Mechanics Of Solids
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Lecture – 14
Hoop Stresses

Welcome back to the course Mechanics of Solids. So, in the last lecture, we have discussed about the analysis procedure for the deformable bodies. So, what are the steps involved there. So, there are three steps basically, the first step was there to study the force and equilibrium requirements, and then the second step was there to study the deformation and geometric fit of the compatibility, and the third step basically you are establishing the relation between the force and the deformation.

So, based on that we came out with a relation for the uniaxial system that is the delta is equal to pl by AE that we have derived in the last lecture, and that is nothing but your simple Hooke's law, whatever you have learnt in your 10 plus 2 in physics.

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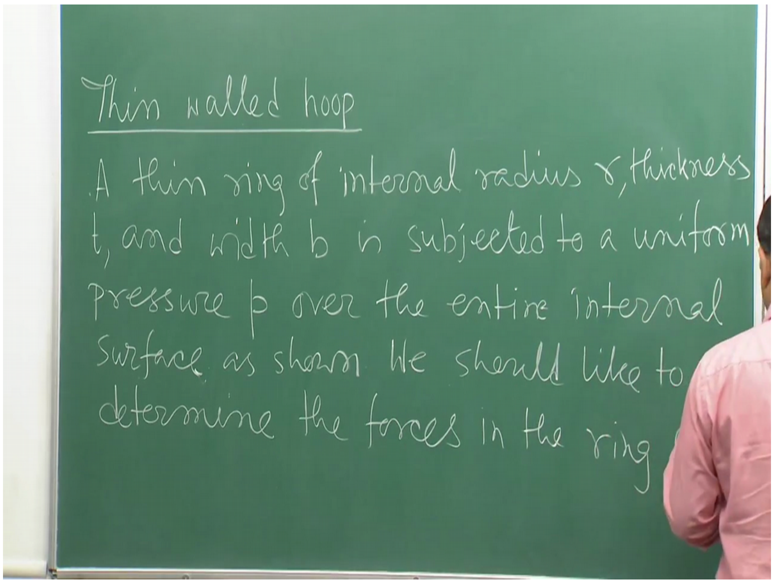

So, now today we will be talking about the analysis of thin walled hoop. So, this is very thin walled hoop. Generally, you see for the pipe line which is transferring say fluid or gas or whatever. So, these kinds of pipe is considered or say some pressure vessel right or you might be you might have seen the oil tanker, so those things can be considered as thin walled hoop.

So, the analysis: so we should understand how you can analyze the deformation characteristics as well as the force characteristics of this kind of structure. So, let me write down this a thin, so this is the problem actually. So, this problem we will try to analyze. So, a thin ring of internal radius r, thickness t and width b is subjected to a uniform pressure p over the entire internal surface as shown. I will show the figure. We should like to determine the forces in the ring and the deformation of the ring due to internal pressure.

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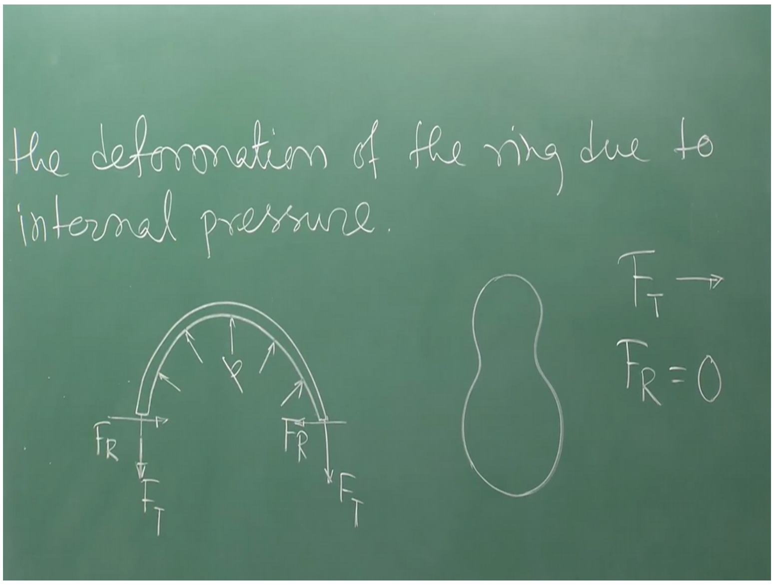

