Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 20
Plane Stress

Welcome back to the course Mechanics of Solids. So, in the last lecture we are talking about the stresses right, we defined the normal stress, we define the shear stress right and we have seen that if you want to define the state of stress at a particular point, so you need nine stress components right in the 3 dimensional state of stress. Now basically in most of the mechanics problem we whatever will be dealing in this particular course that will be taking care of the 2 dimensional state of stress; that means, say suppose for example, if you pull a thin sheet thin I mean something like this. Let me draw it.

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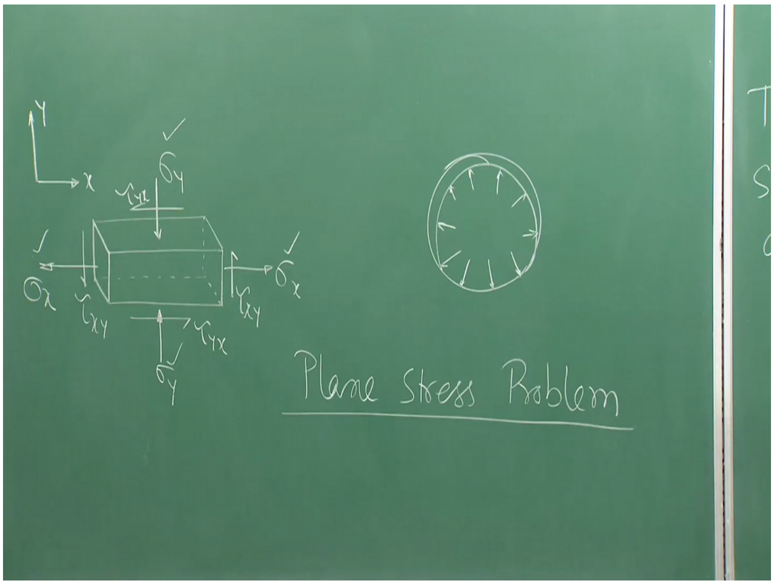

So, if I pull a thin sheet rubber sheet or metal sheet or any kind of sheet right. So, if I pull a thin a sheet. So, basically the z direction stress you will be whatever you will be observing, so there is no variation in that z direction stress. So, z direction stress is not coming to the picture right.

So, basically whatever stresses you are getting here. So, they are the planar stress; that means, in x y plane whatever stresses are acting. So, that will be coming into the picture. Similarly if you consider a circular ring, so this is a circular ring where all round say internal pressure as you have observed when we discussed about the thin wall pressure vessel right. So, if you consider a circular ring and inside the circular ring you have all round say pressure internal pressure then this kind of problem will converge or will come to a state where you have only 2 dimensional state of stress. In this case only the x y plane only along the on the x y plane you have the state of stress, similarly on this particular circular case if you consider the polar coordinate systems the z direction. So, normal to this board there is no variation in the stress. So, all the stress components are confined in the r theta coordinate system. So, this kind of problem is known as plane stress problem.

So, the formal definition of the plane stress problem is that the stress is existing in a particular plane.

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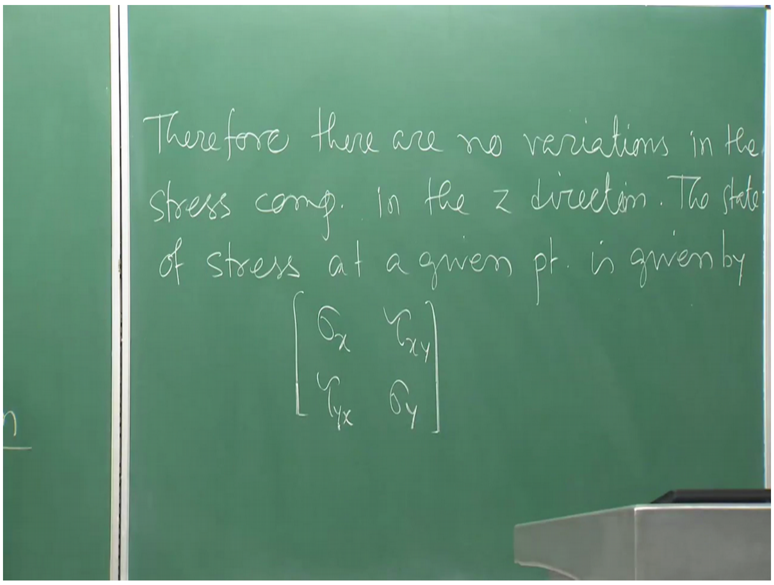

