Mechanics Of Solids
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Lecture – 16
Tutorial on Force Displacement Relationship and Geometric Compatibility – 1

Welcome back to the course Mechanics of Solids. So, in the last lecture we were talking about the thin walled pressure vessel that is and we found out the force developed in that thin walled pressure vessel as well as so we have found out the radial expansion due to the application of the internal all round internal uniform pressure. And then we talked about the mechanism of the belt around a wheel, so the when the wheel is not frictionless, so there we found out the tangential force developed at one point of belt will not be same at the other points. So, because of your friction, it will be that force will be increasing in the exponential order.

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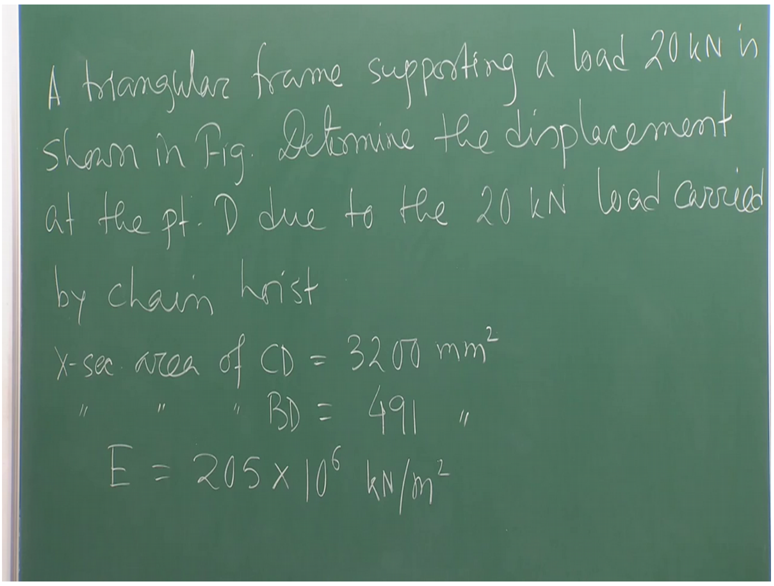

Now, we will be taking couple of numerical problems. So, first problem it says a triangular frame supporting a load 20-kilo Newton is shown in figure. I will show the figure later on. Determine the displacement at the point D due to the 20-kilo Newton load carried by chain hoist.

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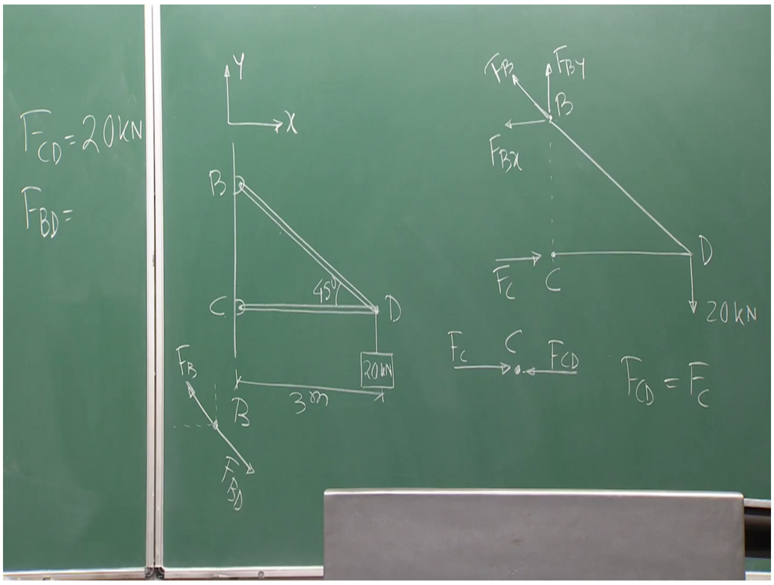

The problem says, so this is the figure. So, the problem says a triangular frame supporting a load this triangular frame is this. So, B C D is supporting a load of 20 kilo Newton as shown in the figure. So, determine the displacement at the point D. So, this is displacement of point D due to the 20 kilo Newton load carried by the chain hoist. So, this is the chain hoist because of that you need to find out the displacement of D, where it is given the cross sectional area of CD equal to 3200 millimeter square, cross sectional area of BD is equal to 491 millimeter square. And the modulus of elasticity of both the members is made of same material, so E is given as 205 into 10 to the power 6 kilo Newton per meter square.