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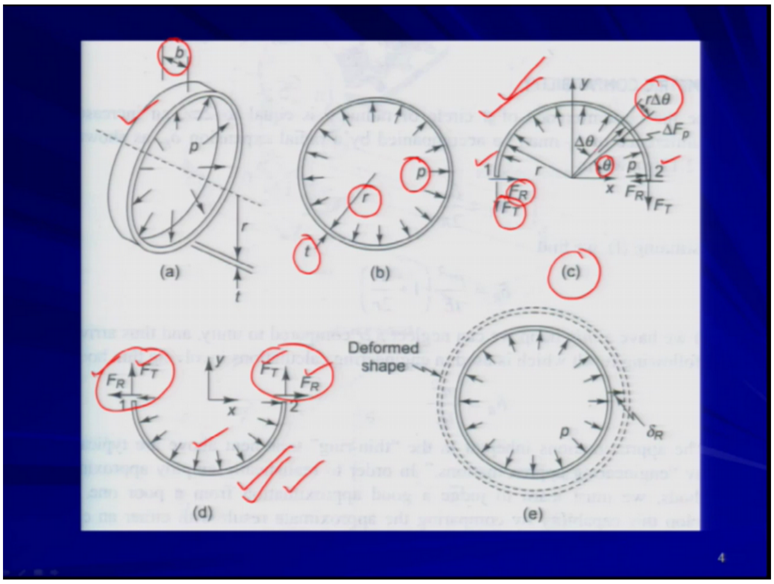

So, if you look at, if you come back to this figure that figure says the problem actually that will talk about the problem. So, a thin ring: so this is a thin ring of internal radius r, and thickness t, and with b. So, this is a thin ring. You can think of some wheel or that kind of thing very thin wheel and that is subjected to a uniform pressure p internal pressure. Suppose, if this ring is subjected to the internal pressure due to fluid or due to say gas or whatever due to oil it could be like that uniform pressure all round over the entire internal surface as shown in the figure. Then what we are going to do we are going to determine the forces in the ring. So, in the ring, what are the different forces are getting developed, so that we are going to analyze and we are going to find out the deformation of the ring due to this internal pressure. So, force and deformation characteristics or the response of this thin walled ring we are going to find out.

So, now as I told you what should be a first approach first approach to draw the free body diagram. Now, what we are doing we are cutting the whole hoop in two halves, this is your upper half and this is your lower half. So, figure c is giving you the upper half that means, you are cutting through the horizontal plane and this is your upper half, this part is your upper half and this part is your lower half. Now, when you are cutting the upper half and lower half the this internal pressure p will be balanced this, this, this internal pressure means this p small p is nothing but the externally applied load, it is giving the all round force radial force on the internal surface of the hoop.

So, if that is happening then of course, when you are cutting the hoop then we will be getting the forces which will be trying to balance this externally applied force and that internal force one will be in the tangential direction that is along the hoop; another one will be the radial direction F R along this. So, if you look at if you see this, so this is your hoop thin walled hoop you are cutting in two halves. So, this is the upper half say. So, all round you have the external pressure; external pressure means that is internally applied pressure, but external force it is giving this that is basically the applied load.

Now, to balance this, you should have one tangential force F T and one radial force F R. So, basically if you resolve that thing in the radial direction and the tangential direction basically we are taking we are considering the polar coordinate system. So, basically these forces will try to balance the externally applied force due to the internal pressure p fine; up to this there is no issue.

Now, if you look at this figure when you are considering the lower half then also basically this internal pressure will be getting balanced by these forces as already we have decided. So, in at point one, so this is the point one, this is the point two. So, at point one, if you look at in the upper half, F T is tensile in nature; F T is going away from or going out from the plane. So, F T is tensile in nature. It will try to pull the hoop. To balance this in the lower half it will be just opposite equal and opposite, so that is getting satisfied by this. So, both upper half and lower half, if you look at F T, the nature of F T is tension, so that is not say making any contradiction. So, in the upper half it was tension; and you have taken the equivalent opposite sign in the lower half that also giving the tension. So, F T is fine.

