime. So, these are the things we covered or we have derived rather in the last lecture.

And then we are discussing some graphical representation of these equations. So, there we introduced the Mohr's circle and we initiated the graphical construction.

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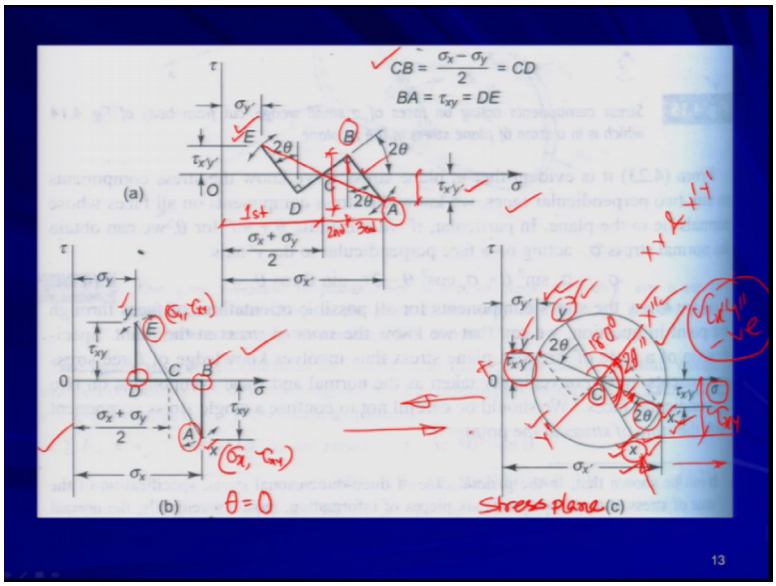

So, if you come back to this figure again. So, whatever figure we saw or whatever figure or whatever graphical construction we initiated in the last lecture. So, if you see this equation the first equation right. So, if you see the expression for sigma x prime the first term of this equation can be given by this, right. So, this we have seen. So, this is your first term.

And, the second term of the same equation sigma x minus y by 2 cos 2 theta that is nothing but the horizontal component of this C B, so that we have seen. So, this is your second term and subsequently the third term will be the horizontal component of B A right and I am not going to repeat again at how you have got these graphical construction because that in the last lecture we covered that.

So, what we have got from this graphical construction that sigma x prime is nothing, but the distance O A which is nothing, but the abscissa of point A right in this graphical construction if you follow the graphical construction. So, if you if you measure that the distance O A or the abscissa of O A point. So, that will give you the magnitude of sigma x prime. Similarly if you come back to the second equation that expression for sigma tau x prime y prime the first term if you look at, the first term is nothing, but the vertical component of this B C this one this is the vertical component of B C because B C or C B whatever you say that is given by this expression. So, that is a vertical component and

the last or the second term that is tau xy cos 2 theta. So, that is nothing, but the vertical component of this B A right.

So, the vertical component of B A if you detect for this part from the vertical component of B A you will be getting the y axis of point A that means this is. So, this is nothing, but tau x prime y prime. So, what you have got from this graphical construction that if you know the save coordinates of A point then you will be knowing the stress components sigma x prime and tau x prime y prime. So, similarly this E point if you look at the last equation the expression for sigma y prime if you follow the same logic. So, you will be getting the point E where the abscissa A point E will give you sigma y prime whereas, the y axis of E point will give you tau x prime y prime. So, E point basically will tell you sigma y prime tau x prime y prime; that means, the stress on y prime plane.

Now, if your theta is becoming 0 if your theta is becoming 0 then virtually you should get sigma x prime is nothing, but sigma x sigma y prime is nothing, but sigma y and tau x prime y prime is nothing, but tau y because if theta is 0; that means, x prime axis is coinciding with x axis and y prime axis will coincide with y axis right. So, virtually you should get sigma x sigma y and tau xy if theta becomes 0 from the equation also you can you can check it. So, whether that thing we are getting it or not for that we are doing the same graphical or we are basically making theta equal to 0 here in the graphical construction.

So, if we make theta equal to 0. So, B point this B point will be coming on the sigma sigma axis. So, that is my B point, similarly the D point will rotate and will go to sigma axis right and here is your E point and here will be your A point again if you look at this graphical construction. So, A point coordinates will be giving you sigma x tau xy whereas, your E point coordinate will give you sigma y. So, A point coordinate will give you sigma x tau xy and E point coordinate will give you sigma y tau xy right you can check it by following the same logic whatever we are we are discussing.