So, therefore, there are no variations in the stress components in the z direction where z direction is nothing, but the normal to the board. So, there is no variation in the stress component in the z direction. So, therefore, the state of stress at a given point is given by. So, if you want to define the state of stress at a point in the 3 dimensional state of stress then how many stress components you had, in the last lecture we discussed 9 stress components right, but that 9 stress components will be coming to the 4 stress components in plane stress problem. What are those? Sigma x I am not writing the second subscript as I told you in the last lecture now onward we will not be writing then tau xy, tau yx and sigma y. So, this is the state of stress in the plane stress problem when you have 2 dimensional state of stress; that means, the stress is acting only in x y plane similarly if you consider the polar coordinate system. So, that will be a sigma r that is in the radial direction sigma r tau r theta tau theta r and sigma theta something like that anyway.

So, now this kind of problem will be dealt in this particular course frequently and we need to understand that; what is the specialty of this particular say type of or particular class of problem? So, now, here in this figure if you come back whatever we have seen in the last lecture right.

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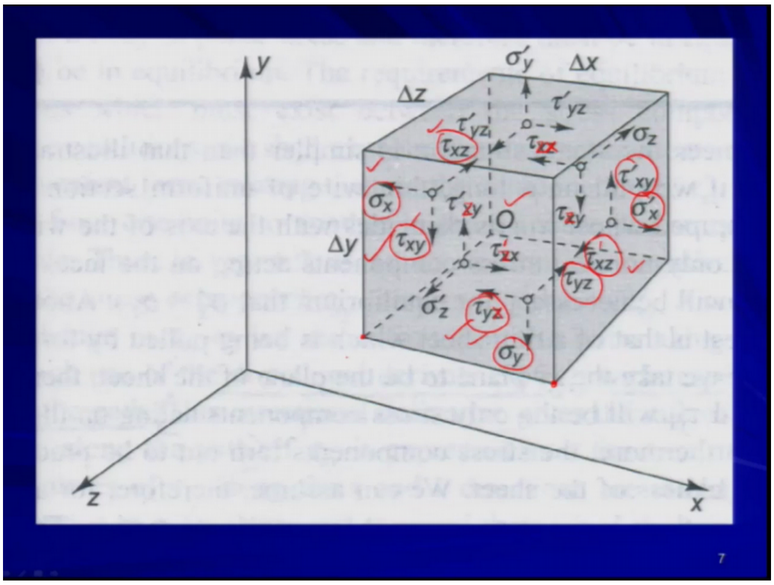

So, this is defining your 3 dimensional state of stress, but when it is coming to the plane stress problem that will take this form.

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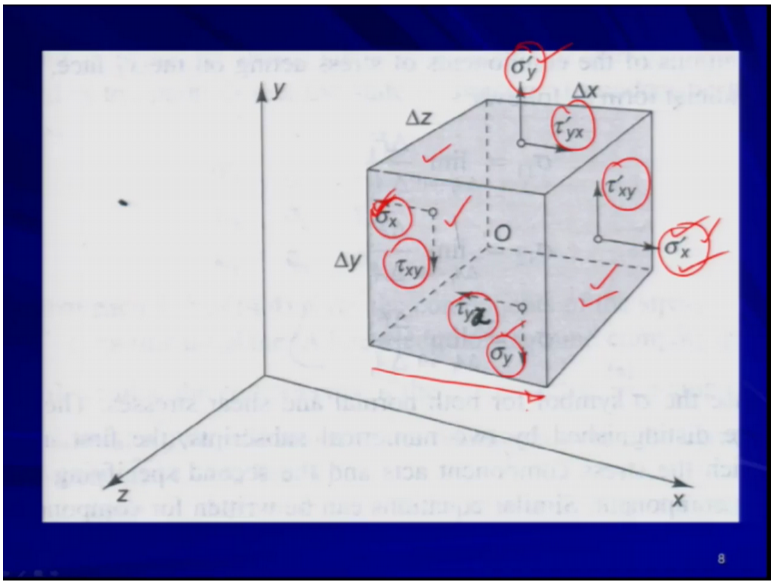

So, in the z direction or the on the z plane basically there is no stress fine; that means, sigma z tau y z tau z y those things are not coming into the picture. Now only you have the stress on x plane this is your negative x plane, this is your positive x plane. So, you have the stress components acting on these planes. Similarly this is your negative y plane this is your positive y plane the stress components are acting. Here what we have considered? We have considered one parallelepiped around the point O as we have done in the last lecture right. So, you have got 2 x phases and 2 y phases.