Now, if you look at F R, in the upper half, if you look at the upper half, so upper half F R is acting I mean inside, inside the hoop; to balance it in the lower half it should be outward. So, F R is inward in the upper half, whereas in case of lower half F R is outward you look at here. So, now, what you are understanding from this. So, basically if I considered F R really, so my hoop will take the shape like this. In the upper half the F R is inward, so that is why it will try to squeeze or try to deform in the inward direction; and in the lower half, it will be expanding because it is going in the outward direction of F R, so that I mean it might take the shape like this.

Now, the question is that under the fluid flow suppose water is flowing through the pipe or oil is flowing through the pipe or even gas is flowing through the pipe, have you ever seen under that kind of flow, have you ever seen this kind of shape is happening in the pipe, it never happens. So, if you try to satisfy the symmetry condition then what you can conclude, you must have F R equal to 0. If F R happens to be there, then you will be getting this kind of asymmetric condition and that is totally contradictory say visualization for this problem.

So, therefore, what we can conclude F T should be there, but F R must be 0; otherwise my symmetry is getting disturbed. So, under the fluid flow or the gas flow or this kind of flow, you never get this kind of shape like your groundnut kind of shape of the pipe, you never you never get that kind of thing, because F R is not there. If F R is there then you might get this kind of thing, but F R cannot be there, because your symmetry has to be satisfied. So, this is coming from your study of forces.

Now, the next job is to quantify this F T because the F R is there F R is 0 that is that we have got it. Now, F T is there, but I do not know what is the magnitude of F T. So, let us find out the magnitude of F T. Now, if you come back to this figure now we are going to find out the magnitude of F T. So, we are considering a small element. So, this is the angle theta and this is a small angle delta theta. So, therefore, this small element is nothing but r into delta theta. Now, on this small element, if small p that is the internal pressure is acting then how much force is getting developed that if you try to write down, so that will be nothing but that delta F p as shown in the figure.

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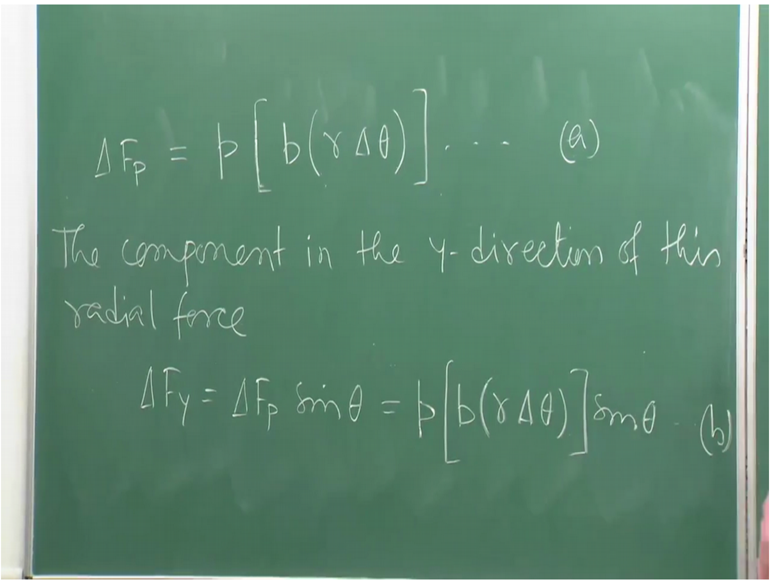

If you look at the figure, you will see it the delta F p is the radial force. So, this is your delta F p that is nothing but your radial force acting on the small segmental area and that can be quantified as delta F p is the pressure that is the internal pressure acting on that small area. And what is that area that area is given by b that is the width of the hoop multiplied by r into delta theta. So, this I am calling as equation a. So, this is your delta F p that is the radial force.

Now, if you take the component, the component in the y-direction as shown in the figure x y direction is shown. So, in that figure, if you look at x y direction, so if I take the component in the y-direction of this radial force that means, this delta F p then I can simply write delta F y that is a y component is nothing but delta F p sin theta if you refer the figure you will get it. So, delta F p sin theta is the y component, so that can be written as p b r delta theta into sin theta that is a equation b.

