**Mechanics Of Solids**

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**Lecture - 22**

# Mohr’s Circle representation of plane stress

Welcome back to the course mechanics of solids. So, in the last lecture if you recall, we have derived this 2 equations right by considering the equilibrium condition along x prime direction and along y prime direction.

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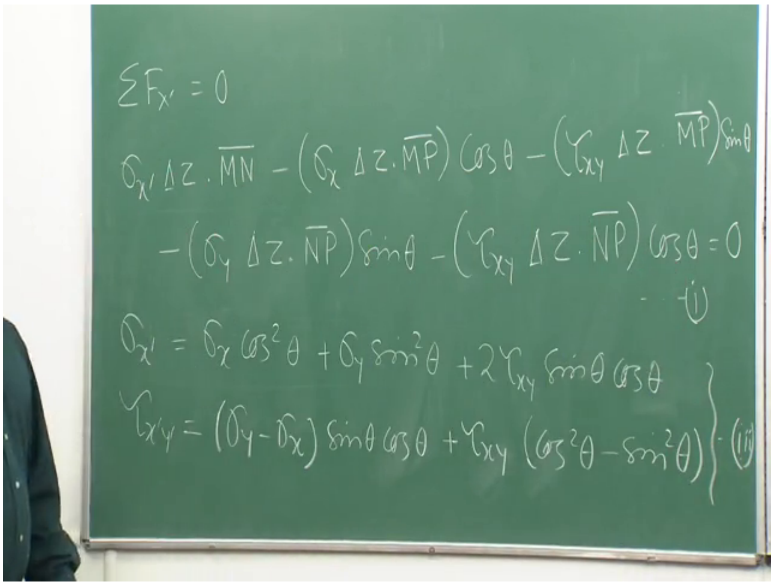


Basically what was our objective? Our objective was to find out the state of stress in some arbitrary oriented coordinate system if we know the state of stress in x y coordinate system. So, this is the equation whatever we have obtained by exploiting or by satisfying the equilibrium condition along x prime direction and this is the equation whatever we have got that is coming after satisfying the force equilibrium along y prime direction.

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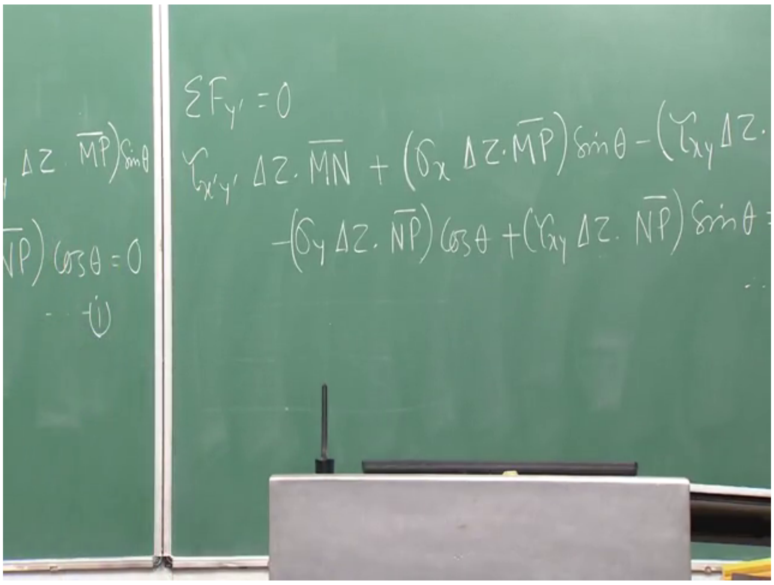


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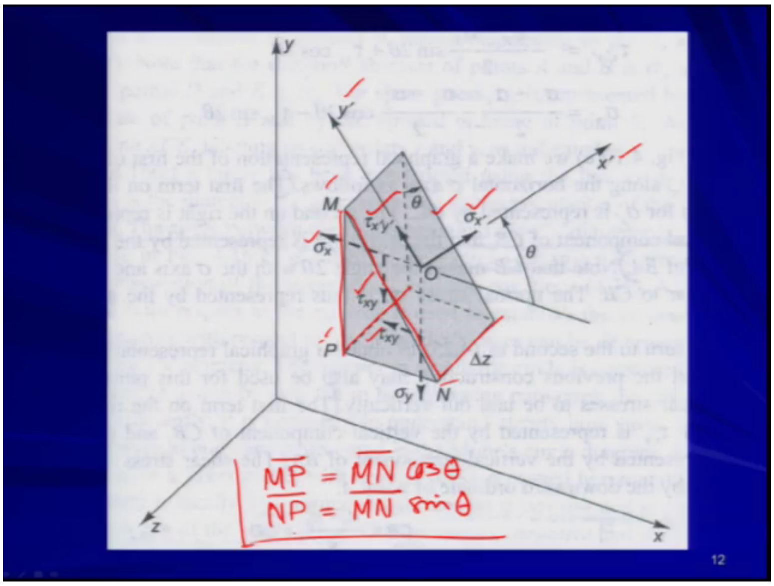