Now, at this stage we are supposed to define the direction or the sign convention for the stresses. At this moment we are going to define the sign convention of the stresses. Now if you come back to this figure the normal stress sigma x or sigma x prime, sigma y prime or sigma y whatever you consider if the normal stress is acting on a particular plane if it is, if I say this normal stress is positive the positive normal stress is nothing, but the stress the normal stress which is acting on a positive plane towards the positive coordinate axis. What does it mean? That means, this sigma x prime if you consider this sigma x prime is defined as the positive normal stress why it is positive normal stress because this is acting on positive x plane and towards the positive x direction. So, therefore, this is positive normal stress.

Similarly, the normal stress will be positive if that is acting on negative plane and towards the negative coordinate axis. What does it mean? If you see is the sigma x sigma x is acting negative on negative x plane right, but towards the negative x direction. So, therefore, this sigma x is again positive. So, if it is reverse then that will be defined as the negative normal stress. So, please let me, I mean repeat once again. So, if the normal stress is acting on positive plane towards positive coordinate axis then that is positive normal stress or if it is acting on negative plane towards negative coordinate axis then that also will be positive normal stress. So, that means, if you think of, if you come back to this figure, if you come back to this figure this sigma x this phase is my negative x phase this phase is my positive x phase. So, on both the, I mean this is the positive x phase and towards the positive x direction. So, therefore, this sigma x is positive similarly on this negative x phase towards the negative x direction sigma x is active. So, that is also positive.

So, what kind of nature you are getting by this normal stress that is a attention. So, as per our convention that the tensile normal stress is always positive, because that is in line with the definition of or the sign convention whatever we just talked about; so the tensile normal stress will be always positive because that will satisfy that condition that is on positive plane positive direction on negative plane negative direction. Now what about this sigma y? This plane is your positive y plane; this plane is your negative y plane for example, right. So, on this positive y plane sigma y is acting in the negative y direction right. So, as per our definition sigma y will should be negative.

Similarly if you look at this part this sigma y is acting on the negative y plane, but in the positive y direction; that means, on positive plane negative direction or negative plane positive direction that normal stress will be always defined as negative normal stress fine. So, similarly the shear stresses will be defined like that. If it is acting on positive plane positive direction then that will be positive or negative plane negative direction that will be always positive, but if they are work if they are acting on positive plane negative direction or negative plane positive direction then they are negative. So, this is the sign convention will be following throughout the course, I hope you have understood. So, I have spend enough time on the sign convention because they sign convention people generally do the mistake. So, I do not expect this kind of mistake will come from me.

So, now after this definition what we can think of, now as I told you that on this negative x plane you have got sigma x tau xy, but on this positive x plane you have got sigma x prime tau xy prime right. So, you have got the variation right why this variation because you are travelling from one point to another point and that that is defined that variation is defined by the directional derivatives right.

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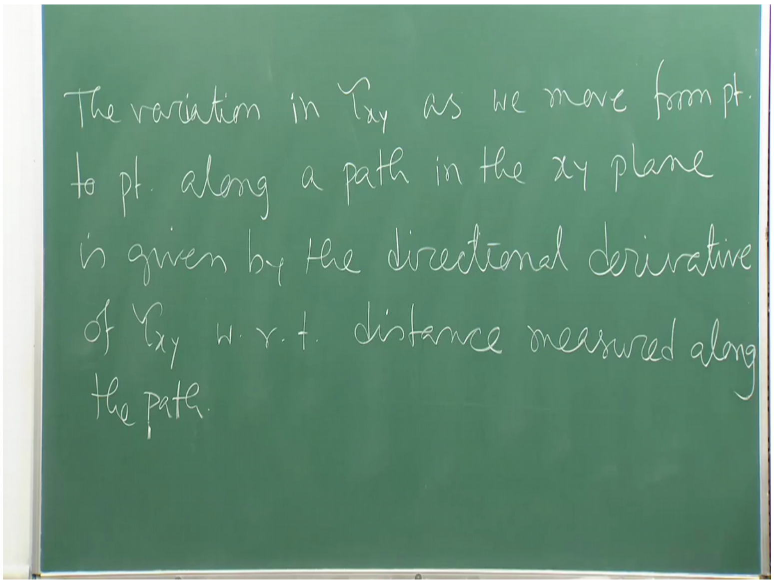

