Mechanics Of Solids
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Lecture – 13
Force Displacement Relationship

Welcome back to the course Mechanics of Solids. So, in the last few lectures we have covered how to draw the free body diagram and then different 2 force member three force member then in the equations of equilibrium that is the law of mechanics and then we solved the truss problem, so different types of trusses and then basically different methods by which you can analyze the truss and then finally, we ended with the friction, right.

So, now in the next topic whatever we are covering in this particular chapter that is force displacement relationship and geometric compatibility. So, of course, we will be considering small deformation problem. So, we are not talking about any large deformation, anyway with illustrations through simple problems on actually loaded members and thin walled pressure vessels. So, these are the things we are going to cover in this particular chapter. So, basically in this chapter we will be talking about the deformable bodies now from the previous discussion in the last few lectures we have seen that how you have analyzed the force moments and all those things and how you have drawn the free body diagram.

So, after drawing the free body diagram you can calculate the support reactions and all those things and now will see that for any system if you have that system will behave as per its own geometric compatibility or geometric fit, right. So, will see that based on that that geometric compatibility we can find out the displacement or the deformation in that deformable model. So, you are talking about the deformable body as we talked in the very first class that will be having 2 kinds of motion that is a movement or deformations right anyway. So, we will be talking about the deformations.

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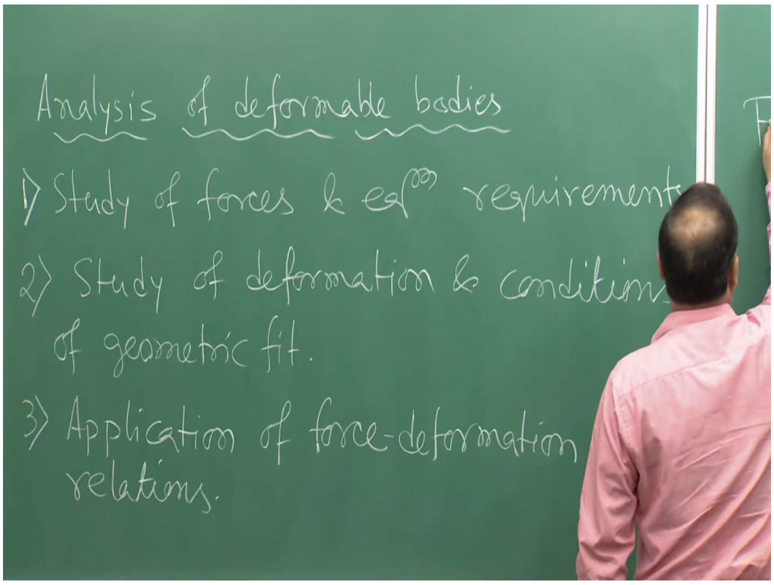

So, we are going to talk about analysis of deformable bodies. So, what are the different steps involved in this analysis. So, there are three steps involving this analysis. So, we will write down all the steps and then we will take one simple example by which we can we can see that we can I mean exhaust all the three steps. So, first one is your first step is study of forces and equilibrium requirements this is your first step right, study of forces and equilibrium requirements.

So, any system if you get first you study what are the different forces are coming. So, for that actually you need to draw the free body diagram. So, once you draw the free body diagram they need to be very clear that what are the forces are acting on that particular body or system and then of course what are the different equilibrium requirements by which you can satisfy the conditions of equilibrium.

Then the next step is study of deformation and conditions of geometric fit or geometric compatibility right, we will see that. So, study of deformation will study that how the system is getting deformed because we are dealing with the deformable bodies under the action of all the forces whatever we have studied in the first step and then we will study the deformation and after I mean and then of course, the conditions of geometric fit. So, basically what I mean, the deformation cannot be arbitrary right I mean if I give these are deformation or if I allow some deformation in some deformable body so that deformation cannot be arbitrary it will follow some rule or follow some guidelines and that is nothing, but your geometric compatibility. So, that deformation should be compatible with the whole system.

Then the third step is application of force deformation relations then. So, in the first step we are analyzing the forces and the second step we are studying the deformation and then in the third step basically we are applying the force deformation relation and then basically the thing is that once in the first step you are studying the cause, in the second step you are studying the effect and the third step you are just relating between the cause and the effect, so something like that. So, let us take one example and then basically it will be clear to with that how you can exhaust all the three steps.