So, now, if you come back to this figure; so, what we can establish that MP that MP is nothing but in terms of the inclined surface MN cos theta. Similarly your NP is nothing, but MN sin theta. So, these 2 relations we are going to include in these 2 equations and these relations are coming from the geometry of the wedge. So, this wedge we have considered in the previous class as you remember. So, now, is in place of MP we are putting MN cos theta and in place of NP we are putting MN sin theta that is coming from the geometry simple geometry. So, if I put that then what I will be getting from equation 1. So, from equation 1 I will be getting sigma x prime is equal to sigma x cos square theta plus sigma y sin square theta plus 2 tau xy sin theta cos theta and from equation 2 what we will get? We will get tau x prime y prime equal to sigma y minus sigma x sin theta cos theta plus tau xy cos square theta minus sin square theta. So, these 2 equations I am calling as equation 3.

So, what do you get sigma x prime in terms of known parameters, sigma x because we started with the known parameters that is sigma x sigma y tau xy and theta because you know the orientation of the arbitrary coordinate system. So, in terms of sigma x, sigma y tau xy and theta you can express sigma x prime similarly the tau x prime y prime can be expressed in terms of sigma x sigma y tau xy and theta again right. Now one more stress component is left out that is sigma y prime. So, how to obtain that?

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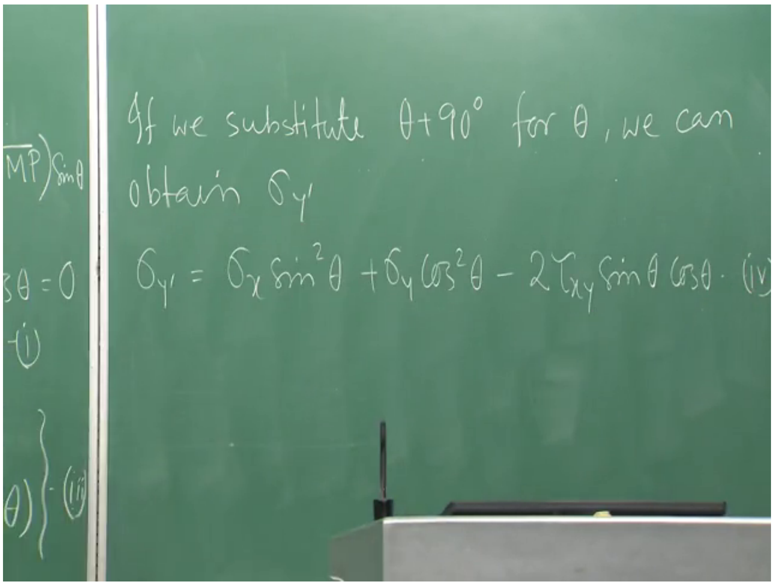


So, if we consider. So, if we substitute theta plus 90 degree for theta we can obtain sigma y prime ok. So, in this figure if you come back, in place of theta we are just putting theta plus 90 degree because we are rotating the axis. So, x prime will go, will take the position of y prime and y prime will go to the position of negative x prime. So, that is something like that. So, if you look at this figure. So, in this process basically you will be getting sigma y prime by simply putting theta plus 90 degree in place of theta listent it that is simple geometry actually. So, what I am doing? I am just rotating the axis coordinate system and I am going to find out the normal stress component sigma y prime.

So, by this process I can get sigma y prime in this equation the first equation of equation three if we put theta plus 90 degree in place of theta I will be getting sigma y prime which is equal to sigma x sin square theta plus sigma y cos square theta minus 2 tau xy sin theta cos theta. So, we will say this equation as equation 4. So, virtually our objective is fulfilled; that means, we have got all three unknown stress components like sigma x prime sigma y prime and tau x prime y prime in terms of known stress components sigma x sigma y and tau xy and the orientation of the arbitrary coordinate system that is theta.