So, what you are getting from this, this x point basically or this x point or the other A point this A point is basically representing the state of stress on x plane where as your E point or y, y point both are saying. So, E point is basically representing the state of stress on y plane right in this figure whereas, in this figure you are A point is representing the state of stress on x prime plane and E point is representing the state of stress on y prime

plane now I am drawing one circle considering C as the center x y. So, in from this construction we are coming to this construction. So, considering x y as a diameter and C as the center I am drawing one or I am constructing one circle.

So, that this x point will basically define the state of stress on explain y point will basically define the state of stress on y plane as we have got in this figure in this figure right. So, in this circle if you look at and if you join A C E now, those points basically A C E point if you join A C E something like that that will be this line which is nothing, but this point is x prime point and this point is y prime point which are basically this x prime point is nothing, but sigma x prime the coordinates of x prime point is sigma x prime tau x prime y prime and similarly the y prime point coordinates are sigma y prime and tau x prime y prime. So, and the angle between these 2 lines basically is twice of theta the angle between x y and x prime y prime line x y and x prime y prime line the angle between these 2 lines these 2 diameters rather is twice of theta.

So, now what you are getting? So, I am just constructing I am just representing or I am just putting these equations in the graphical construction and from the graphical construction what I have got what are the informations I am getting the first information is that this x point and y point if you know the x point and y point which are diametrically opposite then I will be knowing the complete state of stress in xy coordinate system; that means, if I know the location of point x and y I will be knowing sigma x sigma y and tau xy. Similarly if I know the location of point x prime and y prime then I will be knowing the state of stress in x prime y prime coordinate system that is sigma x prime sigma y prime and tau x prime y prime. So and how did I get it, how did I get this x prime y prime line? Just following the same sense of rotation, see as we have seen in the last class this is your xy axis and this was you x prime y prime axis and this angle was theta. So, this is my physical plane.

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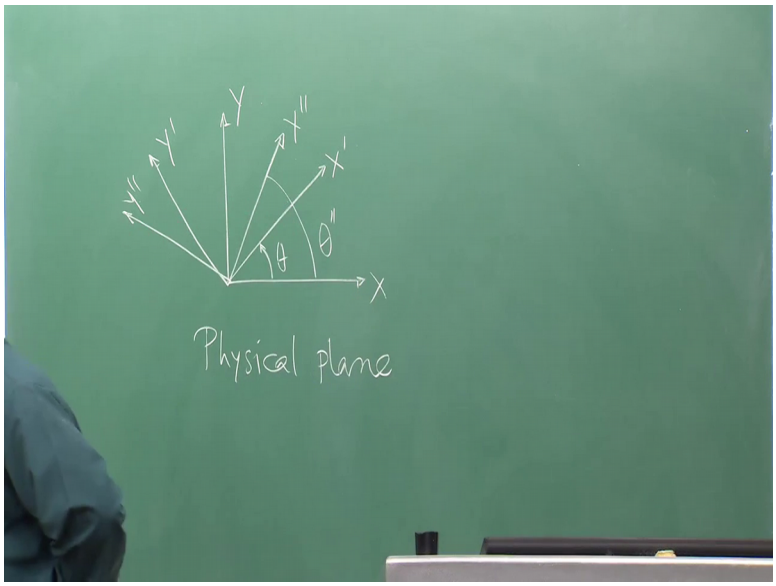

So, I will now onward I will refer this as your physical plane or as this will be your stress plane. So, this physical plane we are going to reproduce that thing in the stress plane and frequently will go from stress plane to the physical plane. So, will go to and pro from physical to stress, stress to physical plane. So, now, what you are getting if you know the state of stress in x y coordinate system; that means, if you know the point x and point y then by using the same sense of rotation; that means, if you look at from x axis the x prime axis is basically anti clockwise direction rotation theta.

So, that is the angle between these 2 axis. So, that is in the anti clockwise rotation anti clockwise direction. So, if we follow the same sense of rotation; that means, anti clockwise twice of theta I will be getting x prime y prime line which will be giving me the state of stress in x prime y prime coordinate system. Likewise you can have infinite number of points on this circle, infinite number of points on this circle which will give you the state of stress on infinite number of coordinate systems with respect to the x axis. I can have one more diameter I mean something like that. So, this is my x is a double prime this is my y double prime say and the angle between x and x double prime say theta double prime twice theta double prime. So, I can have one more say coordinate system x double prime y double prime and this angle is, this angle is theta double prime.

