Mechanics Of Solids
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Lecture – 15
Mechanism of belt around wheel

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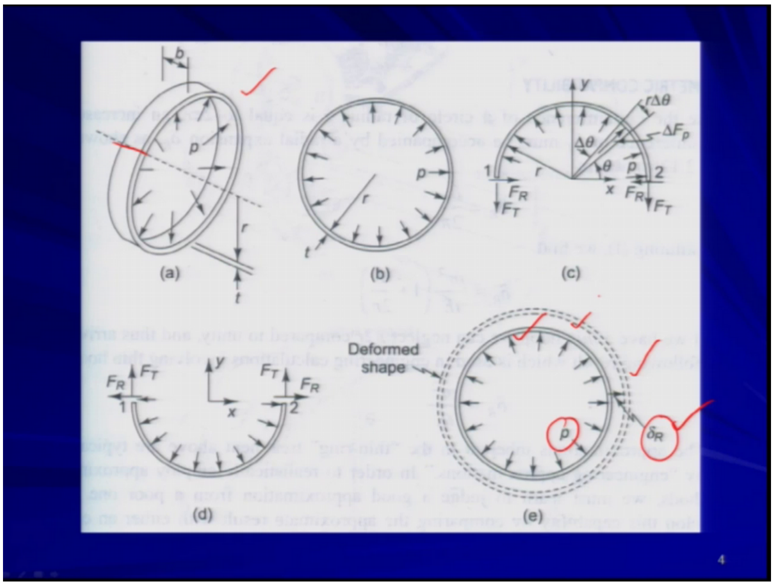

Welcome back to the course Mechanics of Solids. So, in the last lecture, we stopped when we were discussing about the thinned walled pressure vessel or the hoop. So, if you look at in the last lecture whatever wherever we stop, so basically what we considered, so we considered this kind of say hoop. So, where the b is the width, and say r is the internal radius, and t is the thickness and you have all round uniform internal pressure say a small p. We are discussing about this thing and we found that F R that is a radial force must be 0 to satisfy the symmetry of the system, whereas, the tangential force were calculated that F T we have got the expression for F T. And then we are talking about the I mean due to the application of this internal pressure our now our objective is to find out the radial expansion of that particular hoop, so as shown here right. So, this is your delta R that is the radial expansion so that we are going to find out.

So, this radial expansion when we are talking about. So, for that what we did we as we decided is that we are cutting this hoop here and we are stretching it out, we are making it a flat plate under the action of the force F T that is F T is I mean the tensile force which is acting on this flat plate right. And that for that application of the force, we got the expansion or the increase in the circumference.

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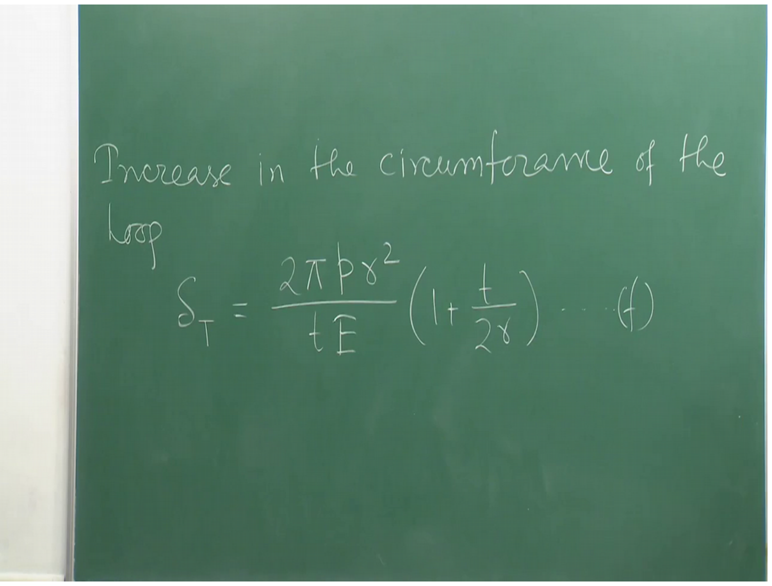

Increase in the circumference of hoop was calculated as delta T. If you go back to your previous lecture, so basically you will see that how we have got delta T. So, delta T was coming as 2 pi p r square by t E into 1 plus t by twice r, so that was equation if we have defined. So, what is this expression? So, this is the expansion of the or the increase in the circumference that means, as I told you that if you cut this hoop, and if you make this hoop as a flat plate and then when you are making it flat plate under the action of say force F T, then this will be the increase in that flat plate.