So, now basically if we know the stress components for all possible orientation of phases right. So, you considered a point as we considered here that is point 2, if you considered a point O and if we know the stress components for all possible orientations of phases through this point and then we can say that we know the state of stress act that particular point if we do not know any one of the stress components then we cannot say that we know the complete state of stress at that particular point. That means, if I know sigma x sigma y tau xy in case of course, plane stress condition if I know the sigma x sigma y tau xy all stress components at a particular point then I will be saying I know the state of stress at that particular point because any possible orientation of the coordinate system if you know the orientation; that means, if you know the orientation means you know the value of theta if you know the value of theta you can find out the other stress components right. That is one thing.

Another thing is that you should not be confused with the state of stress and the stress component sigma x sigma y tau xy all these are stress components sigma x is one of the stress components, sigma y similarly is one of the stress components and tau xy. But collectively sigma x sigma y and tau xy collectively that will talk about the state of stress at that particular point. So, do not get confused dealing during this course, I mean frequently I will be telling stress component or state of stress. So, stress component means one particular stress I mean parameter, but when I am talking about the state of stress; that means, I should I know all the parameters all the stress components at that particular point.

So, now with that whatever we have got from equation three and equation four from there we are going to establish some important and very handy and helpful graphical representation of the state of stress and that is known as Mohr circle representation. So, that was proposed by scientist Mohr and that that is found very handy and very helpful actually in the analysis of your state of stress without going, to going I mean through the rigorous analysis or rigorous calculations like rigorous mathematical calculations you can very simply get the state of stress on any arbitrary coordinate system.

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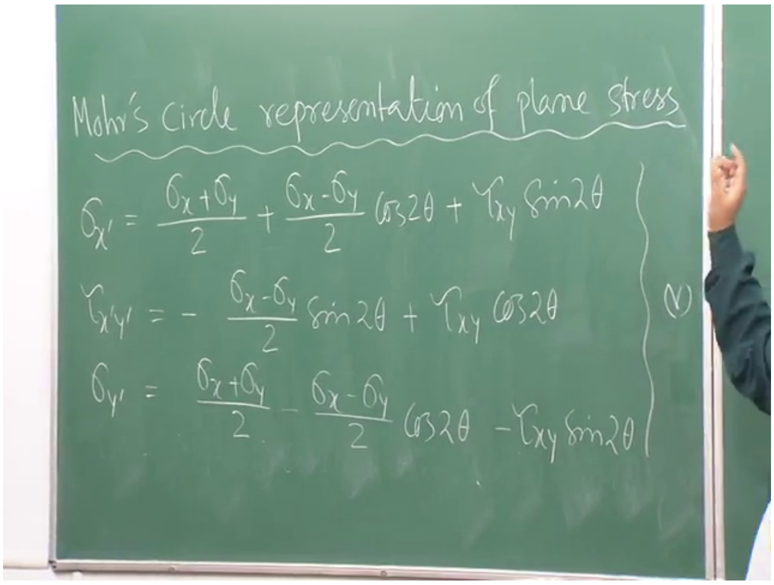


So, let us establish that graphical representation and let us get the advantage or let us understand the advantage of that particular graphical representation of the Mohr circling. So, Mohr circle representation of plane stress now if you look at equation 3 and equation 4 once again you can simplify this equation by using your trigonometric formulation and you will get the equation, equations like that - sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos 2 theta plus tau xy sin 2 theta. Similarly tau x prime y prime is given by minus sigma x minus sigma y by 2 sin 2 theta plus tau xy cos 2 theta and you will be getting sigma y prime equal to sigma x plus sigma y by 2 minus sigma x minus sigma y by 2 cos 2 theta minus tau xy sin 2 theta equation 5 say I am not doing anything new.

So, previously derived equation 3 and equation 4 if you consider and if you do little bit simplification based on your trigonometric formulation and other things algebra formulation you will be getting these three equations they are the I mean revised version of your equation 3 and equation 4. Well, now, if you look at equations the first equation of equation 5; that means, this that is the expression of sigma x prime in terms of sigma x sigma y see tau xy and theta.