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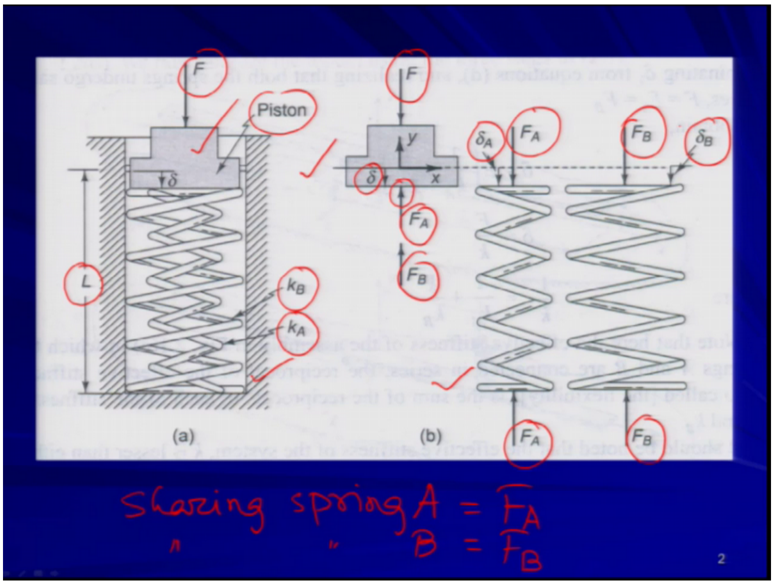

So, very simple example we are taking here. So, what example we are taking here we are taking there is one say a cylinder inside the cylinder there are 2 springs. So, spring A and spring B, defined by the spring constants k A and k B and you know what is your spring constant right, force required for unit displacement so that is nothing, but the spring cause. So, there these are 2 different springs spring A and spring B, which are giving me different space spring constants and there is one piston here. So, this is your one piston and this piston is getting pushed by a force F. So, F is the externally applied force on the piston and the total length of the spring before deformation before application of the load or the force F the length of the whole system or the configuration was say L, capital L.

Now, what will happen from your intuition or from your say concept right, what will happen? If you apply the force F on top of this piston the piston will try to push the spring right, the piston will try to push the spring in the downward direction. So, now, I mean if you try to draw the free body diagram for the piston as well as for the springs, then how it will look like? If you try to draw the free body diagram of the piston, now one thing is very clear from this figure that if you apply a force F on top of the piston and if we consider the piston is giving our piston is uniformly display, I mean that uniform displacement is happening then what we can consider that because what are the things will take care of this applied load F the spring A and spring B that will share the load right, this the sharing business will be coming into the picture that will share the externally applied force F, now it will be shared by I mean as per its own property of property of the spring.

So, now if I say sharing of spring A is a F A force and sharing of spring B is a F B then if you try to draw the free body diagram of the piston that will look like this, the total force is applied on top of the piston A and that is getting say balanced right by the equal and opposite forces is F A and F B coming from 2 different springs, spring A and spring B. Now if you try to draw the free body diagram of the spring A that will look like this. So, F A is in the upward direction on the piston as shown here. So, therefore, F A will be downward direction at this interface and F A will be taking the direction like this; that means, the spring will be under compression and that is nothing, but your 2 force member kind of thing right, already you know the equivalent opposite direction. Similarly spring B will be experiencing compression with the force F B.

And now what I am saying? I am saying that the piston is moving by an amount delta. So, this delta is nothing, but the vertical deformation or the vertical displacement or the deflection of piston. Similarly this delta A is nothing, but the deformation vertical deformation of spring A because spring both the springs have to be deformed otherwise the piston will not move down right the piston is moving down because springs are deforming. If the springs are not deforming, if the springs are say rigid body then this kind of movement is not possible right, springs are allowing some movement so that is why this delta is possible for the piston the piston will go down.

So, and delta A is the displacement or the deformation of the spring A, similarly delta B is the deformation of spring B. So, everything is clear. So, now, basically what we are going to find out from our, I mean force. So, first will what is the first step? First step is the study of forces.