Now, this increase is happening in the circumference that means, you are cutting the hoop and you are stretching it out in a flat plate, now this is the expansion is happening in the total circumference. Now, with that increased circumference now if you are making the hoop, so then you will be getting the radial expansion. So, what I mean to say this delta T will cause ultimately this a total increase in the circumference. So, this increase in the circumference will actually mapped to the radial expansion. So, what I can say that my radial expansion delta R.

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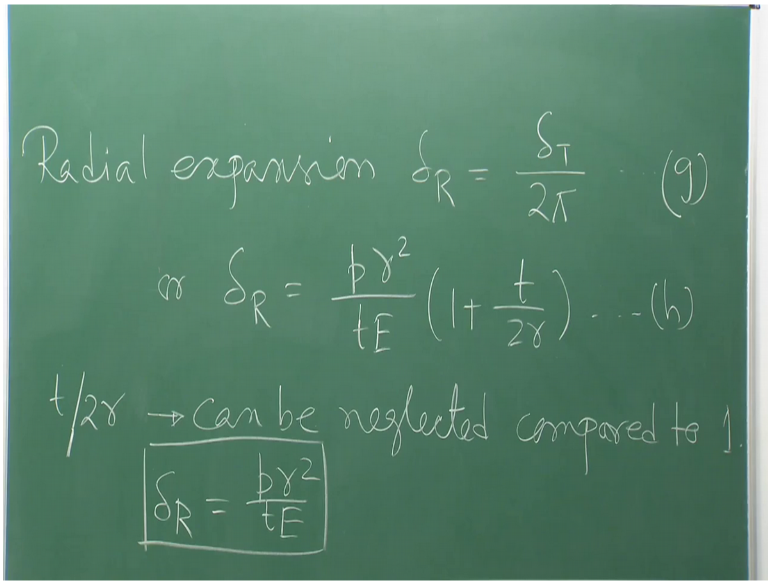

This is my ideal expansion delta R can be written as delta T by twice of pi. So, this is the equation g. So, this delta T is the total increase in the circumference length as obtained from equation f, and this delta T will be uniformly distributed to get the radial expansion by this equation. So, delta R and delta T are related by this expression that is very simple from the geometry itself. So, we can write our delta R is equal to p r square from this expression by t E into 1 plus t by twice r. So, this is the actual expression of delta R. Now, this t if we considered a very thin as I told you that we are talking about very thin hoop or the thin pressure vessel thin walled pressure vessel. When we are talking about thin walled pressure vessel that means, the t is very, very negligible compared to your diameter of the hoop compared to the diameter of the hoop your thickness of the wall or the thickness of the say pipe or the vessel is very, very negligible.

So, if it is so then we can for thin wall pressure vessel can be neglected compared to one. So, therefore, we can simply write delta R equal to from this expression. So, t by twice r is very, very negligible. So, we can neglect this part. So, simply it will be coming as p r square by t E. So, this is your radial expansion of the hoop as you can see from this figure. So, this is yours delta R as you can see. So, under the action of this internal pressure, this was the original hoop. Now, after application of this internal pressure, the hoop will be taking the configuration of this and this delta R can be calculated by this.

So, if you know how much pressure is acting, if you know the initial radius of the hoop as well as thickness of the wall and the material property is nothing but the modulus of elasticity. If you know these things you can find out delta R. Now, one thing is you can see from this your b that is the width of the hoop is not participating here to find out the radial expansion. So, the radial expansion is independent of your width of the hoop that is b.

