Mechanics Of Solids
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Lecture – 19
Concept of stress

Welcome back to the course Mechanics of Solids. So, in the last lecture we just started the new topic that is concept of stress and there we have seen that how we can define the stress at a point right and then basically we are discussing something and regarding the planes right. So, any plane can be defined by its direction normal and the plane which will be defined as the positive plane; that means, the direction normal on that particular plane will be directed towards the positive direction of the coordinate axis. Like if I say this is my positive explain; that means, the direction normal on that particular plane is towards the positive x direction right. So, we defined the positive faces as well.

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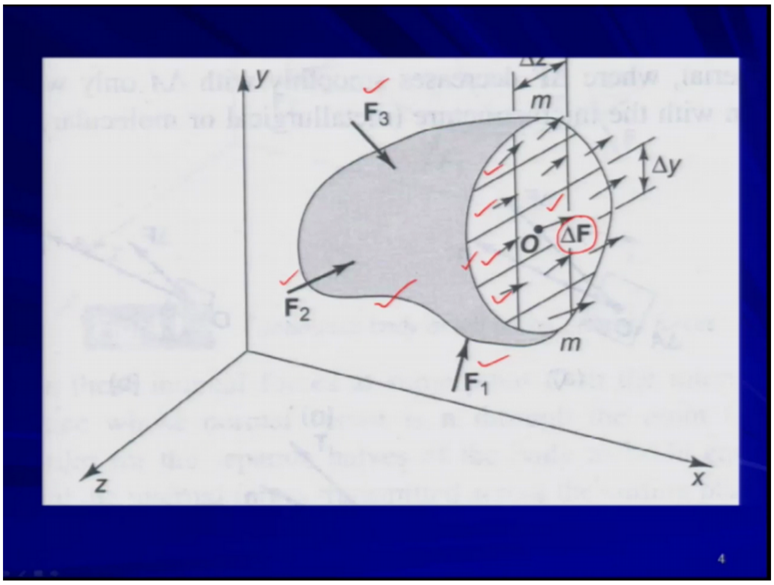

So now if you come back to this figure basically, so there is a body and this is the body and the body is under different external forces like F 3 F 2 F 1 and all, and now we are cutting this body in 2 halves. So, this is the plane the cutting plane which is normal to the x axis so; that means, it is the explained. So, as shown in the figure, this plane is nothing, but your positively x plane as per the figure. Now on that plane particular on that plane if we discretize this different say segments right like this is one segment this is another segment this is another segment likewise several segments are there, so if you discretize that and if we consider one say segment like this is the segment which is containing the point O and on that plane actually on that particular segment your delta F that is the force is acting. It is not necessary as I told you in the last lecture that is the not necessary that delta F will be always perpendicular to the plane right; that means, towards the direction normal it may not be or may be right. So, in more general way if you think of so it is not normal to the plane.

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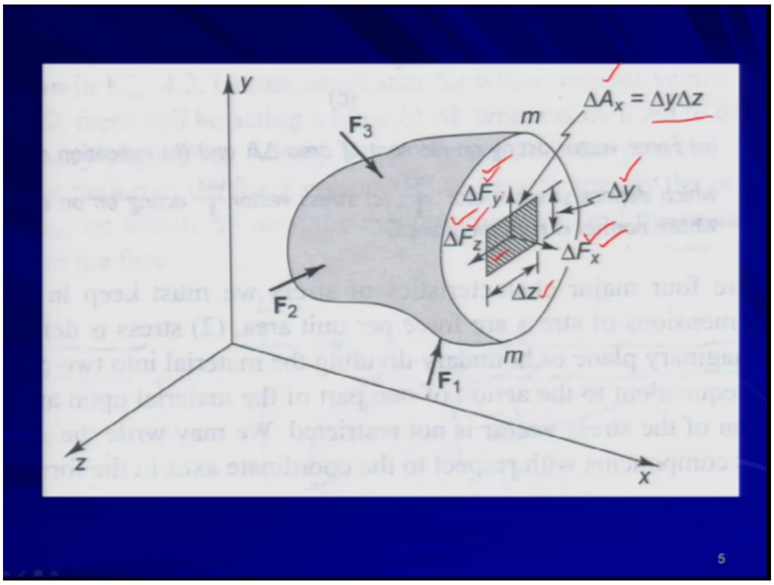

So now if we take out if we separate or if we take out this plane and if we show only that particular plane or particular say segment, so which is containing point O. So, this is the, this faded part is that particular segment whatever we are considering. So, this is the I mean depth of the plane is say delta y and length of the plane is say delta z and this is the x plane positively explained. So, the area of the plane or area of I mean small segment will be z delta ax which will be equal to delta y into delta z that is very simple right.