So, if I know the state of stress in x y coordinate system again I can find out the state of stress x double prime y double prime coordinate system from this graphical construction.

Likewise I can draw in number of or I can consider infinite number of points on the circle and if I draw the diameter then I will be getting the state of stress in that particular coordinate system. So, this is the ad one and this is known as Mohr's circle of stress, Mohr's circle of stress.

So, this is very handy and very helpful you will you will find it out I mean once you construct the Mohr's circle any diameter you consider you will be getting the state of stress in that particular orientation of coordinate system. Now one thing; obviously, is coming to your mind the actual angle between x and x prime is theta in the physical plane, but when I am transferring that thing to the stress plane it is taking twice of theta because for set, I mean satisfying these equation basically ultimately that angle is coming twice of theta similarly the angle between x axis and x double prime axis is theta double prime in physical plane, but in the stress plane; that means, in the Mohr's circle that is taking twice of theta double prime.

So, what conclusion you can draw. So, in the physical plane whatever angle will be there that angle will be simply twice in the stress plane because this xy point whatever you are getting here this x and y points what does it mean these 2 points, what basically they are representing, they are representing 2 planes - x plane and y plane. So, x point is representing explained and y points is basically representing in the y plane in the physical system or the physical plane right. So, actually in the Mohr's circle the angle between x y is 180 degree this is the angle. So, this is the angle 180 degree, but actually in a physical plane what is the angle between x y - 90 degree. So, this 90 degree is taking the value of 180 degree in the stress plane.

So, we can conclude that whatever angle we are getting angle means angle between the axis, whatever angle we are getting in the physical plane that will be simply twice in the stress plane. So, now, the obvious question is coming in our mind that here x is downward, here x prime is also downward - downward means below the sigma axis, but here x double prime is going above the sigma axis. Now are the same I mean sense of a more the sign convention and all those things are same or they are different. So, we need to define the proper sign convention for our, this graphical representation.

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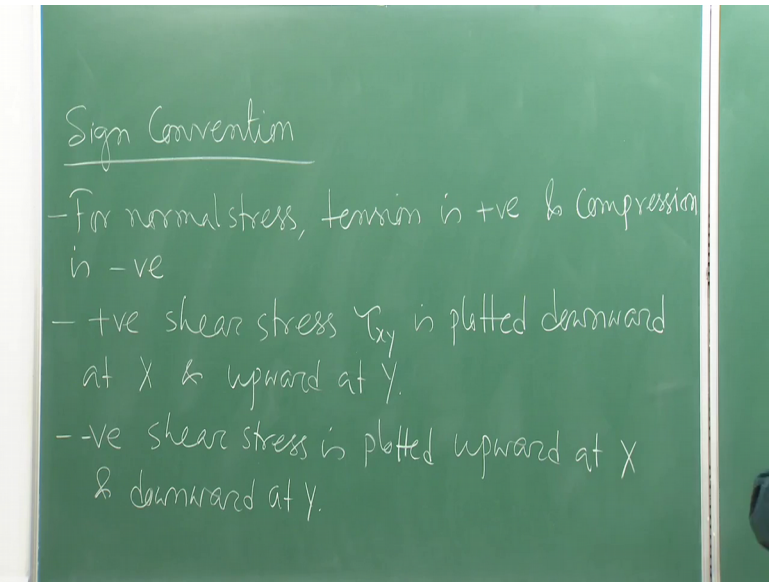

So, let us define the sign convention first - for normal stress tension is positive and compression is negative. So that means, we whatever sine convention we are going to use in the stress plane basically if your normal stress is tension then that will be shown in the. So, because this is your normal stress axis x axis is your normal stress axis. So, if the normal stress is tension in nature then that will be positive; that means, that will be plotted in this direction if the normal stress is compressive or the compression then that will be protect in the negative sigma direction fine. So, there is no ambiguity in the or there is I mean that is very simple actually. So, the sign convention for the normal stress is pretty simple. So, if it is tension positive it is compression negative.

Now, the complexity is involved in the sign convention of your shear stress. So, positive shear stress tau xy is plotted downward at x and upward at y and negative shear stress is plotted upward at x and downward at y. So, this sign convention we are going to use for this particular course; however, if you refer different books you will be getting different sign conventions, but we are going to stick to this sign convention and whatever discussion will be going on, those discussion will be based on this sign convention.