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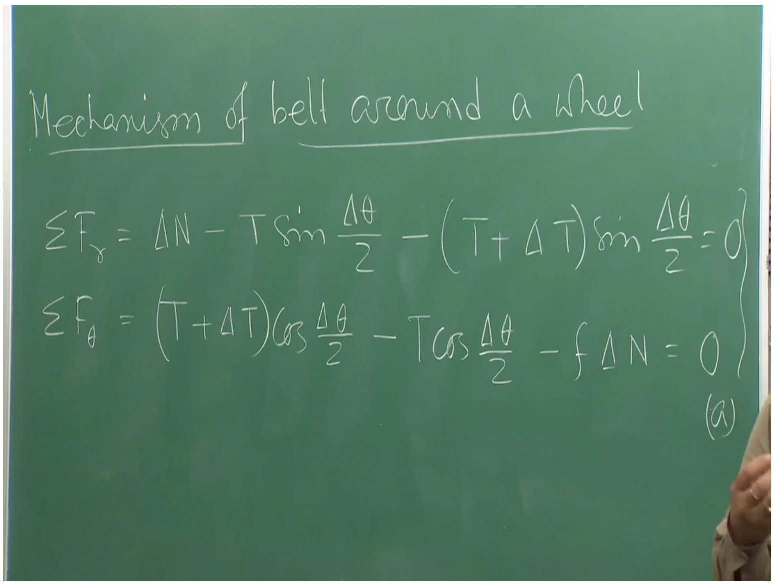

So, now we will talk about the next topic. Now, our next topic is mechanism of belt around a wheel. Now, what for this is required and why you want to study this mechanism? Now, you might have seen this kind of machine components or the machines.

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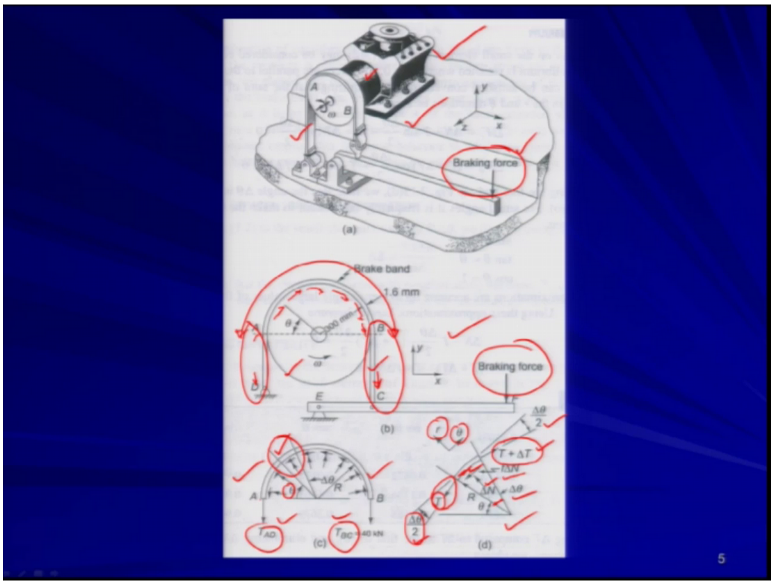

So, this is the machine part, this wheel is rotating. Now, if you want to stop this wheel; that means, if you want to apply the brake the brake generally is made of some belt - rubber or leather kind of belt. So, this belt is here. So, this is called brake band. What will happen, you will apply the braking force here by pressing it, once you press it the belt will be tightly fastened on the wheel. So, the movement or the rotation of the wheel will be stopped by the action of the or the friction developed at the interface between the belt and the wheel. So, this is the common thing or common phenomena we generally see in our day-to-day life in different mechanical questions or the mechanical system.

Now, for that actually we need to because we need to find out or we need to design this belt. So, when we are talking about the design of the belt that means, we need to know that how much for this belt is carrying. So, if I do not know the how much tension because when you are applying the load on the when we applying the braking force basically that will give the tensile that will generate the tensile force in the belt and that tensile force basically the tangential force if you see I mean we will come to that point, so that tangential force will be will be balanced by the frictional resistance. And that in that way actually in that process basically the belt will try to resist the rotation of the wheel. So, it will be tightly touching or it will be tightly fasten the wheel.

