Mechanics Of Solids
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Lecture - 21
State of Stresses

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall we just took the equilibrium condition the moment equilibrium condition and we arrived this, arrived to this relation; that means, the cross shear is equal right tau yx is equal to tau xy, similarly tau xz equal to tau z x and so on.

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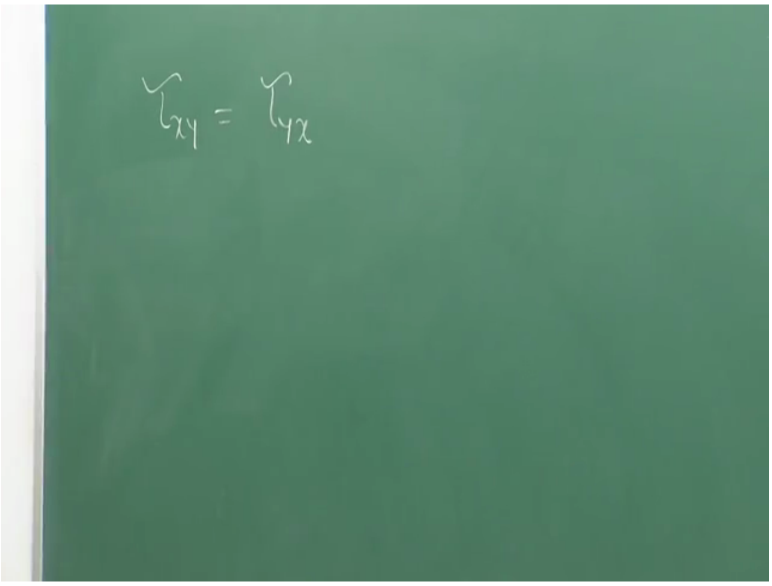

So, if you get the cross shear which are equal then basically the 9 components will come down to 6 components basically right. So, you need 9 stress components to define the state of stress at a point already we have seen in the last lecture.

So, the 9 components 9 stress components will come down to 6 stress components if you consider this equality of the cross shear. So, that is at the biggest advantage because from your 9 unknown components unknown stress component now you just coming down to the 6 unknown stress components right and that is the biggest advantage of taking or considering this equilibrium condition.

Now, in case of 3 dimensional state of stress basically you need 6 stress components to define the state of stress whereas, as I told you as we discussed in the last lecture the plane stress condition right. So, in the plane stress condition now your unknown stress components are 3, so sigma x sigma y and tau xy if the state of stress in x y plane right.

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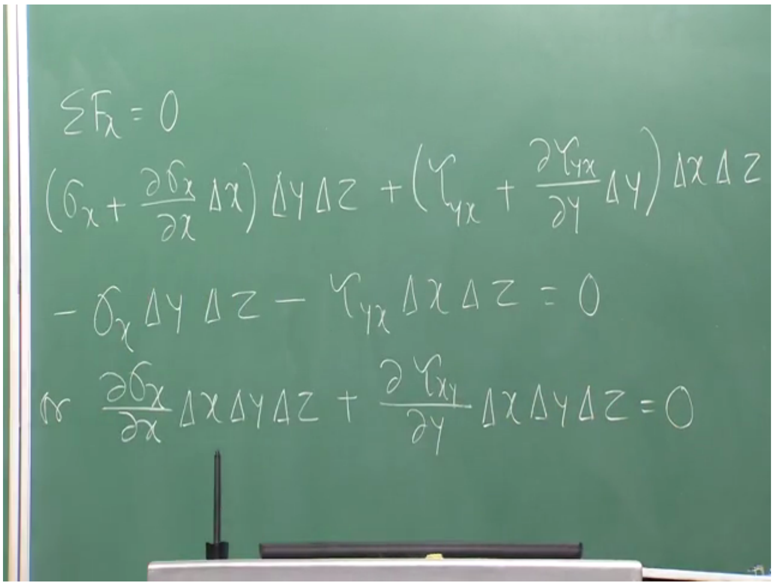

So, now if you similarly if you consider the force equilibrium that is F x equal to 0; that means, all the summation of all the forces along x direction is 0. Then basically you will be getting. So, if you come back to this figure basically. So, first we identify what are the forces are acting in x direction.