So, before going to find out the geometry compatibility and all what is your first step first step is to study the forces by exploiting or by satisfying the equilibrium conditions. So, let us draw the free body diagram of this frame. So, the free body diagram will look like this. This is point C, you will be having the force F C from the support, this is point D, this is point B. So, similar kind of problem already we have solved earlier. So, you will be having the reactions F B y, F B x and this will be the resultant FD.

Now, again for the sake of completeness, I can ask you that why I have considered we can discuss this thing that why we are considered F C in the horizontal direction and F B along the member BD. So, these things already should be known to you already we have discussed this thing earlier when we are discussing similar kind of problem when we are talking about the I mean when we are discussing or when you are determining the support reactions anyway.

So, now basically what we are going to do we are going to find out the support reactions, because without knowing the support reactions basically you cannot proceed so that is why the free body diagram has been drawn. And then support from the support reactions basically once you know the support reactions, then you can consider different joints and you can find out the member force. So, once you know the member force then only you will be able to find out the geometric deformation. The second step is that study of deformation, is not it? The first step is the study of forces.

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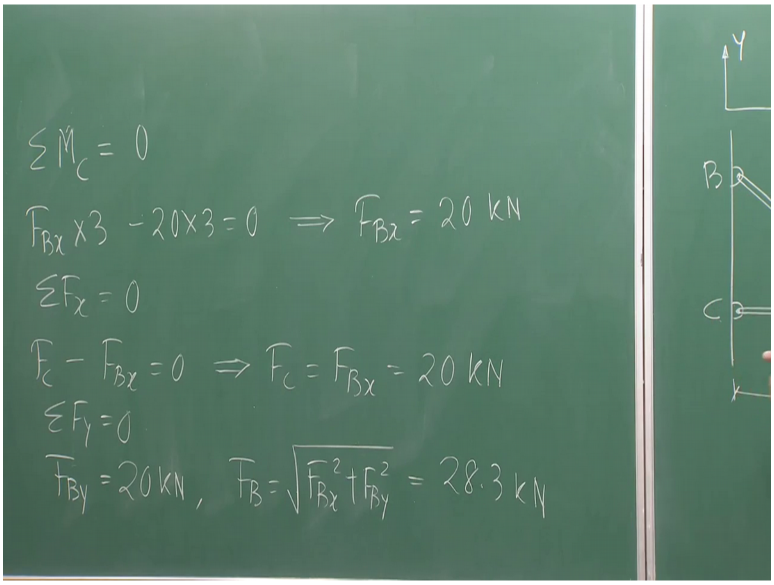

So, now we are studying the forces. So, deformation will be coming later on when you know the forces are acting on different members. So, now, considering the equilibrium that is we are considering movement equilibrium that is summation of moment with respect to point C is 0. So, what we can write F B x into 3 minus 20 into 3 is 0. So, from that we can get F B x equal to 20 kilo Newton.

Similarly, we can exploit this condition summation of F x where you can think of this is my x y direction, summation of F x equal to 0. So, I can simply write F c minus F B x equal to 0. So, from which I can get F c equal to F B x equal to 20 kilo Newton. Similarly, I can exploit this condition as well summation of F y equal to 0 from which I can get F B y what are the forces acting in y direction F B y and 20 kilo Newton. So, F B y will be simply 20 kilo Newton.

So, therefore, we can get F B equal to root over F B x square plus F B y square similar thing we have already done for earlier in the lecture, so that will be giving me 28.3 kilo Newton. So, now, we will try to find out the forces in each member. So, what are the members I mean do exist in the problem member BD and member CD. So, we need to find out the forces carried by each member of the frame. Now, we know the support reactions; that means, F B and F C both are known to me. Now, if we consider each joint, so joint C if we consider or if I consider joint B, I can find out the member force in CD and BF.