Now, the idea is that, so if you look at this complicated system in very simple say line diagram. So, this is the wheel and this belt is around the wheel this is the brake, brake band. This brake band is around the wheel. And you are applying the braking force here. So, due to that you will be getting you will be applying some tension here and here also some tension will be developed and then ultimately you will be getting the all round say if it is rotating like that. So, you will be getting the friction force will be developing at the interface between the wheel and the brake band. So, we need to find out that how much tension is carried by the brake band then only we can design this brake band or rather belt. So, this is the this is the requirement or this is the objective actually to study this mechanism.

So, now, if you look at this figure, so as I told you that here there will be a tensile force will be acting here there will be a tensile force will be acting. So, if we consider or if you assume that the half circle that means, from A to B that is the half circle of the wheel half of the wheel. If the half of the wheel is completely touched by the belt or the brake band then basically this is the free body diagram of the belt or the brake band. So, now one word I will be talking about I will be calling as belt. So, this belt will be under the action of this tension and this tension. And because the wheel is moving in this direction, so you will be getting radial direction you will be getting the development of the normal force and due to the interface friction between the belt and the wheel you will be getting the frictional force.

So, now if you look at so now, I am considering a small element, I am considering a small element and I am blowing it up here this small element is taken theta angle from this horizontal. And this I mean the element angle angular element is delta theta as shown here this is angle. So, now on this side basically so what you are observing that here T AD is here say suppose at this point we are having say T AD here we are having T BC. So, they are not same say right. So, T BC why you are calling T BC, so because this is the part as ad and this is the part BC, so therefore, T AD. So, T AD is not equal to T BC.

Now, why it is not equal earlier you might have solved different problems frictionless pulley or frictionless wheel there you considered the tension on this side and tension on the other side both are same. But now this is not your frictionless wheel we are considering the friction in between the wheel and the belt therefore, that the frictional force will try to make the difference between the tension on both the sides that means, T AD is becoming different than T BC.

Now, so if I consider a small element like this so on this phase we have a tangential force t. So, on the other phase, we will be having some differential increment in the tangential force that is t plus delta T, agreed, because the tension is not same. Now what is causing this difference as I told you the frictional force. Now, this belt will try to give some radial force right radial normal radial force means that will be normal on the belt. So, that is say we are calling as delta N. And due to this radial force you will be getting the development of your frictional force that is F into small f into delta N, where small f is the friction coefficient fine, up to this there is no problem. So, this is the free body diagram.

So, the wheel is moving in this direction. So, the frictional force will be acting in this direction. So, this is the direction of frictional force. So, now and this is the direction of r; that means, radial direction this is the direction of theta that is the hoop direction. And this angle of course, will be delta theta by 2 from the geometry, this angle because this angle is delta theta. So, therefore, this angle will be delta theta by 2 that will be coming from the geometry.

Now, we are going to find out or we are going to establish the force balance for different directions. So, first one is we are balancing the force in the radial direction because that will be on equilibrium. So, we have to satisfy the conditions of equilibrium. So, this is the first thing is that delta F r must be 0. Now, what are the forces are acting in the radial direction. If you look at this figure, so what are the forces are acting in the radial direction, this component of t in the radial direction this component of t in the radial direction, component of t plus delta T in the radial direction and of course, delta N is acting in the radial direction. So, these are the forces are acting in radial direction.

So, if you look at this figure, I can simply write delta n; that means, towards the positive radial direction if I consider the outward direction is positive. So, delta N minus T sin delta theta by 2 plus sorry this will be a minus minus T plus delta T sine delta theta by 2 is equal to 0, is it clear? So, these three forces are acting in the radial direction. Similarly, I can take the force balance in the hoop direction or in the tangential direction. So, there what are the forces are acting T plus delta T cos delta theta by 2 along the tangential direction minus opposite T cos delta theta by 2 anything else. So, these two forces are acting in the tangential direction any force is left out the frictional force right, the frictional force is acting along the tangential direction, so minus F into delta n is 0. So, these two equations I am calling as equation a.