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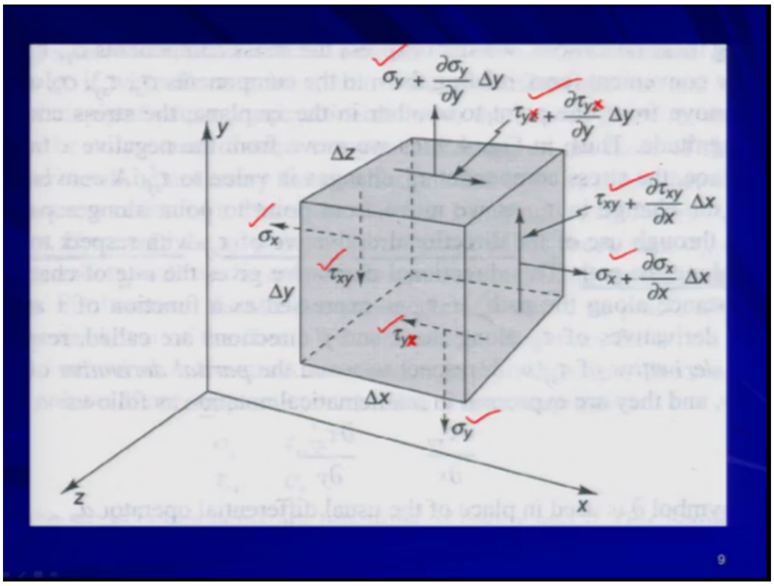

So, let us start from the negative x phase. So, this sigma x is acting along x direction on negative x phase this sigma x plus del sigma x del x delta x that is the variation in sigma x which is acting on positive x phase that is also acting along x direction. Similarly, this tau yx on y plane on say negative y plane now I mean as far our discussion, now it is very clear the tau xy is equal to tau xy tau y x. So, therefore, this is also acting along x direction and this is also acting along x direction. So, these are the stress components which will cause the force in the x direction.

So, let us write down those. So, sigma x plus del sigma x del x delta x and this is the stress acting on positive x phase right on which area, the area is nothing but del y del z right. Similarly, this is the stress tau yx plus del tau yx del y delta y del x del z. So, this is the force acting on positive y plane along x direction. So, these 2 forces are along the positive x direction right say, so other forces other forces which are acting along negative x direction are sigma x del y del z plus. So, this will be minus sorry, this will be minus tau xy or rather tau yx let us write down the actual thing. So, tau yx del x del z equal to 0.

So, these are four force components which are acting along x direction. So, if we simplify this thing we will be coming to this del sigma x del x del x del y del z plus del tau xy I can write because tau yx equal to tau xy already we have established this relation. So, del tau xy del y into del x del y del z equal to 0, right.

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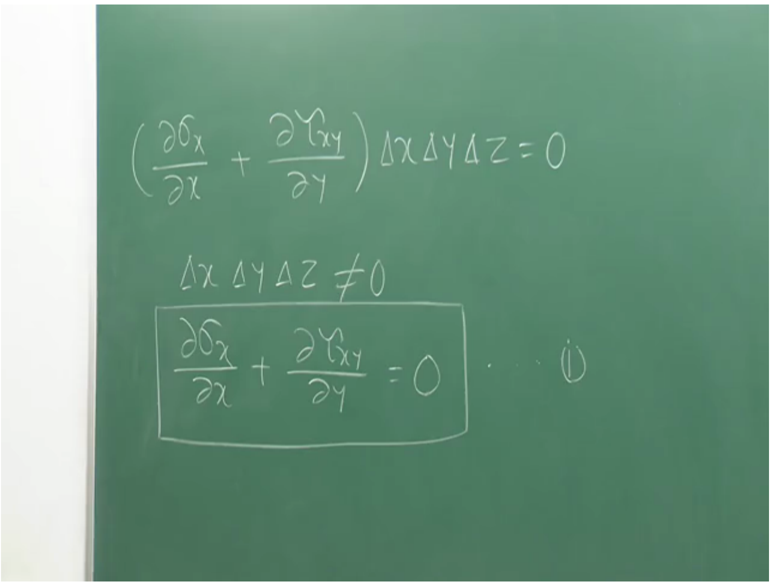