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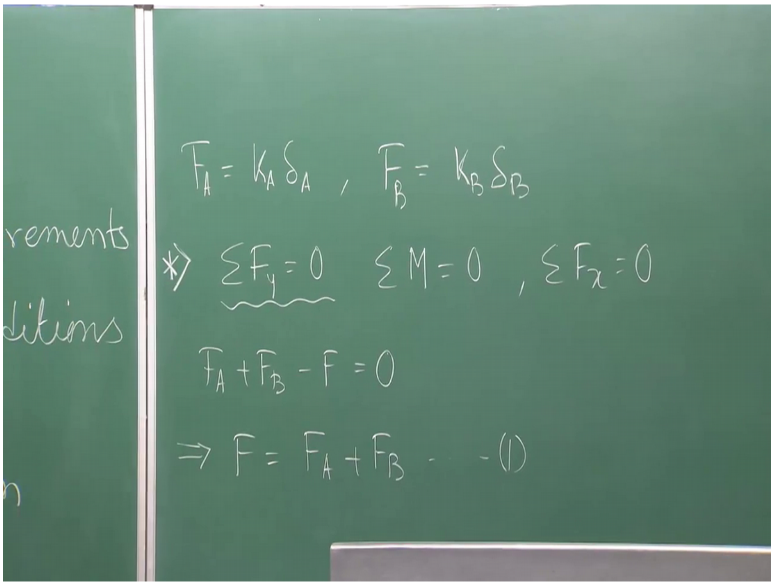

So, when we are studying the forces then F A can be written as k A into delta a similarly F B can be written as k B into delta B, well k A and k B are the spring constants already we have seen this equation the very first or second class right. So, this is the, in this way actually we are studying the forces. And then when we are talking about the first step basically the first, what the first step says? Study of forces and equation and equilibrium requirements, so these are the forces in the spring and now we are going to satisfy the equilibrium requirements. So; what is my equilibrium requirements; that is the summation of F say if I say if it is shown here right if you look at the figure here right x is the horizontal direction, y is the vertical direction.

So, I can simply take F y equal to 0 and summation of M equal to 0 and summation of F x equal to 0. So, this is my equilibrium conditions available with me. So, if we consider this equilibrium condition that is summation of F y equal to 0. So, what we can write? We can write. So, what are the forces are acting, so if you consider the free body diagram of the piston as shown in the slide right. So, F is acting downward whereas F A and F B both are acting upward. So, we can write simply F A plus F B minus F equal to 0, right. So, from this I can simply write F equal to F A plus F B that is say equation 1.

So, after satisfying the step one or after exhausting the step one; that means, study of forces and equilibrium requirements we have got we have established this relation; that means, the total force applied on the piston is shared by both the springs by an amount F A and F B. So, now if you talk about the second stage, now if you talk about the second step, second step what does it say? The second step if you recall the second step says that you study the deformation you study the deformation and condition of geometric field that is geometric compatibility right.

So, in the second step if you try to think about if you try to study the deformation what information you will be getting from this problem? That means, see I am not considering any tilt in the piston. So, piston is here and the piston is gradually moving in the downward direction, uniform or equal displacement. So, the amount of displacement the piston is experiencing and if I am not considering any gap between the piston and the spring after the deformation then basically whatever amount of displacement piston will experience the same amount of deformation will be happening in the spring itself in both the springs, whatever deformation will be happening that it will be equal to the displacement of the piston that is nothing but your geometric compatible the geometric fit. So, that is the condition by which the system will, system will be governed by that condition actually.

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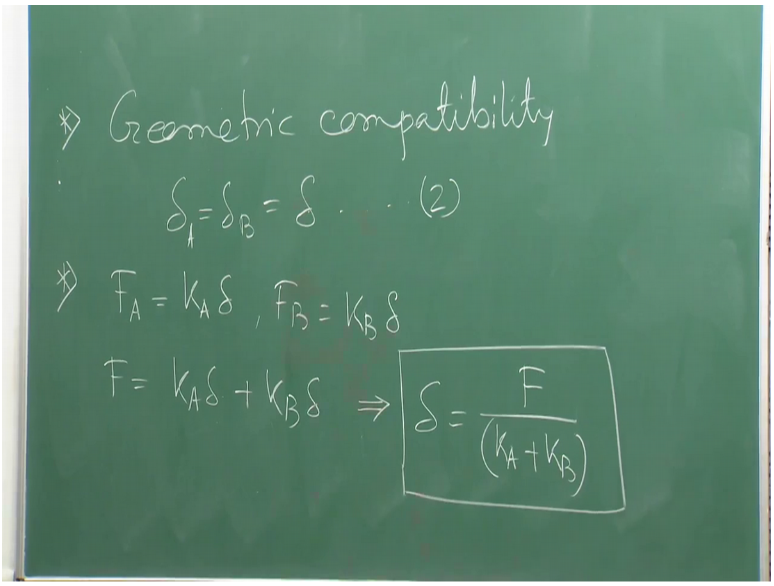

So, what I can write? So, your geometric compatibility that says delta A must be equal to delta B must be equal to delta say this equation I will say equation 2, agreed. So, I hope there is no problem. So, you have to identify this geometric fit. So, the second step is over.