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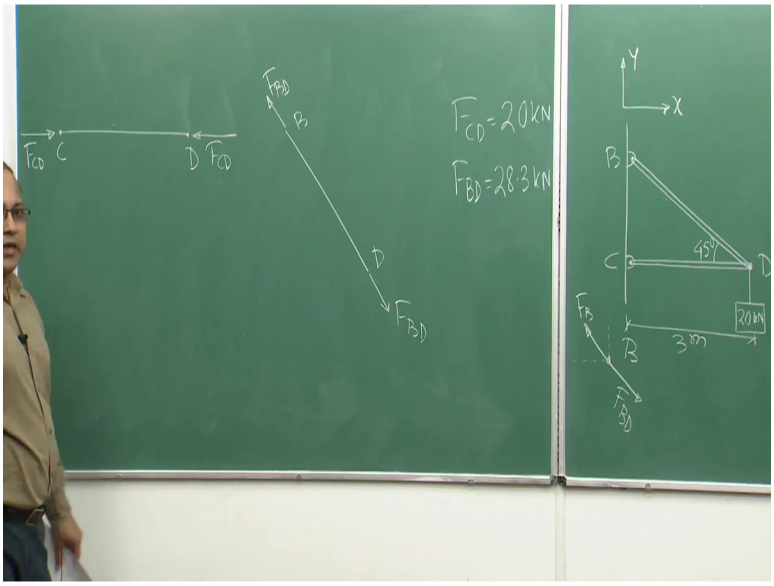

So, let us do that. So, we are considering. So, force now we are talking about and also still we are studying the forces say we are try to find out the member forces. So, this is my member CD, and we are considering the member force in CD is compressive in nature. I mean I told you several times I am telling this thing that you not to think about whether it should be compressive or tensile; right now I am considering compressive that does not mean that I know the answer that is why I am writing. So, you can take tension, if it is really tension then it will become positive; if it is not really tension then it will becoming negative.

Now, for example, if I consider compression and if I get positive value of F C D then it is really compressive; if it is not really compressive then it will becoming negative. So, this thing I mean several times I am saying you. So, I mean you need not to think much about these things the direction of the force. Well, and I am considering BD say tensile. So, I am considering BD is under tension that is F B D is tensile in nature and F C D is compression force. So, now, you need to find out the force F C D and F B D; F C D and F B D.

Do you know the magnitude numerical value of these forces? Yes, you know. How do you know? If you consider the individual joint, so suppose I am considering joint C this is my joint C, this is F C and this will be F C D. So, F C D is nothing but F C, agreed? So, therefore, whatever direction we consider that is compressive force F C D is compression in nature then that comes perfectly fine. So, F C D is equal to F C. So, I can write F C D equal to F C D was F C was 20 kilo Newton. So, this is 20 kilo Newton: and what about F B D? So, you consider joint B, so this your joint B. So, from joint at joint B you are having F B. So, this is the vertical direction, this is the horizontal direction and this is your F B D.

So, from this from the equilibrium of joint B, you can simply say F B D is nothing but equal to F B. So, F B already you have calculated. So, F B D will be F B is nothing but 28.3 kilo Newton. So, my study on the forces is over. So, I have got the information about the support reactions; and from the support reactions, I have got the information about the member forces. So, the study of forces is all over. Now, we will be studying the deformation.

Now, if you look at this member CD and I am calling I am saying that member CD as well as member BD both are deformable bodies right, I am not talking about by rigid body or something like that, they are deformable bodies. Now, under this action of force F C D, the member CD will be compressed. Now, what will be the amount of compression that we can calculate by using your Hooke’s law p l by A E. Similarly, if you consider member BD under the action of this tensile force F B D that member BD will be elongated how much will be the elongation that also we can calculate by using the simple Hooke’s law. So, let us calculate those things and let us get the numerical values.