So, we can further write del sigma x del x plus del tau xy del y into del x del y del z equal to 0, right. Since your del x del y del z cannot be equal to 0 because you are considering one parallelepiped around point two, so they cannot be 0, right. So, the multiplied because you have you are considering some finite volume of the parallelepiped. So, if that is not 0 then of course, these must be 0. So, del sigma x del x plus del tau xy del y must be 0. So, this is one important relation. So, this is equation say one, this is an important relation which has been established by considering the equilibrium condition along x direction.

So, if you perform or if you exploit this equilibrium condition, you will be coming to this relation. So, your stress if you say this body is under equilibrium your stress cannot be arbitrary. So, they should, they should have some proper relation and that relation is established by this equation 1. So, del sigma is del x that is the that is the derivative partial derivative plus del tau xy del y summation of these 2 must be 0 Similarly, if we try to establish the equilibrium condition along y direction. So, in a similar fashion if we take F y equal to summation of F y equal to 0 then what we will get? We will get. So, first let us come back to this figure again what are the forces let us identify what are the forces are acting along y direction.

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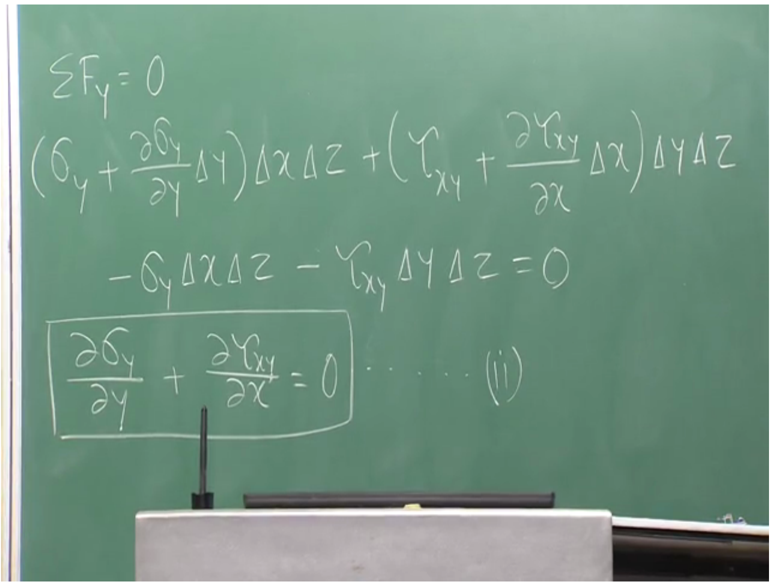

So, starting from say negative x phase, so this tau xy is acting along y direction this sigma y plus del sigma y del y delta y is acting along y direction which is the acting on of course, positive y phase. Similarly, sigma y is acting on negative y phase which is acting towards y direction and this is the stress tau xy plus del tau xy del x delta x that is also acting on positive x phase, but along y direction. So, these are for say this will constitute for force components which are acting along y direction.

So, very similar to the previous operation if we perform, so we will get sigma y sigma y plus del sigma y del y delta y del x del z plus tau xy plus del tau xy del x delta x del y del z minus sigma y del x del z minus tau xy del y del z if we perform the similar simplification then we will finally, get del sigma y del y plus del tau xy del x equal to 0.