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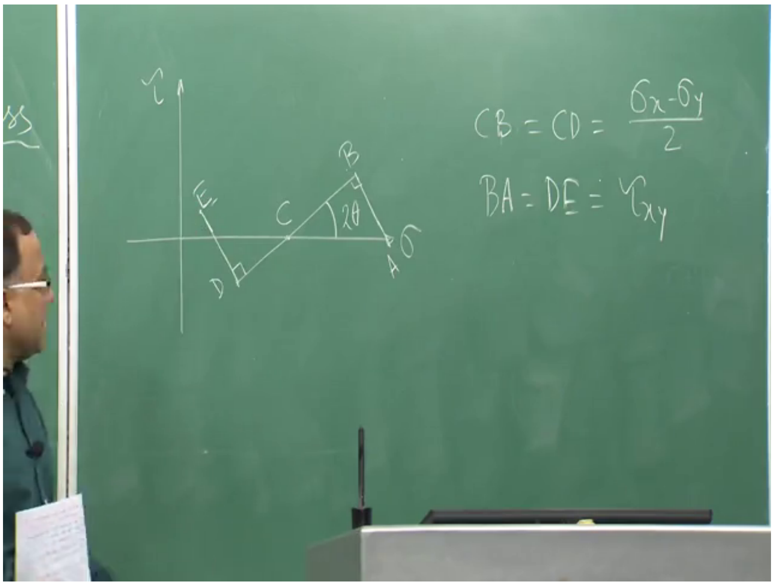


So, now, what I am doing I am doing some graphical construction I am choosing one coordinate system along x axis we are putting the normal stress along y axis we are putting the shear stress.

Some graphical representation or graphically say construction we are performed. So, in that graphical I mean say construction if you look at the first equation of equation 5 I am just taking the first component of equation of this first equation - sigma x plus sigma y by 2 and that I am plotting here say that is a considered say this is the point. So, point, so that point is say I am calling as c. So, c point is giving me this first component of the equation. Then what I am doing? I am do I am drawing one line in such a way that is making an angle twice theta with the sigma axis and this is basically CB and CD I am calling where CB equal to CD equal to sigma x minus sigma y by 2, I can do that because I am doing that graphical construction.

So, now again what I am doing I am just drawing one normal I am drawing one normal perpendicular line on CB and similarly I am drawing another perpendicular line on CD. So, I am drawing one perpendicular line on CB another perpendicular line on CD in such a way that this say BA I am calling and this is I am calling say DE. So, that this BA is equal to DE is equal to tau xy because these are all known term sigma x sigma y tau xy.

So, you can play with those terms right. So, what I have done once again, I have taken a coordinate system space in sigma tau space in that space I am choosing one point C in such a way that C point the abscissa of C point is sigma x plus sigma y by 2 that that is the first term of this equation and then I am drawing one line say D C B in such a way that is making an angle twice theta with the sigma axis and then I am drawing 2 normals BA and DE in such a way that BA equal to DE equal to tau xy, I can do that right.

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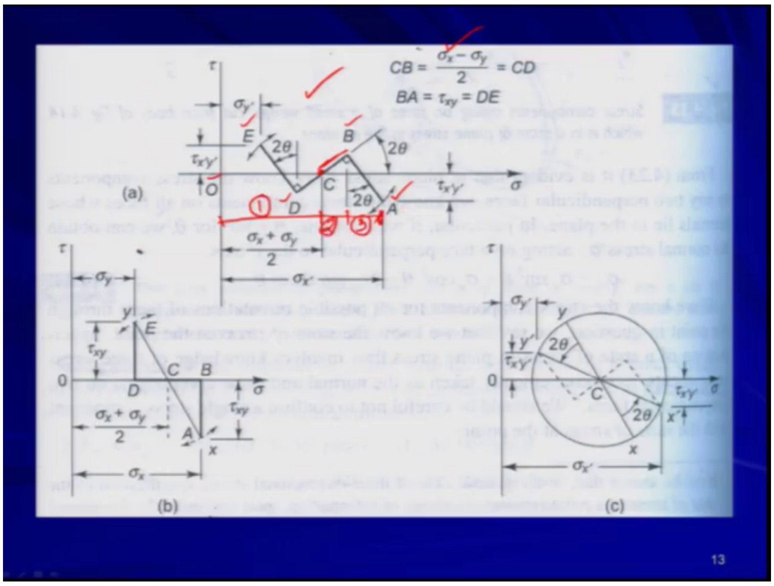


So, let us come back to this figure if you look at this figure on the slide. So, that construction is here. So, this is your B point, C point this is your B this is your A D and E whatever construction we have done. So, that construction is shown in the figure.