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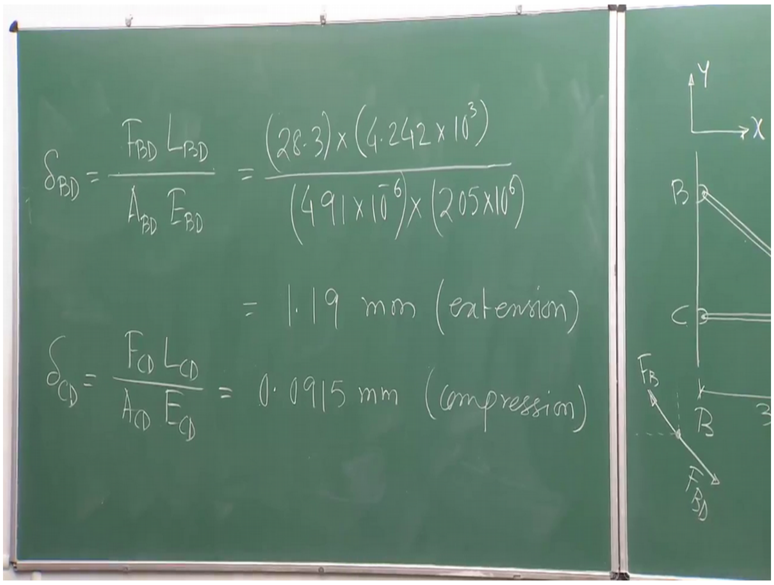

So, your delta BD So, delta BD is the expansion or the elongation or the member BD because that BD member is under tension so that is given by F B D length of BD member cross sectional area of BD member and E B D. So, you put the numerical values here F B D is 28.3 into L B D you can calculate 4.242 into 10 to the power 3 by area is given 491 into 10 to the power minus 6 into E is given 205 into 10 to the power 6. So, from this you can get the value will be coming as if you calculate the value it will be coming as 1.19 millimeter. Similarly you can calculate F S delta CD which will be compressive which will be the shortening of the member CD. So, that is F C D L C D A C D E C D. So, if you put all the values you will be getting 0.0915 millimeter this is you can write extension and this is compression.

Now, whatever values you are getting, what it will indicate that means if you do not have the frame, suppose you have individual member BD and CD under the forces F B D and F C D respectively then member BD will be extended or will elongated by this much amount, and member CD will be compressed by this much amount. if I do not considered any assembly, but when they are actually connected or when they are present in an assembly then basically they I mean these values have to be satisfied, but at the same time they cannot be they cannot behave individually.

What I mean to say the joint suppose if I consider the elongation of BD, the BD will be elongated like that and CD will be compressed like that, then where is the point D. So, now CD after compression D point will be coming here, and after elongation of BD, D point will be coming here that is not acceptable because they are acting in an assembly. So, the D point will be the common point even after deformation; before deformation D point was the common point where CD member and BD member both members are meeting; even after deformation D point will be the common point for both the members, but at the same time they have to experience this much of deformation.

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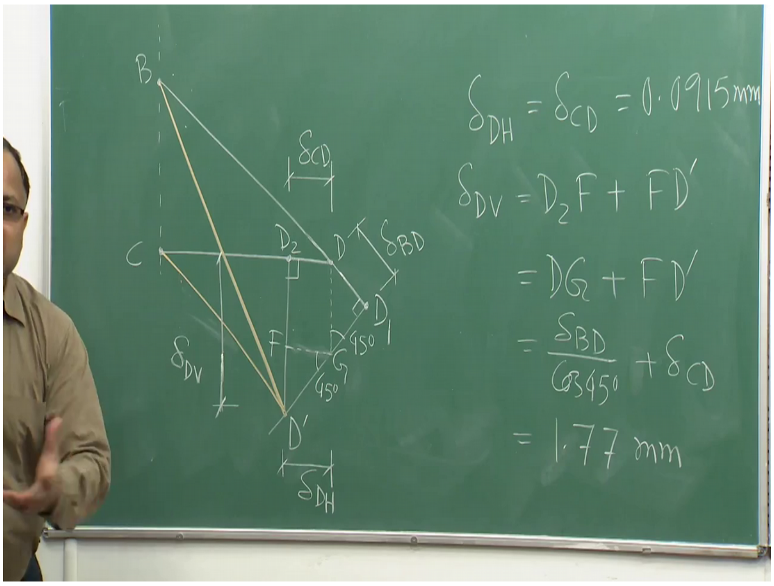