So, we have got another relation which is considering sigma y and tau xy and this relation again is required to get the equilibrium condition of the system. So, your stress components should follow these relations. So, I mean this is nothing, but these 2 equation 1 and equation 2 are nothing, but the equilibrium equations required for your plane stress condition.

So, now if you look at these equations we have not considered the body force, the body force may happen and it is a pretty common thing right. So, if you consider the body force; that means, if you consider the gravitational; that means, a weight or I mean in case of civil engineering we generally get the seepage force or inertia force or something like that. So, if you have the body force then you need to consider the body force into account.

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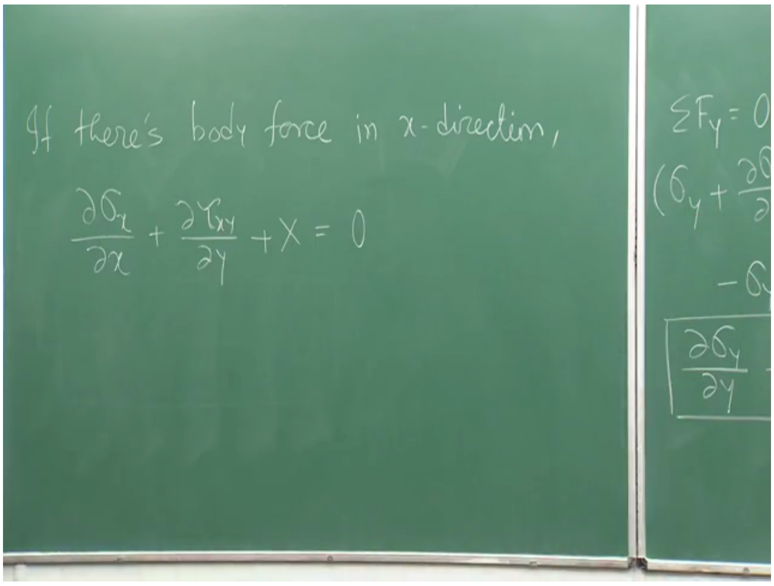

So, if you consider the body force into consideration then what exactly will be getting. So, if there is body force in say x direction then you get this equation 1 will be modified as del sigma x del x plus del tau xy del y up to that we have derived already plus this body force term depending on the direction of course, must be 0. So, this is the extra term which will be coming into the picture due to the consideration of body force.

Similarly, I am not writing this thing again, but similarly this equation 2 will be modified del sigma y del y plus del tau xy del x plus y where y is the body force along the y direction. So, if you have the body force in x and y direction. So, basically these equations of equilibrium will be modified by considering the respective body force.

Now, if you try to I mean we are not going to derive that thing, but you can we are just following the same procedure and say same steps involved in obtaining this equations if we follow for the 3 dimensional state of stress then basically that will consider or that will reveal the following equations.

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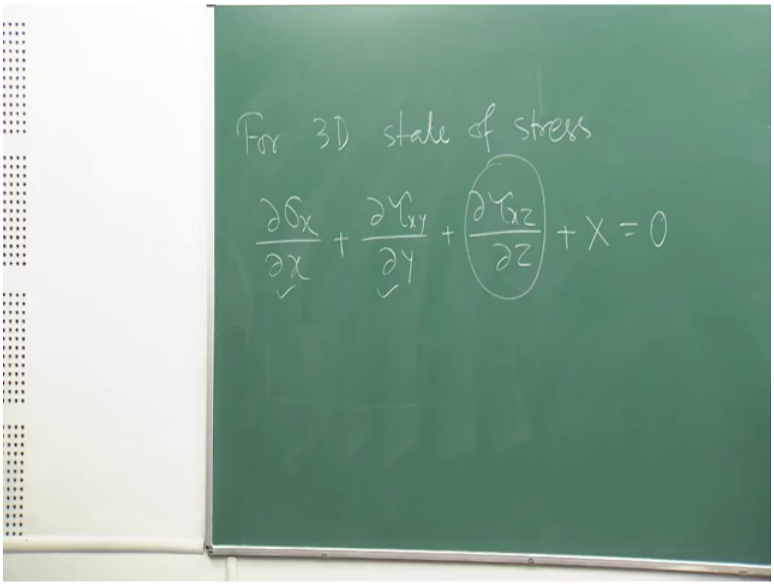

So, for 3 D state of stress your equation 1 will be modified like that del tau xy del y plus del tau extend del z plus body force star equal to 0, is that clear.