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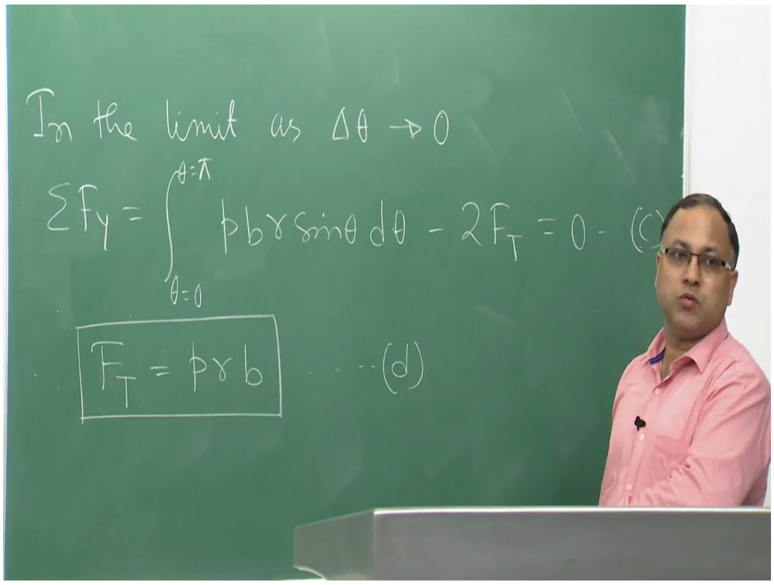

So, therefore, further we can write in the limit as delta theta tends to 0, I can simply write summation of F y equal to theta equal to 0 to pi p b r sin theta d theta minus twice F T. If you look at the upper half, so there are two this side you have F T this side also you have F T. So, 2 F T will be balanced by the y component of your radial force that means, the external force will be balanced. This is your external force. And this is your internally developed force that means, internal developed force means when you are cutting the section at the time you are getting that force which will be balancing the that internal pressure I mean the force arrive I mean where developed due to the internal pressure anyway, so that is basically 0. So, this is equation says c.

Now, if you do the integration, you will be getting F T equal to p into r into b. So, now, we have quantified this thing. So, F T is equal to p into r into b, agreed. So, where p is the internal pressure, r is that inner radius and b is the width of the hoop. So, now, instead doing this kind of integration and very complicated analysis, if you look at this expression, so basically what we are getting, this force F T can be obtained by considering the force on the projected area.

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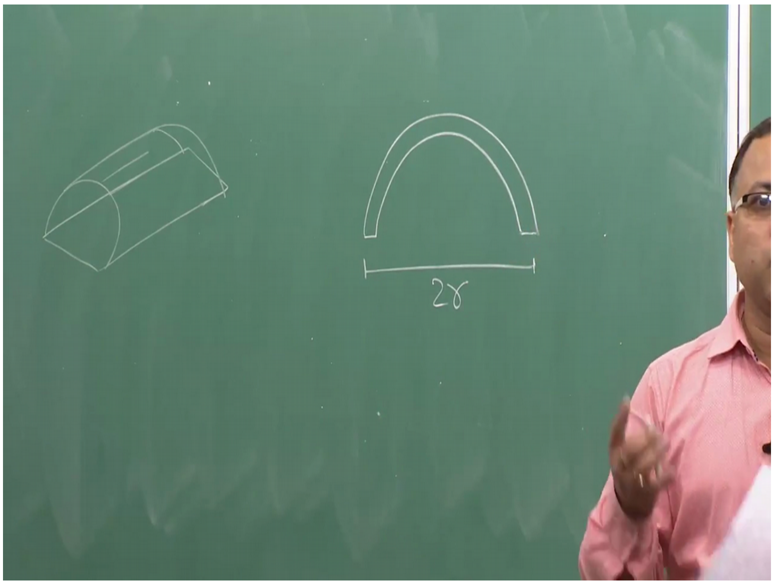

What does it mean? Suppose I have. So, this is your so this is the say hoop. So, this is the plane. So, this plane on that projected area basically what I mean even if you see this, this is your say upper half. So, this is nothing but yours twice r that is the diameter and normal to the board is your b that is a width. So, the projected area of this hoop is nothing but twice r into b. So, on that area if your internal pressure p small p is acting that must be equal to twice of F T. Instead of doing this much of complicated calculation, we can simply considered the projected area. This is my projected area; on that projected area your internal pressure p small p is acting and that should be balanced by the developed force F T.