Now, if you look at these 2 statements the positive shear stress tau xy is plotted downward at x and upward at a y, what does it mean? So, if your shear stress is positive if your shear stress is positive and I mean how to define the shear stress positive that you know from the earlier discussion that point will be calling sigma x or sigma y or the

normal stress positive or negative when will be saying the shear stress is positive or negative, those discussion already we have made. Now if your shear stress is positive then that will be plotted downward at x; that means, downward of x means below the sigma axis. So, here now if you look at here, x is plotted downward or below the sigma axis whereas, y is plotted above the sigma axis. So, here whatever tau xy you are getting for this points. So, this is your tau xy. So, that is positive if x point is coming below the sigma axis then tau xy is positive fine and of course, if x is coming below the sigma axis y has to be above the sigma axis that is the meaning of the second statement.

Now what is the meaning of the third statement? If the shear stress is negative then it is plotted upward at x and downward at y. Now, if you come back to this figure whatever already we have shown. So, x here, x point is here x point is below sigma axis and y is above sigma axis. So, my tau xy is positive. Now what about x prime point? So, x is getting mapped to x prime and y is getting mapped to y prime. So, now, instead of thinking of x now we are going to think about x prime point which is basically similar to x. So, now, x prime point is coming below the sigma axis and y prime point is coming above the sigma axis therefore, this tau x prime y prime is positive for me, clear.

Now, if you look at x double prime y double prime, x double prime y double prime now x double prime is I mean x is getting mapped to x double prime right, but in the stress plane x double prime is plotted above the sigma axis therefore, tau x double prime y double prime is negative as per our sign convention as for our third statement is that clear sign convention. So, I hope that you will not between any mistake in the sign convention your sign convention is very tricky actually you can have your own sign convention, but this is sign convention we are going to follow in this particular course throughout the discussion.

So, now with this we are going to write down the stepwise the construction of the Mohr's circle, so how you can construct the Mohr's circle. So, this is the discussion regarding the Mohr's circle, how Mohr's circle is getting evolved from the algebraic equation whatever we have got earlier, we have derived some algebraic equation, right. So, from that we have got this circle, right.

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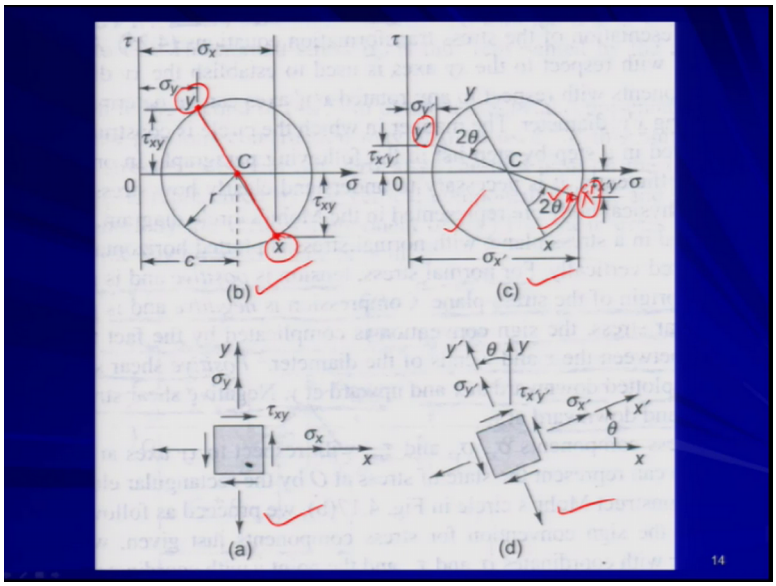

Now let us talk about the construction of the Mohr's circle. Now we are going to list down the steps involved in the construction of the Mohr's circle.

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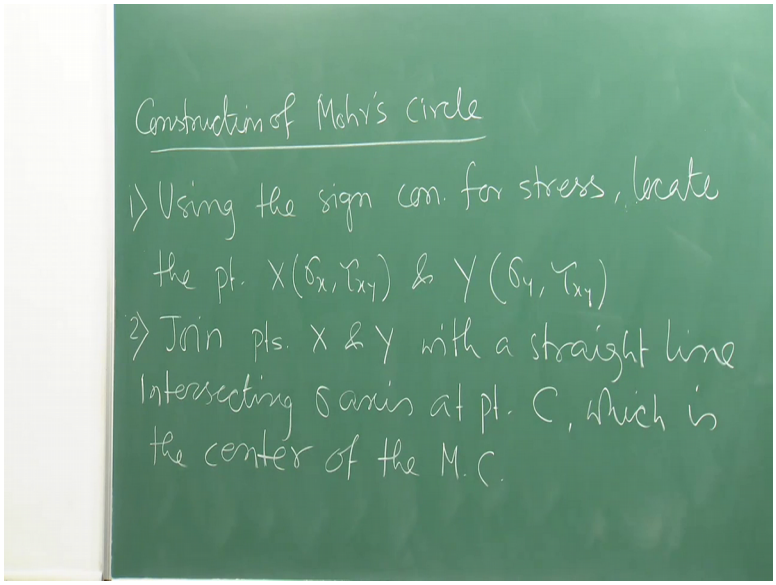