So, if you consider 2 dimensional state of stress then basically these first term and second term over coming already you have derived that right, but once you are considering the 3 dimensional state of stress. So, this is the extra term which will be coming for the consideration of the z direction. So, this is nothing but the equilibrium equation for 3 dimensional state of stress.

Similarly, so this is these will be coming when you are considering the equilibrium say conditions that is summation of F x equal to 0. Similarly if you consider summation F y equal to 0, so this equation 2 will be getting modified by considering another term say tau y z and so on. So, you will be getting 3 equilibrium equations by considering the force equilibrium along x direction, along y direction and along z direction; that means, summation of F x equal to 0 summation of F y equal to 0 and summation of F z equal to 0. So, we are not going to derive that thing. So, you can get or you can derive this thing by your own.

So, let us talk about some new concept. So far whatever we have discussed based on that, some new concept we are going to discuss that is if you know the state of stress in x y coordinate system right, if you know the state of stress in x y coordinate system. So, still we are talking about the plane stress condition. So, as I told you, in this particular course will be mainly confining our self or restricting our self in the calculation or in the analysis of plane stress condition.

So, if you know the state of stress at a point based on x y coordinate system; that means, if you know the state of stress in x y coordinate system; that means, what are the stress components are known to you 3 - sigma x, sigma y and tau xy; and since tau yx equal to tau xy so that will come to one common stress components. So, suppose you know sigma x sigma y and tau xy; that means, you know the state of stress in x y coordinate system can I define or can I get the state of stress in some arbitrary coordinate system, if I know the orientation of that coordinate system with respect to the x y coordinate system.

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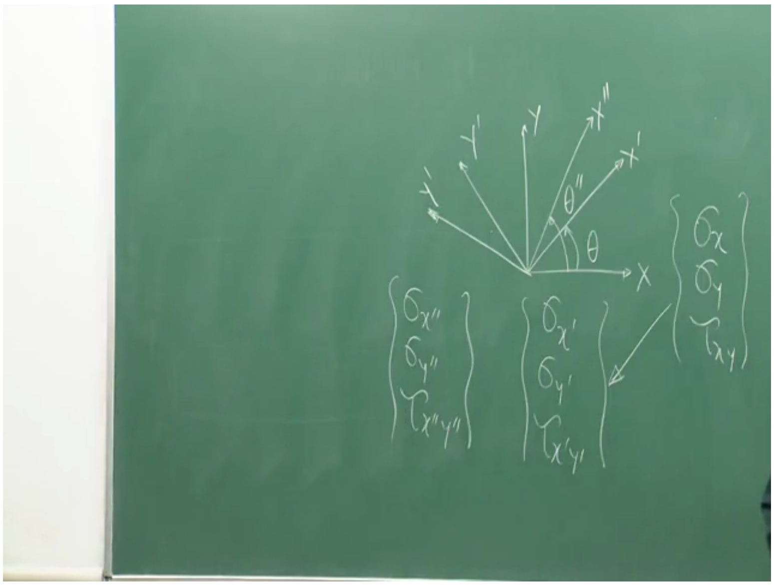

So, what did I mean? Suppose this is your x y coordinate system this is your x y coordinate system. Now, another coordinate system you are considering say x prime y prime which is making this is known to me which is making an angle say theta with respect to x axis now I know the state of stress in x y coordinate system; that means, I know sigma x sigma y and tau xy.