Now, as I told you in the previous figure you what you have you have seen the delta F that is the force is acting on that particular plane on that particular segment and basically at point O say and now if this delta F is decomposed or dissolved in three different say components force components like in x direction in y direction and in z direction then I will be getting delta F x delta F y and delta F z.

Now, this delta F x delta F y and delta F z are the three components of the force actual force delta F and now this delta F x delta F y and delta F z these forces are acting towards the respective coordinate direction. Now if I want to define, now one thing is very clear from this figure that the nature of stress developed by the force delta F x may not be similar to the stress developed by the force delta F y right because their line of action on the plane, the behavior of these forces on the plane would be completely different right, based on that we can define different stress components on that particular plane.

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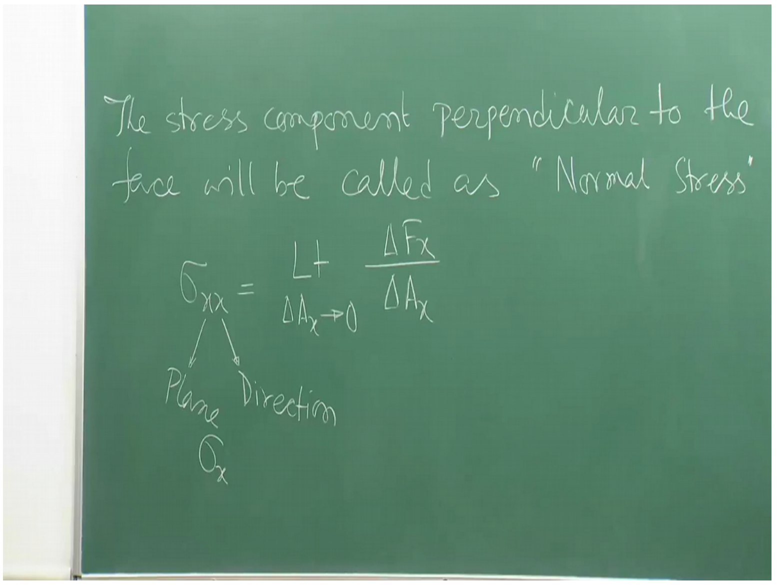

So, let us define that the stress component perpendicular to the face will be called as "Normal Stress". So, we are defining the stress as normal stress when the stress component is perpendicular to the face; that means, whatever plane we are considering if the stress component is normal or the perpendicular to that particular plane then that stress component will be defined now onward we defined as normal stress. That means, if you have this plane like this and if you have the stress components which is completely normal to this plane then that stress component will be defined as normal stress.

Now if you come back to this figure, whatever is shown here you have three components of the force like delta F x delta F y and delta F z. Now this delta F x if you look at, this face is nothing, but the positive x face as per our definition because the direction normal on this plane, on this face basically will be towards the positive x direction right. So, therefore, this face is nothing, but the positive x face. So, once it is positive x face then this delta F x will be acting as the normal or perpendicular to the plane right.

So, normal stress will be defined on that particular face like this sigma xx is equal to the definition will be remaining same delta ax tends to 0; that means, the area of that particular segment whatever I have shown you delta x means delta y into delta z right delta F x over delta A x. So, this is your, this is the definition of normal stress. So, this delta F x is the normal force acting on that particular plane and delta ax is the area on which this force is acting. So, when delta A x tends to 0 then basically that will define the stress and that stress will be defined or will be called as normal stress because it is acting normal to the plane.

So now there is one thing to be noted, it is shown sigma xx. So, sigma will stand for normal stress now onward whenever we will be talking about sigma then that will completely denote, I mean and I mean will not be repeating this thing again and again, but whenever will be writing sigma. So, that will denote the normal stress. Now after sigma I have written 2 subscripts xx. Now, what does it mean? So, please try to understand or please try to remember this thing throughout this course. Now we will be defining the stress right. So, when you define the stress basically you need 2 things - one is that on which plane the stress component is acting and in which direction right. Unless until you would you tell or you mention these 2 things then the definition of that particular stress is not complete. So, what I mean to say that if you write sigma xx; that means, sigma will denote for as I told you normal stress fine.