Now, how to do that? So, let us draw the geometric compatibility figure and then I will explain. Let me draw it. Now, let us understand this figure first. As I told you that if BD and CD are not in the assembly, so this BD is free to elongate up to BD 1 agreed? If what I what I said if BD and CD are not making this frame or they are free to expand or they are free to deform then BD due to the action of F B D that is the member force BD will be elongated up to BD 1. So, this from D to D 1 distance is nothing but the elongation delta BD whatever we have calculated. Similarly if CD is not in the assembly if CD is free to deform then this CD will be compressed under the action of F C D, and it will take the position of D 2 C D 2. So, D will be shifted to D 2.

So, now, CD 2 is the compressed member if we are allowing free deformation of the members. So, D, D 2 is nothing but your delta CD agreed. So, what and for I mean for the sake of completeness, we must think that this problem we are defining with small deformation problem. If we consider this is the same problem with large deformation problem the formulation and analysis will be completely different, but so far as I told you in this particular course we are not talking about or we are not concerned for that large deformation problem, we will be only restricting ourselves to the small deformation problem.

So, if it is the small deformation problem, so the figures of course it is little bit exalted figure to understand in a better way. So, this D D 2 is your delta CD, and D D1 is your delta BD. So, basically BD will be taking the point D 1 and CD will be taking the point D two if they are free to expand or compress, but that is not happening because D point is the common point. So, I cannot have some different point D 1 and D 2. So, they should be in the same point.

So, what is happening then D if you consider D will be taking the arching kind of thing is. So, D point this is the common point for BD and CD this point will be becoming D prime point. Where this BD is getting expanded and CD is getting compressed. Though these angles and all those things are looking not looking like the same angle but for the small deformation problem and as I told you this is a very exerted figure, but those angles will be remaining same, anyway.

Now how to get this D prime point what we are doing here from this point D 1, we are drawing one normal to line BD or the BD 1 other we are drawing we are dropping one normal to this that line is coming like this. And from this CD from point D 2, we are dropping another perpendicular line. So, these two perpendicular lines are meeting with a common point D prime. So, frankly speaking that should happen through the I mean I mean circular arc, but nevertheless if we consider like this dropping the normal that will be also fine I mean that will not create any significant difference because we are dealing with the small deformation problem.

So, what we did we did we draw a normal line or the perpendicular line from D 1 and another one perpendicular line from D 2, which will be meeting a common point D prime. Now, this D prime is the final point after deformation before deformation it was at D now after deformation it is becoming D prime. So, what is the displacement of point D. So, D is coming to D prime. So, this much is your horizontal displacement or horizontal shift of point D and D is coming to D prime. So, therefore, this is your vertical displacement of point D.

So, if it is so then from the figure it is very clear that is delta DH that means, the horizontal displacement or the horizontal shift of point D is equal to delta CD. You just look at this figure is equal to delta CD is nothing but 0.0915 milimeter; and what about your delta dv; that means, the vertical displacement of point D that will be if you look at figure D 2 F plus F D prime. So, D 2 F D 2 F plus F D prime. It can be further written as D 2 F is nothing but D 2 it has D G, you look at here D G this dash line or the dotted line plus F D prime. Now, this DG can be written in terms of delta BD in this session. So, delta BD by cos 45 degree plus. Now, what is F D prime, what is F D prime? So, F D prime is nothing but your FG because this angle is 45 degree. So, if this angle is 45 degree, so FD prime is nothing but equal to FG. Now, what is FG, FG is nothing but equal to delta CD. So, I can simply write delta CD.

So, by putting the values here you know delta BD, you know delta CD the numerical values by putting the values here you will be getting 1.77 millimeter. So, what did you get, you got the horizontal displacement or the horizontal shift of point D is this much and the vertical shift of point D is this much. So, this is the way you solve the geometric, I mean basically this is this figure is coming from your geometric feet or the geometric compatibility because D point cannot be different. So, you cannot have D 1, D 2 two different points after deformation it will be the same common point. So, D has to take the position of D prime

Well, I will stop here today. In the next class, we will take another numerical problem and then we will move to the next topic.

Thank you very much.