So, if I can write the variation in tau xy or whatever tau xy or sigma x y sorry tau xy or sigma x or sigma y or tau yx whatever you consider. So, the variation in tau xy as we move from point to point along a path in the x y plane is given by the, that comes from the mathematics. So, there is nothing to do with that the directional derivative of tau xy with respect to the distance measured along the path. So, that you know from the mathematics right. So, if you follow this then basically this sigma x prime is equal to sigma x plus delta sigma x del x delta x right.

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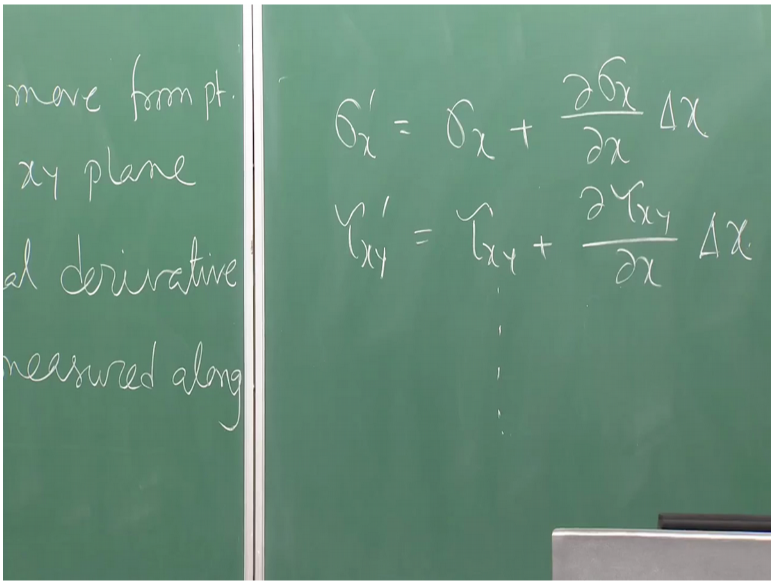

So, if you come to this figure once again. So, sigma x prime we are defining this sigma x prime is nothing, but sigma x that was there on the negative plane and then from negative plane to positive plane you are moving. So, that is giving given by the directional derivative of delta sigma x del x that is the direction derivative of sigma x basically right with respect to the distance delta x. So, you are traveling by a distance delta x right. Similar to that we can write tau xy prime is nothing, but tau xy plus delta tau xy del x delta x and so on. So, other stress components you can define likewise.

So, now if you consider the equilibrium condition, so whether the stress you are calculating or you are analyzing this parallelepiped and all those each and every point is under equilibrium in that particular system, whatever body I have shown you if I say that body is under equilibrium so each and every point in that body will be under equilibrium that that is coming from the definition whatever we have discussed when we are talking about the equilibrium condition right. So, if we exploit the equilibrium condition then basically what we get.

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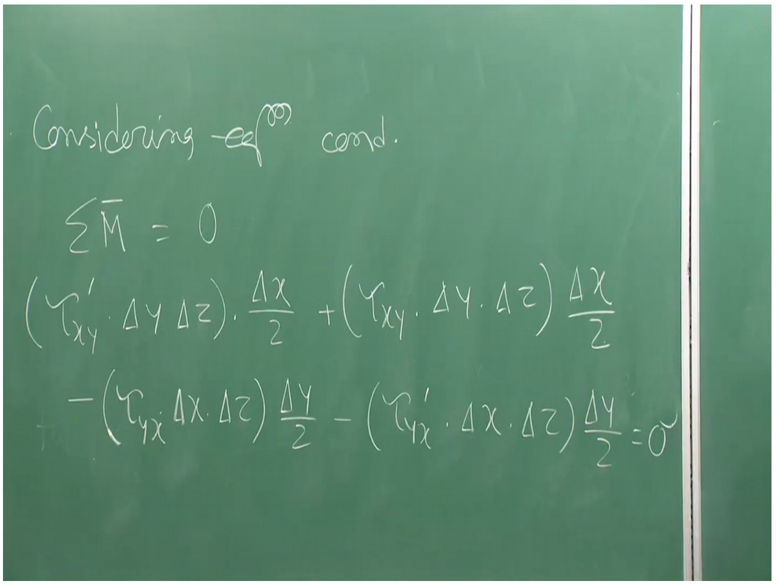