Now, xx means the first subscript will stand for plane on which the stress component is acting and the second subscript will stand for direction. So, normal stress generally when we are defining normal stress. So, we will be having the both the subscripts are same why? Because what is the normal stress normal stress is means on that particular plane along the same direction right. Now if this, this is the normal stress which we are defining for that particular x plane so that means, the all the x plane it is acting and along the x direction. So, x plane on x plane say this is why if it is my x plane along the x direction if the stress is acting then only I will be saying that thing as normal stress, so on x plane along x direction that will be defined by sigma xx that is the normal stress.

So, whenever you are having a normal stress. So, both the subscripts must be same. So, please try to understand now onward whenever we will be using this kind of notation. So, that will define the stress components on a particular plane along a particular direction.

Now for our convenience now onward because as per our discussion that whenever you are defining the normal stress then both the subscripts will be same. So, therefore, we will be simply writing sigma x or sigma y or sigma z. So, if I write sigma x; that means, that is the normal stress on x plane if I write sigma y that will be normal stress on y plane and similarly sigma z. So, we are not going to write now onward because it is quite understood. So, if I write sigma x; that means, both the subscripts are same xx. So, instead of writing twice now onward we will be writing only sigma x or sigma y or sigma z that will define the normal stress on the respective planes, is that clear.

Now apart from that, so what we have defined here? We have defined the stress based on the force if delta F x, now similarly you have other 2 force components one is delta F y another one is delta F z right. So, if you look at the figure if you come back to this figure. So, delta F x was perpendicular to the plane, but delta F y and delta F z these forces are along the plane right are parallel to the plane they are not normal to the plane they are parallel to the plane. So these forces we will be defining some different stress components. So, let us define that.

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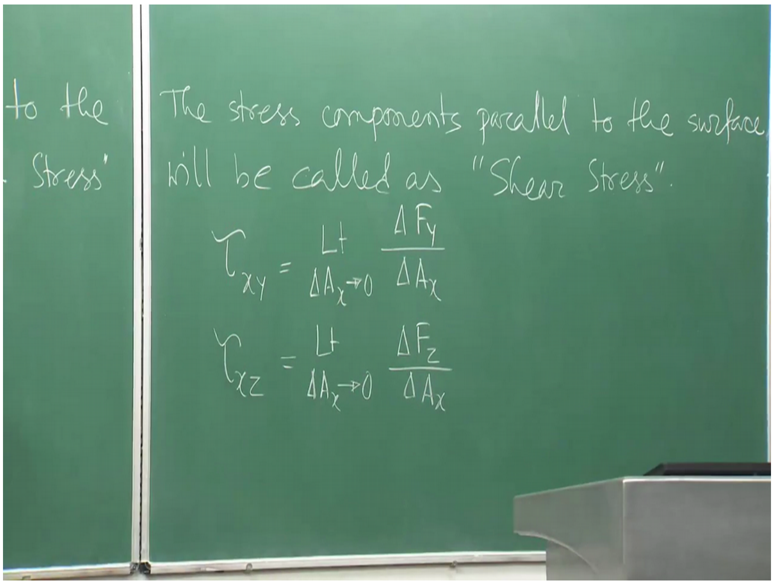

So, the stress component parallel to the surface will be called as “Shear Stress”. So, that will be defined by tau. So, whenever we will be writing the onward tau that will be simply defining the shear stress. So, tau, so if I write tau that will define the shear stress and then I can write x y limit delta A x tends to 0 delta F y delta A x. So, this is the definition of the one shear stress component. So, you will be having 2 shear stress components on a particular plane right. So, if we this is your x plane right normal to the plane you are getting normal stress that is defined by this and you will be getting one shear stress along the y direction another shear stress will be have occurring or will be acting along the z direction right. So, when I writing tau xy, so tau is defining the shear stress it is it is denoted I mean shear stress is denoted by tau as I told you.