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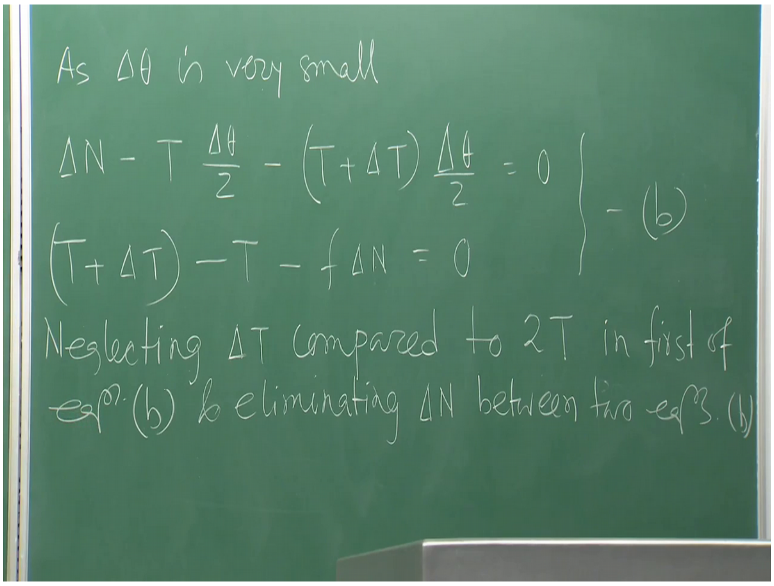

So, as your delta theta is very small because we have considered a small element. So, therefore, we can say simply from equation a delta N, so sin delta theta by 2 will be delta theta by 2. So, sin theta is theta and cos theta is one because theta is very, very small. So, by applying that condition I can simply write T delta theta by 2 minus T plus delta T and delta theta by 2 is equal to 0. And from the second equation, I can simply write T plus delta T because cos theta is 1 minus T minus f delta N is 0. So, these two equations I am calling equation b.

So, now, what we are doing neglecting delta T compared to 2 T in first of equation b and eliminating delta N between two equations of equation b. What we can write what we can write, so what we are doing here, we are neglecting delta T compared to 2 i sub T. So, basically here if you look at say twice of T into delta theta by 2 and that will be delta theta by 2 into delta T. So, we are considering delta T is very, very small compared to twice of T. So, therefore, we can neglect that part and we are eliminating delta n between these two equations.

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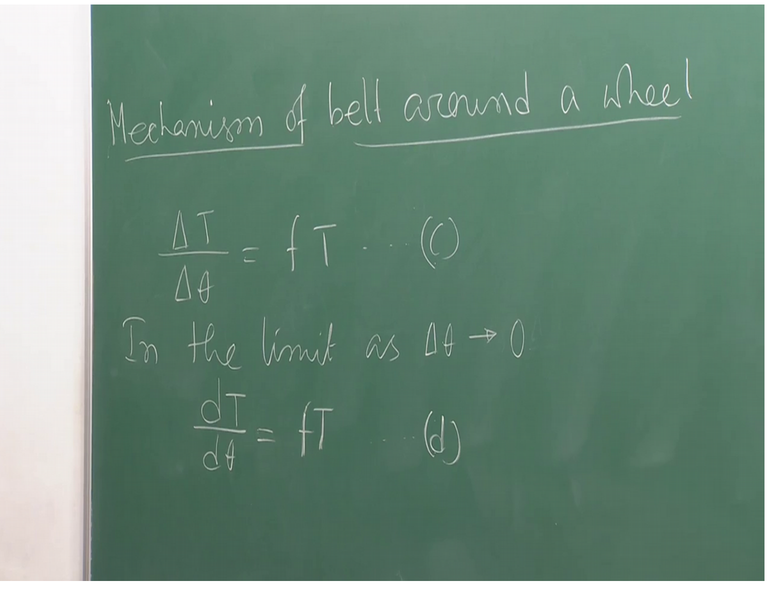

So, if we do these two operations and then we will be getting simply delta T by delta theta is equal to f into T. So, equation c. In the limiting condition in the limit as delta theta tends to zero we can simply write d T d theta is equal to f into T.