Now, in the third step we are trying to establish the relation between the force and the deformation as per our previous discussion right. So, in the third step what we are doing? We are already we know F A equal to k A delta. So, instead of delta A and delta B we can simply write delta because they are equal and F B equal to k B into delta and now I am putting this thing in equation 1 and I am getting F equal to k A delta plus k B delta from which I can simply write delta equal to F by k A plus k B, right.

So, this is very important relation we have got. What we have got? We have established the relation between the applied force F and the deformation delta. So, what we got from this? If you know the force how much force you are applying on the top of the piston if it is known to you and if you know the property because these are the inherent property of the spring the spring constant is nothing, but the inherent property of the spring. So, if you know the property of the spring; that means, what type of or what is the strength of the spring if you know that is k A and k B if it is known, if they are known to you then you can find out how much deformation will be happening in the system that is delta and the same deformation will be happening in the spring as well because as per the second step.

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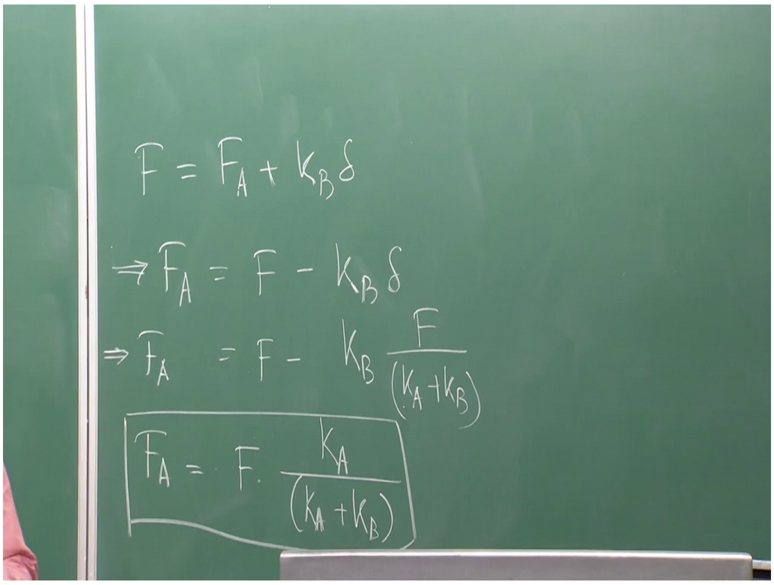

So, now further if we move we can establish the relation that how much force is shared by each spring? So, I can simply write again I can write that F equal to F A plus k B delta. So, in place of from this we can simply write F A equal to F minus k B delta or we can further write F minus k B in place of delta we can simply write F by k A plus k B as given here. So, from which I can simply write F A equal to F into k A by k A plus k B; that means, if you are applying F force on top of the piston and if you know the properties of this spring, so this is the shear taken by the spring A.

Similarly F B can be calculated F B equal to F into k B by k A plus k B that you can check it. So, that is the share taken by the spring B. I hope you have understood the steps. So, these are the steps involved to analyze the deformable bodies and we will solve all, we will see different deformable bodies for which we can find out this kind of relation force deformation relation. Now let us talk about the uniaxial loading and deformation.