Now what is the definition of these 2 subscripts? The first one will define the plane as I wrote here right. So, first one will be defining the plane. So, this tau xy when I am writing the first subscript is x; that means, this shear stress is acting on x plane, but in which direction the second subscript we tell about the direction. So, y is the direction, is that clear. So, whenever your subscripts are not same right. So, if your subscripts are saying xx then; that means, normal stress if your subscripts are not same then basically that will define the shear stress. You look at there, so that means, this stress component is acting on x plane, but in y direction and that is defined by this because which force is causing this kind or which force is developing this shear stress? Delta F y right, so delta F y over delta ax when delta ax tends to 0 that will give you give you the shear stress component along y direction on x plane.

Similarly, you will be having another component of shear stress that is tau x z. So, tau x z means the shear stress acting on x plane along z direction and that is defined by delta F z over delta ax because delta F z is causing that shear stress when delta A x tends to 0. So, what you have got from this, say discussion? You have got that on a particular plane, so along I mean through the point O I have drawn one x plane right we started with that through a point O we had drawn a line or drawn a plane which is nothing, but the positive x plane.

Now, on that particular plane you have got three stress components - one normal stress component and 2 shear stress components right. Similarly if you draw A y plane through point O the same point and if you draw A z plane through point O then you will be getting another 3 plus 3 - 6 components of stress understood. So, when I am drawing one x plane through point O I have got 3 stress components similarly I can draw a y plane through point O on which you will be getting another three stress components and I can draw another z plane through point O. So, you will be getting 3 more stress components. So, you will be getting total 9 stress components which will be defining the complete state of stress at that particular point O, is it clear or not right.

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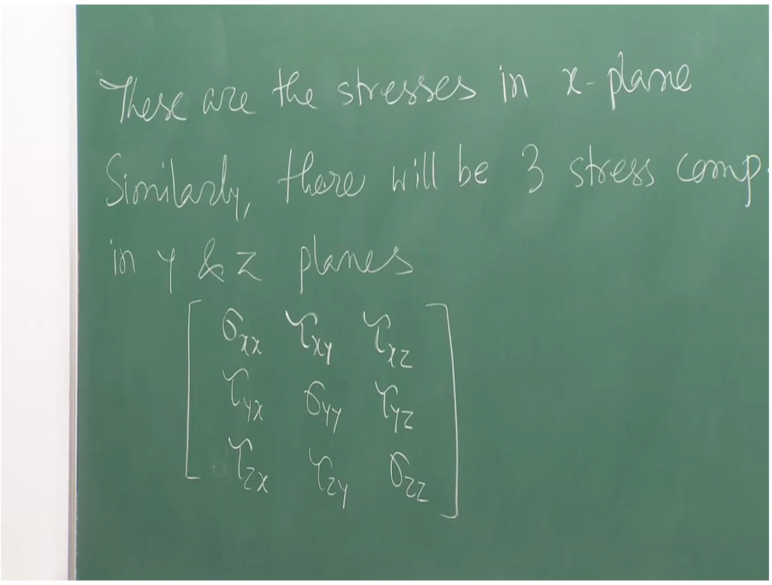

So let us write down that for our complete I mean discussion. So, we can write, so these are the stresses in x plane. So, whatever stress components I have shown you like sigma xx tau xy and tau xz all are acting on x plane. So, if you consider one x plane on which some force say delta F is acting. So, you can have the components of that delta F E along x direction y direction z direction and based on that you can define the stress component. So, you will be getting three stress components. So, these are the stresses are acting on x plane right.

So, now similarly there will be three stress components in y and z planes right. So, therefore, on x plane you have three stress components on y plane you have got three more and on z plane another three stress component will be coming. So, at point O you can define the state of stress with nine stress components. So, let us write down those stress components, so sigma xx tau xy tau xz, tau yx sigma yy tau yz and finally, tau zx tau zy and sigma zz.

So this is your in the matrix form, so this is your complete state of stress at point o. So, whenever somebody is asking you what is the state of stress at that particular point you have to find out all 9 components? So, out of this 9 components 3 are normal stresses which is quite obvious and other 6 components are the shear stress components. Now there is some specialty of this matrix if you look at this matrix along the row if you look at along the row basically it is defining the plane now first row is defining x plane the second row is defining the y plane and the third row is defining the z plane right. So, if you take the components of the first row then basically you are defining the stress components on x plane.