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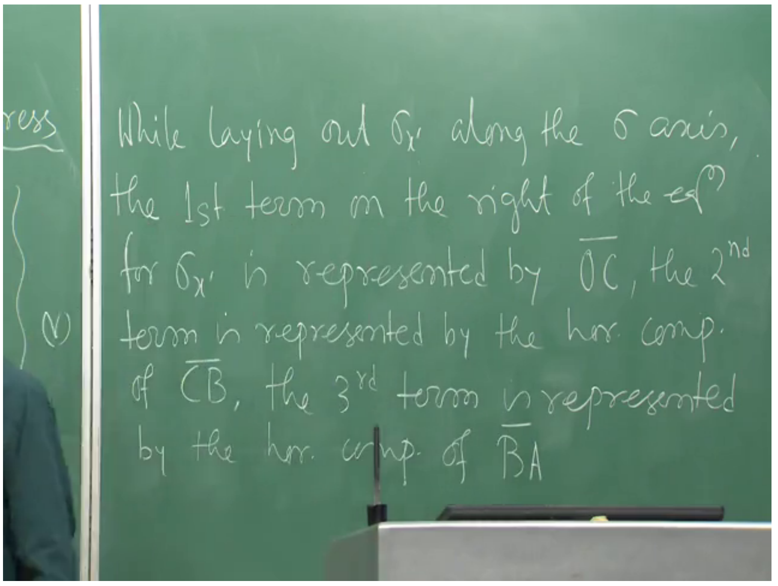


So, now what I am going to do let us write down that, while laying out sigma x prime say along the sigma axis the first term. So, what I am getting? So, I am considering this equation only. So, I am just, so sigma x prime I am just trying to plot on sigma axis. So, sigma axis is nothing, but a normal stress axis. So, I am plot, I am trying to lay out sigma x prime along the x axis the first term; that means, this is the term on the right of equation of the equation or sigma x prime is represented by OC. If you look at the figure you will get it. So, first I plotted the point C in such a way that OC, O is the origin. So, OC is nothing, but your sigma x plus sigma y by 2 right. I drew the, I constructed the graphical representation in such a way that OC is nothing, but the first term.

Now the second term now what is the second term. Now if you look at CB here this is CB now what is the meaning of the second term. So, CB is nothing but sigma x minus sigma y by 2 that is given and in that way we have constructed the representation graphical definition right. So, sigma x minus sigma y by 2 cos 2 theta is nothing, but the horizontal component of CB on the sigma axis agreed or not.

So, that second term is represented by the horizontal component of CB agreed and so this term we have got the meaning or representation of the second term. Now if we consider the third term; that means, this term the third term is represented by the horizontal component of BA, agreed. So, if you look at this figure. So, basically first you have got up to this, this is your first term; then you have got up to this, this is your second term there is a horizontal component of CB and the third term is the horizontal component of BA; that means, up to this part this is your first term, this is your second term, this is your third term, agreed. Now if you have got this now the abscissa of point A is nothing, but the sigma x prime agreed.

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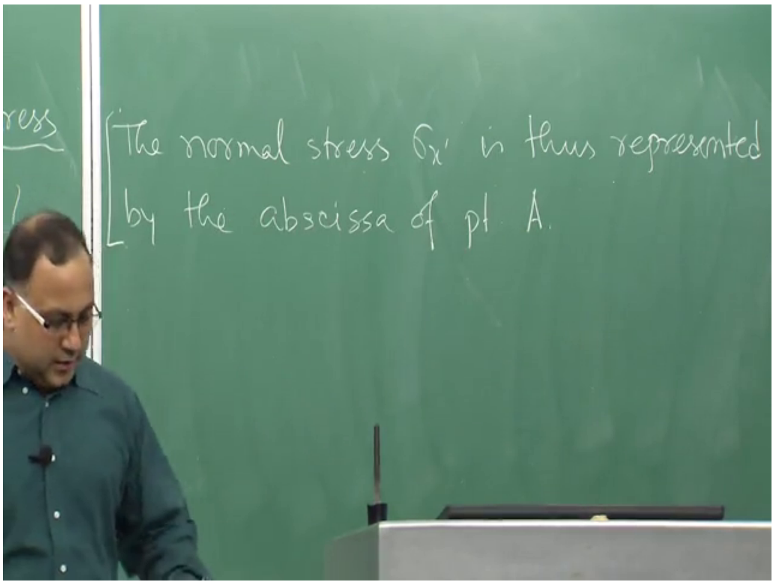


The abscissa of point A is nothing, but the sigma x prime. So, what we can write from this the normal stress sigma x prime is thus represented by the abscissa of point a very very important. Similarly if you look at this equation the second equation, if you look at this second equation we will be getting the ordinate of point A will be giving me tau x prime y prime. So, anyway, so that will be taking in the next class. So, just you think about this equation the first term of this equation and the second term of this equation again it will be coming from the graphical representation and that will be represented by the ordinate the y axis of the A point. So, we will take that thing in the next class. So, I will stop here today.

Thank you very much.