So, the first step using the sign convention just now whatever we have defined for stress locate the point x and y. So, the coordinate of x is sigma x tau xy and coordinate of y is sigma y tau xy because these three things, three stress components are known to you sigma x sigma y and tau xy they are known to you. Now Mohr's cycle will give you the

information about the stresses on stress components on other planes right other coordinate system. So, using the sign convention for stress locate the point x and y.

Now if your sigma x is positive sigma y is positive both sigma x and sigma y are positive then that will be plotted on the right side of the y axis as simple as that if they are negative then they will plotted on the left side of the y axis if one of them is positive then accordingly it will be plotted on the sigma axis. Regarding tau xy if it is positive then x point will be coming below the sigma axis the possibility could be because what you are getting. So, you are just thinking about sigma x and sigma y right.

So, once you know sigma x and sigma y and as well as tau xy if you do not know the sin of tau xy x point maybe below the sigma axis may be above the sigma axis, but if tau xy is positive then; obviously, as per our sign convention sigma I mean x point will be coming below the sigma axis. Similarly y point will be going above the sigma axis if your tau xy is positive let us tau xy is positive.

So, I am locating x point and y point in the sigma tau space now what is my next step next step is join points x and y with a straight line which is intersecting sigma axis at point C which is nothing, but the center of the Mohr's circle, is not it. So, what we are doing? So, after identifying these 2 points x point and y point we are joining these 2 points with a straight line and that straight line is intersecting the sigma axis at point C and that is eventually your center of the Mohr's circle.

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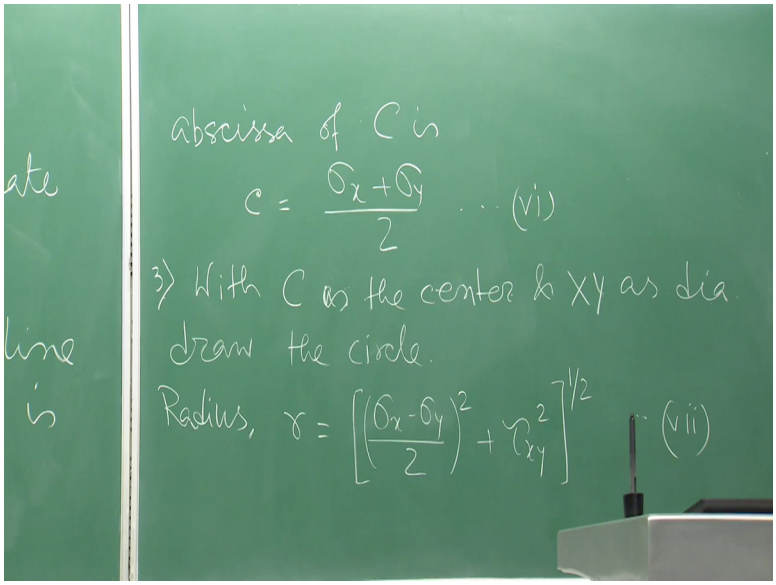

So, once you know the center of the Mohr's circle then your abscissa of point C, abscissa of C is C can be calculated because you know the coordinate coordinates of x point and y point. So, therefore, you can find out C abscissa of C is simply sigma x plus sigma y by 2 6, equation 6.

Now with C as the center and xy as the diameter I am drawing the Mohr's circle. So, with C as the center next step is with C as the center and xy as diameter draw the circle. So, radius of the circle is, radius of the circle is you can calculate that will be coming as sigma x minus sigma y by 2 whole square plus tau that comes from the coordinates of x and y points to the power half. So, this I am following equation 7.