Similarly, the second row will defining the will define the stress components on y plane and likewise the third row will define the stress components on z plane. Now if you look at the column wise what you are getting here column wise you see this, these one normal stress and 2 shear stress components. So, they are defining the stress components along x direction that the first column will define the stress components along x direction it could be on different plane. So, it is not necessary that on x plane only x direction force will be there all stress will be there right. So, on y plane you can have x direction stress that will be nothing, but your shear stress. So, you are getting along column I mean one basically you are getting the stress components which are acting along x direction.

Similarly, the second column will give you the stress components are acting along y direction and likewise the third column will give you the stress components which are acting along the z direction. Now one more thing you need to remember or you need to see here that is where noting, along the diagonal this diagonal of the matrix is giving you will the normal stress components the diagonal of the matrix will give you the normal stress components.

So now with this discussion, if we define the x axis as 1 axis, y axis as 2 axis and z axis as 3 axis. So, 1 2 3 axis system if we define, then basically that will give me the stress components like this.

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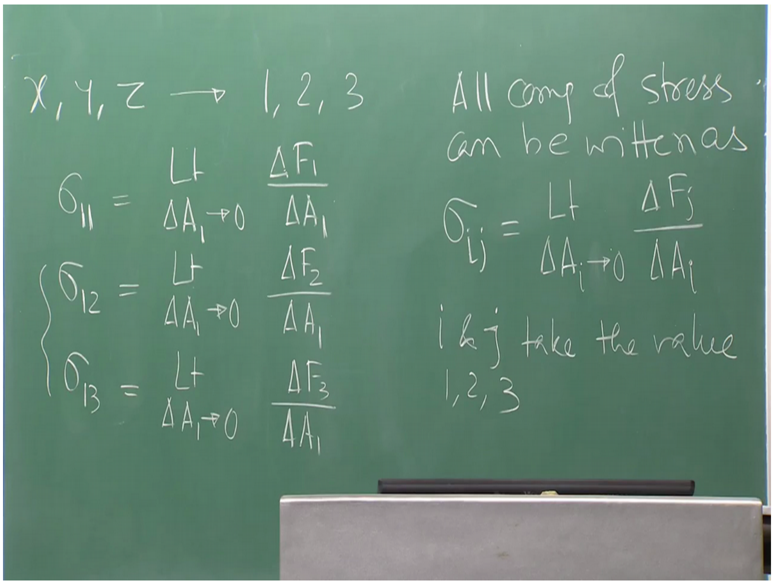

So, if we define x y z as 1 2 3 axis system then basically the on plane 1; that means, actually on plane x the stresses are acting sigma 11 limit delta a one that is the area of plane 1 tends to 0 delta F 1 delta A 1. So, that is the definition of the normal stress on plane one what is plane one basically we are doing, we are just mapping I mean this is the initiation of your indicial notations though that is not included in your syllabus, but I just I mean give you some indication or some kind of hints by which you can think of the in these a notations later on when we will be going through the advanced level mechanics course anyway.

So what we are doing here we are just mapping x y z to 1 2 3 coordinate system and then we are defining sigma 11 by this delta A 1 is nothing, but the area of plane 1 and delta F 1 is the force acting towards one direction. Similarly I can write now instead of tau I am writing sigma there is no problem because in the individual notation I mean when you will be writing your computer code or when you will be going through the advanced level mechanics course you will see that everywhere it is defined as sigma I mean stress is defined as sigma. Now if the subscripts are same then that is normal stress, if the subscripts are different then that is shear stress.

So, here your subscripts are different. The sigma 12 what does it mean what does it mean? That means that this is the shear stress on one plane along 2 directions. So, that is defined by limit on the same area tends to 0 delta F 1 delta V 1, similarly sigma 13 that is the shear stress acting on one plane act towards third direction three direction that is given by limit delta A 1 tends to 0 delta F 3 and delta A 1 agreed. So, these are the stress components one normal stress and 2 shear stresses right. So, these are the stress components acting on plane 1.

Similarly you can define other stress components the three stress components on plane 2 and plane 3 right. So, what we can write all components of stress can be written as. So, all component of stress can be written as in the indicial notation sigma i j is equal to limit delta A i equal to sorry; tends to 0 delta F j delta A i where i and j take the value 1 2 3.