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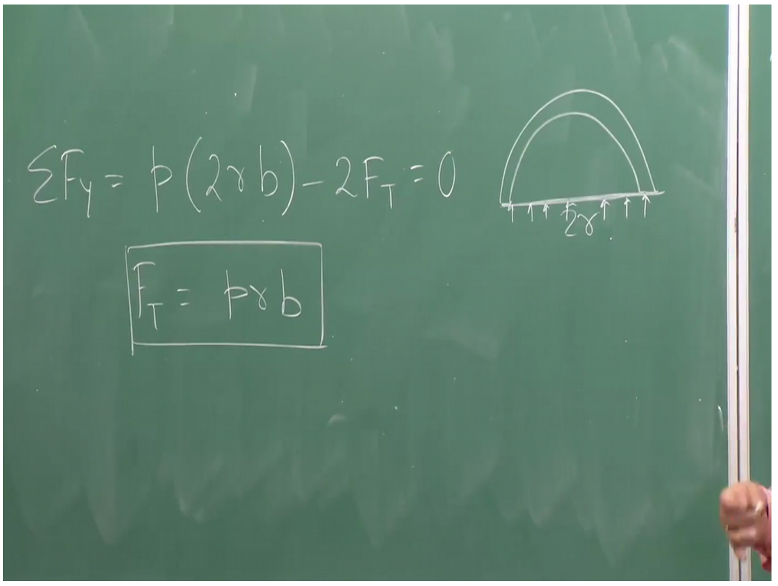

So, what we can simply write instead of writing that integration form we can simply write that is if we take the equilibrium or if we try to satisfy the equilibrium condition along y-direction, so I can think simply write p into the projected area is twice r into b. So, p is the force. So, as I told you this is the hoop. So, this is the plane, this is the projected projection plane normal to the board, this kind of plane is there along the plane, this is b and this is twice r. So, this is twice r, so twice r into b. So, on that plane a p is acting internal pressure p is acting along y-direction, so that is the force given by this minus twice of F T is equal to 0.

From this, I can simply write F T equal to p r. So, we are getting the same solution, but before I mean without going too much complicated calculation, so I showed you this calculation because that is the actual thing I mean coming from your actual configuration or the actual analysis. But if you do this analysis this is on the projected plane if you calculate the force then also you will be getting the same solution. So, later on we will be talking about this thing already because that is giving me the same solution whatever I have got from these analyses.

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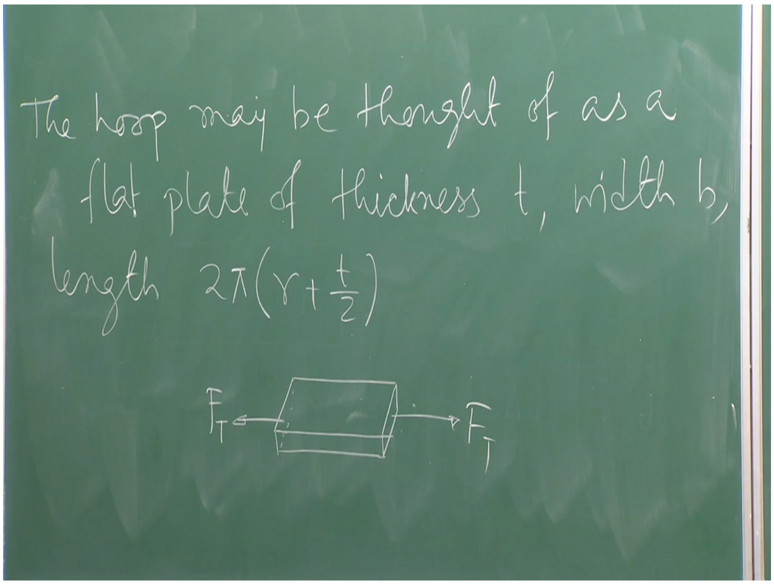