So, this is known to me. So, if that is known to me can I find out the state of stress in x prime y prime coordinate system; that means, sigma x prime sigma y prime and tau x prime y prime. So, if I know this can I get this, this is my question. So, we will we will we will try to try to prove that yes if I know the state of stress in x y coordinate system I will be getting any arbitrary oriented coordinate system x prime y prime where the set of stress can be defined in terms of sigma x, sigma y and tau xy and the orientation angle theta.

Similarly, if you define another coordinates another arbitrary coordinate system say x double prime y double prime and say this angle say theta double prime. So, by knowing sigma x sigma y and tau xy, can I get sigma x double prime sigma y double prime and tau x double prime y double prime? Yes, we can get it. So, that is my objective. So, by knowing the stress components in one coordinate system can I get the stress components in other coordinate system. So, that is my objective.

Now, let us talk about that thing now and this is very much required I mean if you if you deal with any system any mechanical system right you whether it is civil engineering system or aerospace engineering system or mechanical engineering system. So, wherever you go basically you know, suppose you know the boundary condition you know the state of stress in some coordinate system and you want to find out how much stress is getting developed if you rotate the axis on some different plane if you want to find out the stress components how you will find out. So, it is very much required that is your stress components knowing in some arbitrary coordinate system.

So, if I look at this figure. This is one thin sheet or thin plate whatever you say which is under plane stress condition which is under different externally applied forces F 1 to F n, this is your x y coordinate system because z direction we do not have the variation because this is the plane stress condition as we know.

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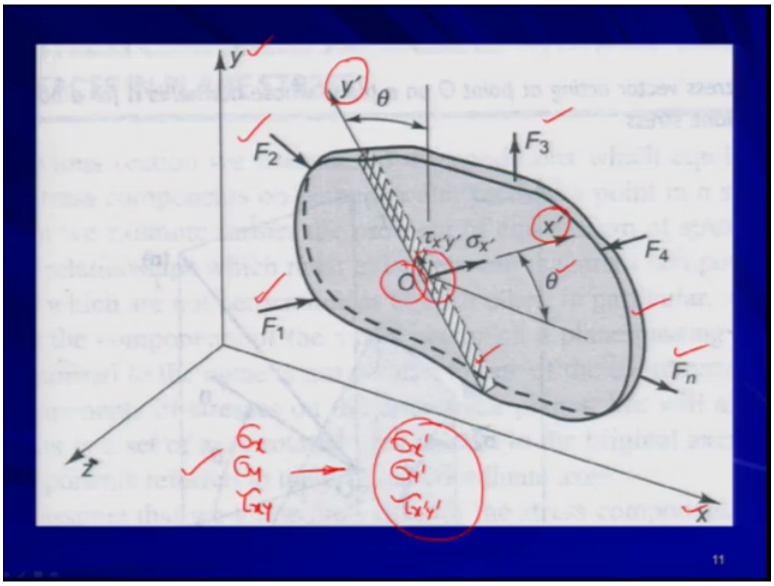

So, in this situation we know the state of stress in x y plane; that means, I know sigma x sigma y and tau xy. So, these 3 things are known. Now, I am choosing one arbitrary plane some inclined plane which is considering O as a centre and this arbitrary plane is defined by the I mean the plane will be defined by I mean direction normal. So, this plane is nothing, but your x prime plane and similarly y prime plane is also defined, so that is along the plane. So, now, what I am considering I have the body and I am considering another plane inclined plane which is inclined by an angle theta. So, with respect to your x axis your x prime axis making an angle theta right. So, that means, the angle between x and x prime axis is theta. So, I am defining one arbitrary plane x prime that is defined here and y prime that is another axis of course, the x prime y prime axis will be there. So, my objective is to find out from this sigma x prime, sigma y prime and tau x prime y prime. I hope you have understood the concept, why we need that, I know the state of stress in x y coordinate system. Now I want to know the state of stress on some arbitrary plane say x prime plane.

So, for that you need these stress components fine. So, let us do that. So, what I am doing I am just considering this part and I am blowing it up and I am getting. So, this is my x prime axis, this is my y prime axis this is the inclined plane.