So, now if you look at this, this is the basically initiation of your indicial notation kind of thing. So, that is that is very much there in your advanced level mechanics; however, we are not I mean this is, I mean I just wanted to say this thing at this stage so that you will understand the genesis of your indicial notation; however, we are not going to discuss this thing further in this particular course. So, sigma i j is your stress components if i is equal to j then basically that will be giving you the normal stress, if i is not equal to j then that will give you the shear stress as per our discussion and then the sigma i j the stress components is given by this equation whatever we have derived from this where i and j will take the value of 1 2 3 fine.

So now if you look at the stress components acting on different planes, so, if I consider one point O, so this is my point O, this is my point O and around point O if I consider a parallelepiped, I am drawing one parallelepiped in the x y z coordinate system and then we can find out these are the stresses acting on each plane.

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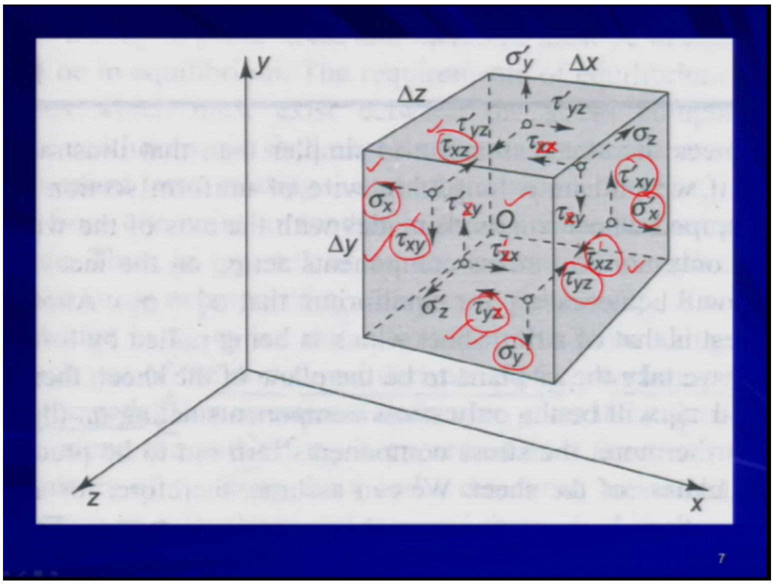

So, how many planes are there 2 x planes, 2 y planes, 2 z planes - 2 x plane means one positive x plane, one negative x plane right; similarly one positive y negative y and one positive z and negative z planes right. So, there are six planes. So, on this 6 planes, this is the point O right. So, we are taking we are drawing the parallelepiped around point O.

So, what is this plane? This plane is nothing, but your negative x planes on that negative x plane these are the stresses are acting sigma x tau xy and tau xz right. Similarly if I consider this is my negative y plane right this plane this plane is the negative y plane. So, on that plane you are having sigma y tau yx and tau yz and likewise all the stress components are acting on the respective planes.

Now you may ask me; what is this sigma y prime. So, on this plane on this negative x plane you have sigma x tau xy and tau xz, but on the positive x plane you have sigma x prime tau xy prime and tau xz prime right because they will be different. Why they will be different? Because you are moving from this point to this point this much of distance you are moving. So, there will be some variation of stress so that I will be coming later on when we will be talking about the equilibrium condition and all. So, basically due to this variation you are getting the difference in the stress components, but their definition will be remaining same - one normal stress and 2 shear stress components.

So I will stop here today. So, in the next lecture we will be talking about the plane stress condition that is nothing, but the special kind of stress set of stress than the, I mean special kind of state of stress when you are talking about the 2 dimensional state of stress. So, that we will be discussing because this whatever we have discussed so far, this whatever is discussed here, basically that will give you the three dimensional state of stress right when you have all three coordinate axis or all three I mean the stresses are acting in all three directions, at that time whatever stress components you are getting.

So, that is defined by this matrix, but we will we will be restricting most of the mechanics I mean mechanics problem at least in particular, in this particular course we will be dealing with the 2 dimensional state of stress and there will be getting either plane stress or plane stress when we will be talking about the strain at the time it will becoming as plane stress. However, in the next lecture we will be talking about the plane stress condition which will be coming out as a special case from this general condition, general state of stress. So, I will stop here today.

Thank you very much.