So, now the hoop may be thought of as a flat say plate of thickness say t, width b and length. So, this hoop you, have the hoop- circular hoop. Now, you are cutting the hoop and you are stretching it out. If you are stretching it out then basically the thickness of the hoop will be remaining same that is small t, width of the hoop, suppose I mean this is this is the say, this is the hoop, now I am cutting it, I am stretching it out. So, thickness will be remaining same, width will be remaining same that is b, the only the length will be the perimeter of the hoop. So, that perimeter will be defined as length will be 2 pi into r plus t by 2. If you considered the mean radius, so that is the perimeter. So, when you are stretching it out, so that will be the length of the flat plates.

So, what I am doing I am just cutting the hoop and I am making it a flat plate. So, this is these are the dimensions of the flat plate. I am not doing anything new, only we are cutting the hoop and making it flat plate. So, now, this flat plate as you have seen this flat plate will be under the tension of F T will be under the force of F T. So, F T basically you have seen the F T is a tangential force. When you are cutting and you are stretching it out, so F T will be the tensile force which will be acting on the flat plate, so that means, as if a uniaxial member, uniaxial flat plate is under the tension of F T is under the force of F T. So, as if it will be looking like this, as if the flat plate is under the tension of F T like this.

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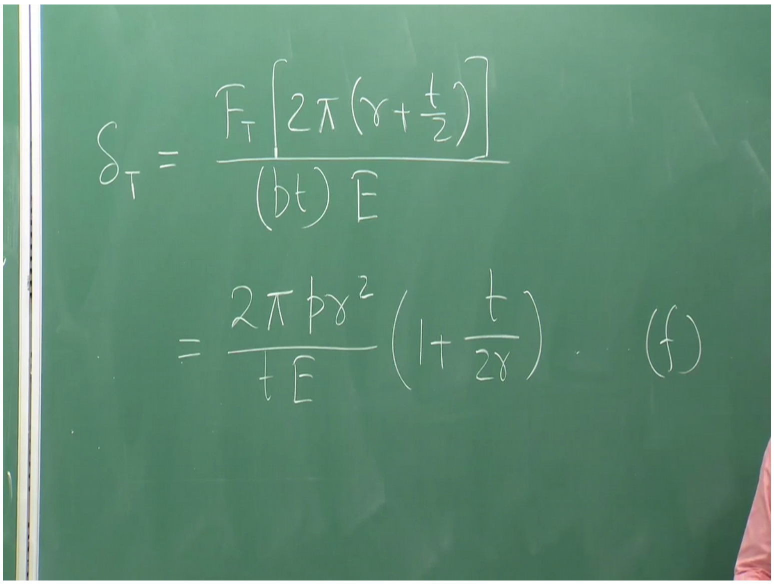

So, if it is so then what I can write already we have discussed the force deformation relation for the uniaxial loading pl by A E delta equal to pl by A E. I hope that you can recall that. So, by applying that equation, I can say how much elongation is really happening in this flat plate that I am going to find out and that is say delta t is equal to F T that is the force. And what is the length, length is 2 pi r plus t by 2 that is p l. So, this is your l that is a length and that is your force F T by a. What is the cross section, cross section is b into t. So, b into t is your cross section and of course, the material property that is the modulus of elasticity of the hoop, so that can be obtained as and that can be written as 2 pi pr square by t into e into 1 plus t by twice r, so equation f.

Now, what we have done, we have got the hoop we have stressed it out we have made the hoop as a flat plate and that flat plate is under the action of this tensile force F T just we are just cutting this and we were stretching it out. And then basically what we are getting, we are getting the deformation in the flat plate which is nothing but delta t by this expression. Now, we have to convert this deformation, this axial elongation delta t in the radial expansion, so that we will do in the next class. So, I will stop here today. In the next class, we will take this and we will see how we can get the radial deformation that means, that hoop was like this and then it will be expanded, so that expansion will be seeing in the next class.

Thank you very much.