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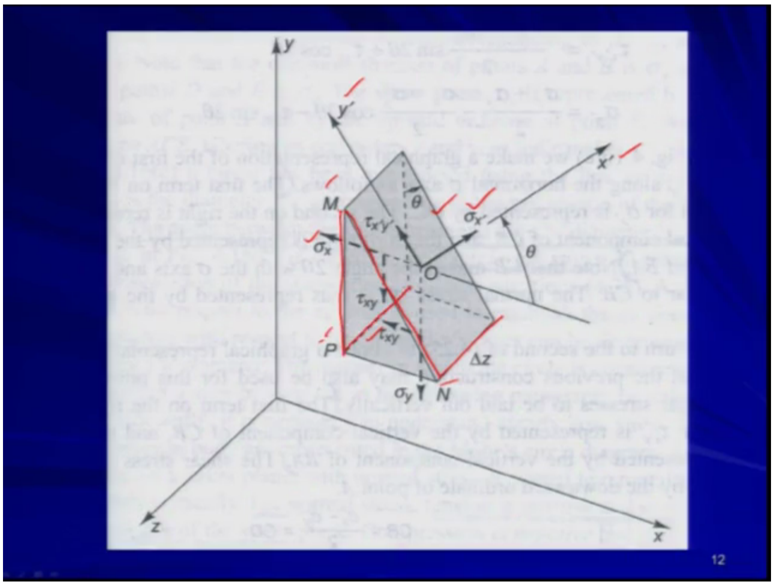

So, M N is the inclined plane and whereas, M P is your x plane and your N P is your y plane fine. So, I know the state of stress in x y coordinate system I want to know the state of stress on x prime y prime coordinate system; that means, sigma x prime tau x prime y prime and sigma y prime are required to know.

So, now what we can do in this figure if you look at. So, this figure if you consider as I told you that when we are going to discuss this state of stress and all, so every each and every point of that system is under equilibrium. So, now, in this figure whatever stresses or force I mean ultimately the stress will be multiplied by the area and you will get the force. So, that force will; all the forces together will satisfy the equilibrium conditions.

Now, what I am going to do, I am considering the equilibrium condition along x prime direction and along y prime direction. What I am going to do? I am going to take or I am going to satisfy the equilibrium condition along x prime direction and y prime direction; that means summation of F x prime equal to 0 and summation of F y prime equal to 0. So, let us do that.

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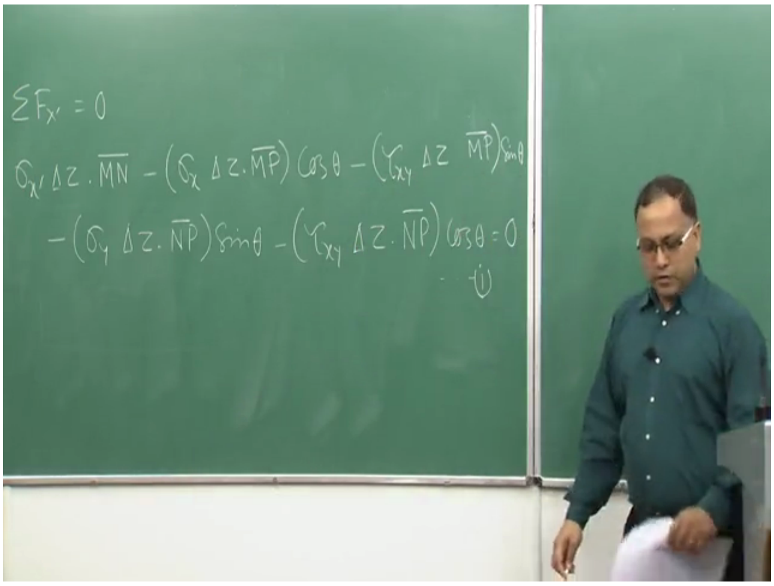

So, if I consider this is 0; that means the force summation of all the forces acting along x prime directions are 0. So, what are the forces are acting along x prime direction? So that you will be getting from this figure, so let us identify those stress components which will be causing the forces along x prime direction. So, first one we will be sigma x prime that is the normal stress acting on a M N plane that is the inclined plane along x prime direction.