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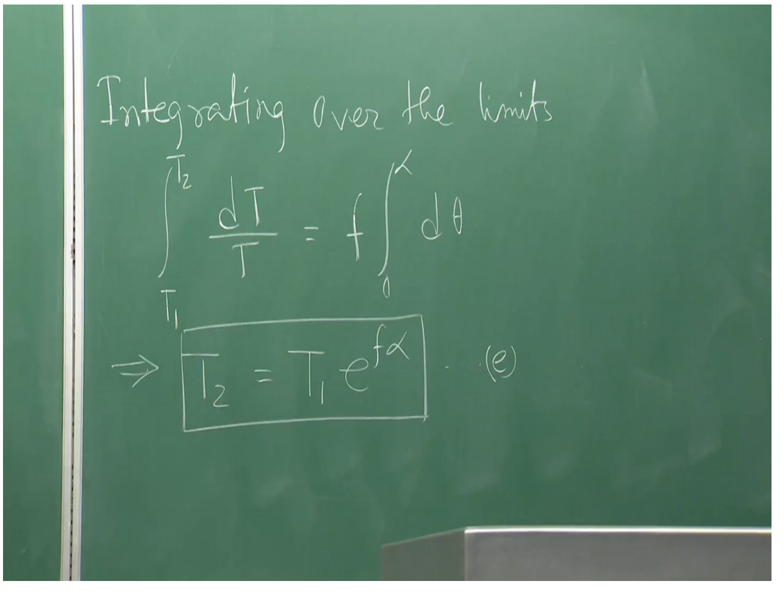

So, now, if we are integrating, if we integrate over the limits say T 1 to T 2 dt by T is equal to f into 0 to alpha d theta. So, from this I can simply get T 2 equal to T 1 e to the power f alpha.

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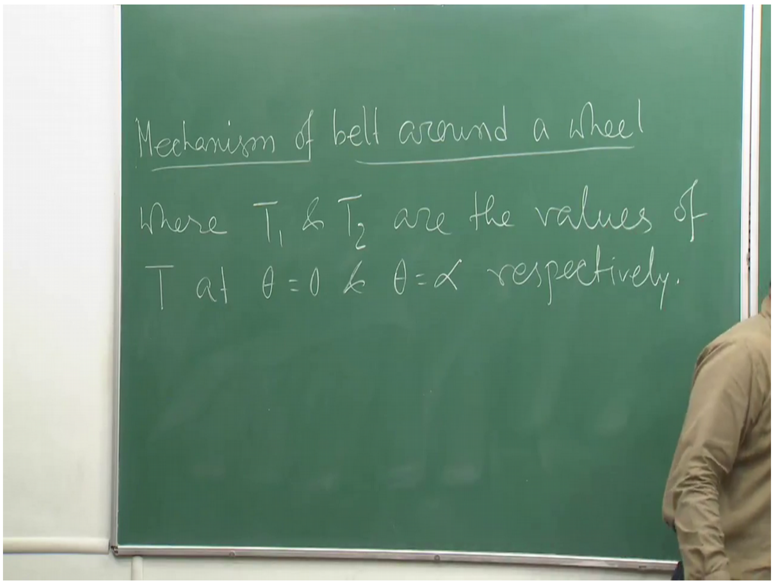

Where T 1 and T 2 are the values of T that is a tensile force T at theta equal to 0; and theta equal to alpha respectively. So, from this equation if you look at this equation if your f is 0; that means, if we do not consider any friction or say frictionless wheel you might have heard this stall several times in your physics when we you talked about your pulley problem or something like that you considered frictionless wheel. So, frictionless wheel means f is f you have considers as 0. So, if you consider f is 0 then basically T 2 will be always equal to tone then no matter wherever you consider the belt or the what section you are considered the I mean what section of the belt e is considered, so that is immaterial.

So, basically you will be getting all round same amount of tension, but if the pulley is not or if the wheel is not frictionless. Then basically your initial say tension if it is T 1 that means, the theta equal to 0. The initial tension is T 1 then that tension or that tangential force will be will be increasing that is T 2 basically at some point say theta equal to alpha if you are going to find out the tangential force T 2 then basically that would be t one multiplied by e to the power f alpha. So, it will be exponentially increasing. So, it will be exponentially increasing so that you can you can see and based on that you can basically design the brake band or the belt and at the same time you can you can find out that how much tension will be getting developed at any particular location where the belt is touching the wheel. So, this is all about your next chapter the chapter two where we are talking about the geometric compute with the other things. So, I will stop here today. So, in the next lecture, we will be solving couple of numerical problems.

Thank you very much.