So, we are just considering we are just, considering equilibrium condition and we are taking the moment, moment equilibrium with respect to point O or the center of the element. Now, if you come to this. So, if the point O is at the center of the element. So, we are considering moment equilibrium with respect to the center of the parallelepiped or the element and that must be 0. So, that gives me if you look at this figure whatever is shown whatever shown now just now, so in that figure tau xy prime is acting del y del z into del x by 2. So, tau xy prime del y del z is nothing, but the force tau xy prime del y del z is nothing, but the force acting on x plane towards acting on positive x plane. So, that is nothing, but this right. So, this stress multiplied by del y, this is del y, this is del z. So, that gives you the force all the x positive x plane along the y direction right. So, that force multiplied by delta x by 2 is giving you the moment.

Similarly, the next part tau xy delta y delta z that is the force which is acting on negative x plane along y direction multiplied by del x by 2 minus if you look at the figure you will get it tau yx del x del z into del y by 2 plus sorry minus tau yx prime into del x into del z into del y 2 equal to 0. So, now, if we expand tau xy prime in terms of tau xy and tau yx prime in terms of tau yx right as we have done by using your directional derivative and the distance when basically finally, you will be getting this thing. 2 tau xy minus tau yx plus del tau xy del x delta x minus del tau yx del y delta y equal to 0, because we are not getting any moment from the normal stress right, we are not getting any moment from the normal stress because the normal stress is passing through the center of the element well.

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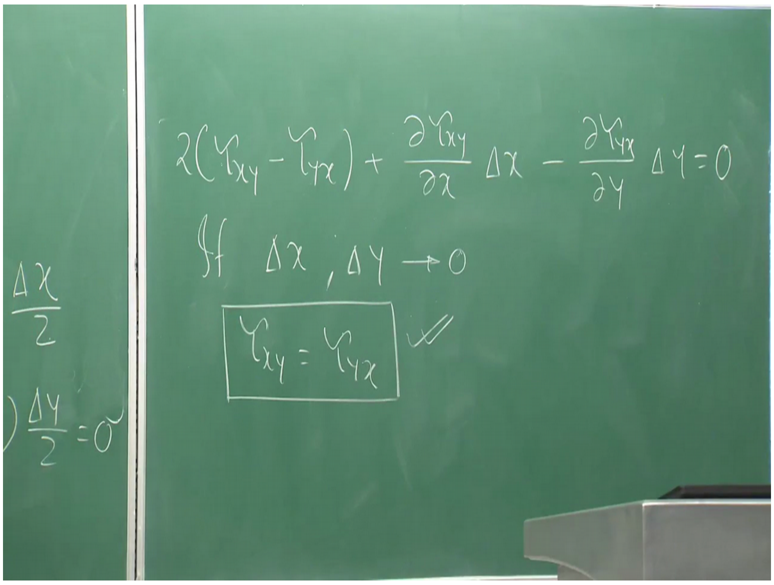

So, now if del x and del y tends to 0; that means, we are converging to the point O then basically these 2 terms will vanish and then finally, you will be getting tau xy equal to tau y this is a very important and useful relation tau xy is equal to tau yx right; that means, the cross shear is same tau xy is equal to tau yx similarly tau x z should be equal to tau z x and so on. So, all cross shear components are equal right. So, that gives you basically instead of 9 components right instead of 9 stress components whatever you considered in case of 3 D state of stress right. So, that will be coming down to 6. So, this is very very important relation we have established that is the cross shear is equal.

So, I will stop here today. So, in the next lecture will be discussing about or will be taking or will be exploiting other equilibrium condition that is the force equilibrium condition and then we will see that what equation we can derive from that.

Thank you very much.