Now, this sigma x will be having the sigma x will be causing the force in x direction, but of course, I can take the component of that force along x prime direction. Similarly tau xy that is also acting or along I mean along your this on this plane it is acting on along y direction and this plane it is acting along x direction right. So, I will be getting the components of that stress also. So, if I take all the components. So, it will be coming like that. So, sigma x prime delta z into M N this is the, so sigma x prime is the stress acting on x prime plane and what is the area of the x prime plane? M N this is the side M N multiplied by delta z. So, this is the area on which sigma x prime is acting and that force is acting along x prime direction.

Similarly, sigma x is acting on x plane right. So, sigma x now what is the area of x plane? Delta z into M P; what is that? So, this is your M P and this is your delta z. So, this is the area this is a rectangular area on which sigma x is acting. So, that is the force in x direction. Now I have to take the component of this force along x prime direction.

So, what to do? Simply multiplied by cos theta. So, I will be getting the component of that force along x prime direction.

Similarly, tau xy into delta z into M P right, what is that force? That is the force acting on x plane right that is the force acting on x plane along y direction, because this is because of this tau xy right. So, I have to take the component of that. So, that component will be sin theta. Similarly I will be getting sigma y del z into N P. Please look at the figure you will be getting this. So, this is the force acting on y plane along y direction I have to take the component of that towards the x prime direction minus tau xy delta z into N P and the components. So, these are all forces acting along x prime direction. So, this I am calling as equation say 1.

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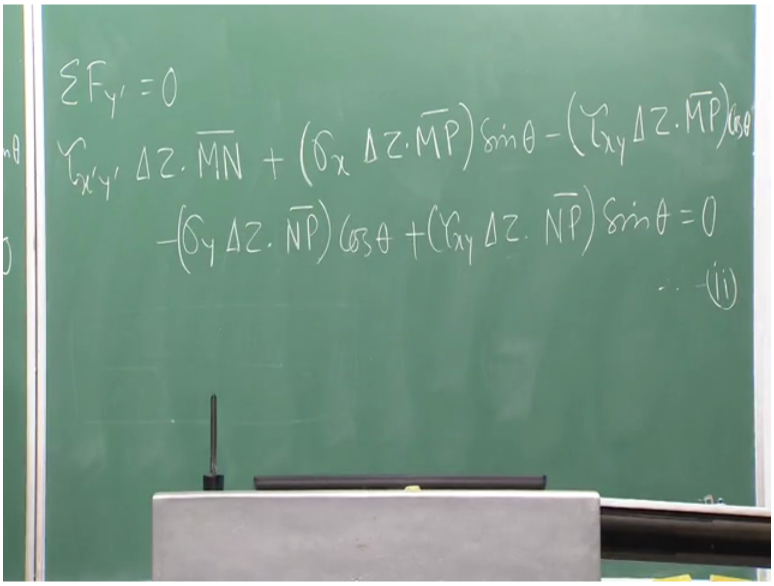

Similarly, I am going to exploit this if y prime, summation of F y prime equal to 0. So, if I do that what are the forces are acting del z into M N. So, that is the force acting on the inclined plane along y prime direction. So, this is due to this component fine minus plus rather sigma x del z M P and the component of that along y prime direction minus tau xy del z M P component of that cos theta minus sigma y del z N P cos theta plus tau xy del z N P sin theta equal to 0 say this equation is 2. So, I have got 2 equations equation 1 and equation 2 by satisfying the equilibrium of forces along x plane direction and along y prime direction.

So, I will stop here today. So, in the next lecture will be taking these 2 equations and we will get our required solution.

Thank you very much.