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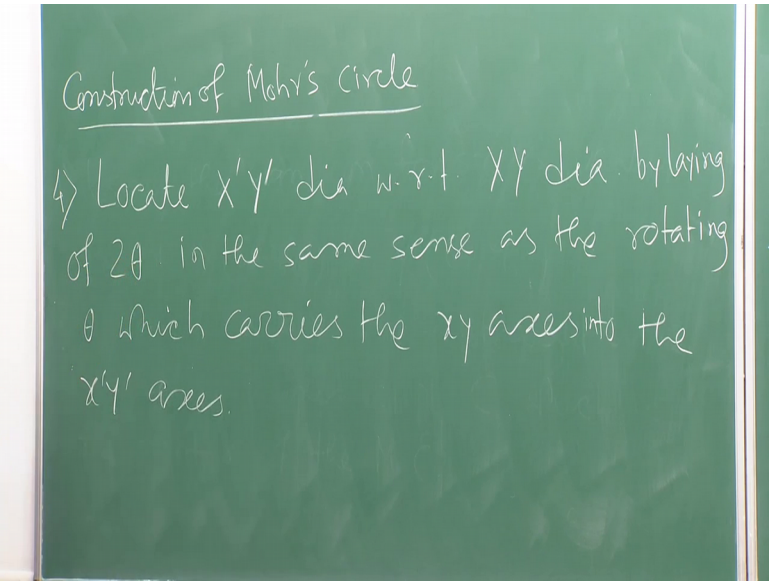

Now, what is your next step? What is your objective? Our object is to find out the state of stress in x prime y prime coordinate system from the Mohr's circle. So, from the known state of stress in xy coordinate system then my next step is locate x prime y prime diameter with respect to xy diameter how you look at it by laying off twice theta as shown in the figure. If you look at this figure, so after construction of the Mohr's circle this is your Mohr circle, now you are locating the x prime y prime diameter in the same sense as the rotation theta which carries the xy axis into the x prime y prime axis; that means, in the physical plane your angle between x axis and x prime axis is theta now you are when you are coming to the stress plane that will be considered as twice as I told you.

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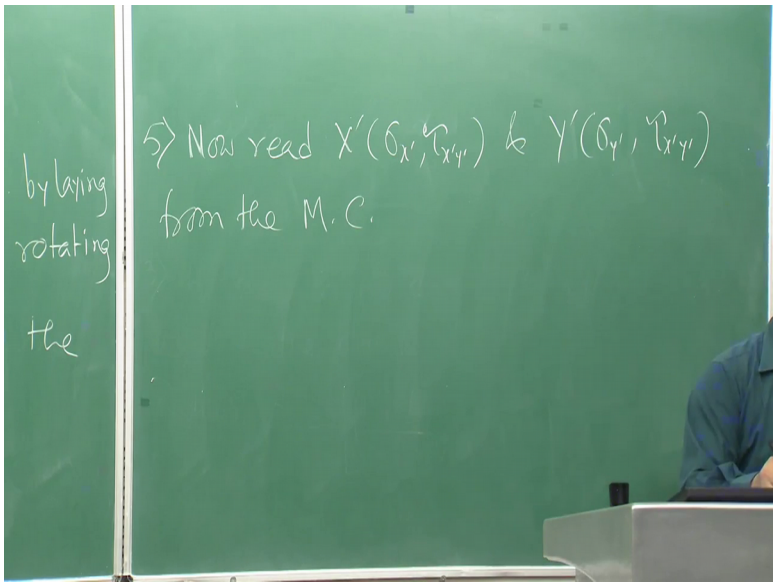

Now, next step is now read x prime point and y prime point from the Mohr's circle. So, this is your stress plane, this is your physical plane. So, what did I do? So, after locating xy coordinate, I mean diameter then I am getting x prime y prime diameter. So, these 2 things I have located, now after locating these 2 things I am just what I am doing if I draw this Mohr's circle as per our scale then basically I can simply measure or I can calculate from the algebraic equation. So, the coordinates of x prime point will give you sigma x prime tau xy prime and coordinates of y prime point will give you sigma y prime and tau x prime y prime.

So, in the Mohr's circle we can use the scaled construction; that means, everything to the scale if we do everything to the scale then basically we can simply measure from the Mohr's circle and we can get the value of the stresses otherwise we can use the algebraic or the mathematical equations to get all the values. So, if we solve some couple of numerical problems it will be clear to you how we can extract the stress components from the Mohr's circle.

So, I will stop here today. So, in the next lecture will be talking about a 3 D state of stress and three and the construction of the Mohr's circle for the 3 D state of stress or the general state of stress.

Thank you very much.