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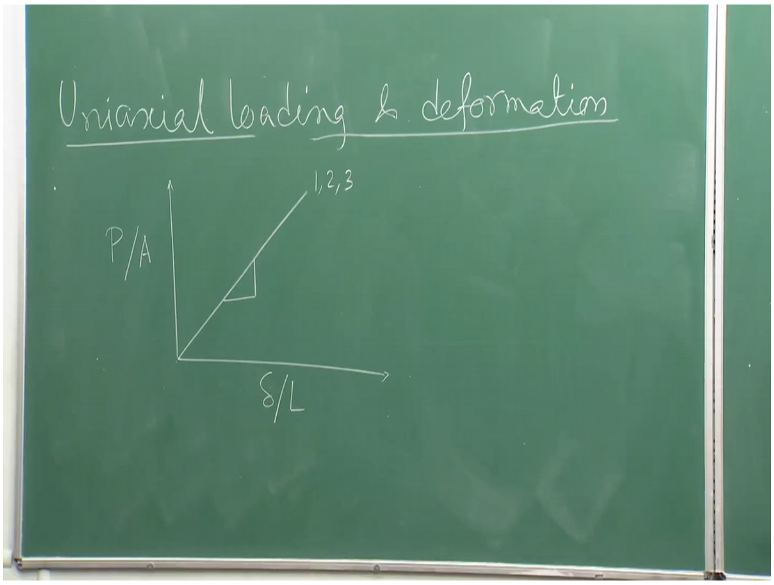

So, we are going to establish the force deformation relation between for a uniaxial system, so uniaxial loading and deformation. So, now, if you look at the figure, so we are taking three different systems.

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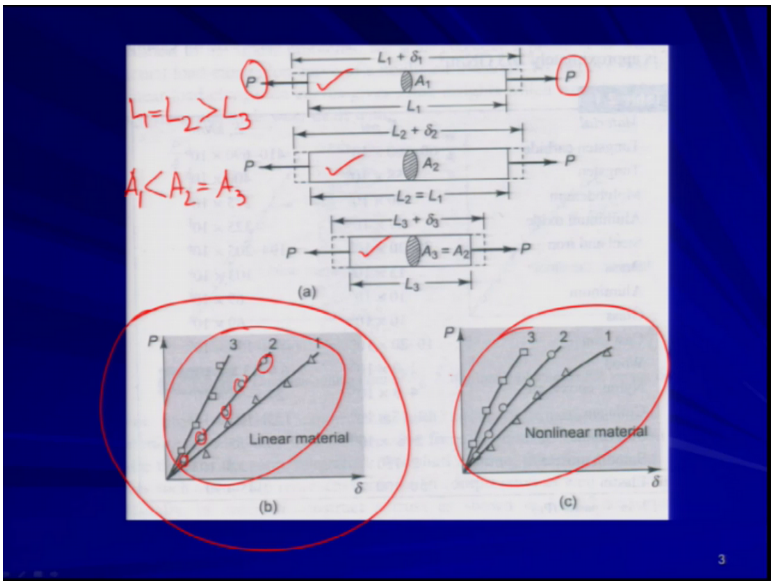

In the first system, so this is some uniaxial member this three are in the uniaxial member. So, in this system basically what we are having L 1 equal to; L 1 equal to L 2 which is greater than L 3. If you look at the figure it will be saying that relation. So, L 1 that is the length of the first member is equal to length of the second member, but they are greater than the length of the third member and this is another relation from the figure we can see that A 1 is less than A 2 and where A 2 and A 3 both are equal.

So, these three say members of the system uniaxial member we are considering and these three members are under uniaxial force tensile force P. So, you see the nature of the force P is tensile. So, that uniaxial member it could be steel rod or copper rod or some uniaxial member the cross section where a is nothing, but the cross section A 1, A 2, A 3 are the cross section of those members and L 1, L 2, L 3 are the length of those members and P is the axial force, axial tensile force acting on each member. So, P is constant for all the members. Now suppose you are gradually applying P; that means you are not applying the total P at any instance you are gradually applying P.

So, suppose P is divided by setting 10 factors or 10 times. So, you are getting, so delta P something like that, you are applying delta P then you are observing the deformation; that means, elongation it will be elongated right, that uniaxial member will be elongated. So, you are you are observing the deformation then you are applying the second increment you are observing the deformation, third increment and in that way if you proceed suppose you are you are doing the experiment like that and if you proceed like that gradually you are applying the delta amount of load and if you are going to the total load say P then basically you will be establishing one plot like that P versus delta. So, you are getting different points as I told you. So, these are all different or discrete points for say each increment of load. So, you are giving each increment of load and you are observing the deformation.

Now if the material is linear; that means, if the force deformation is linearly related; that means, I mean directly proportional. So, F that is the force is directly proportional or the P here it is P the force P is directly proportional to delta if it is happening and if it is linear then basically you are getting the linear variation you see, that is the complete linear variation between P and delta. But most of the times you will see that the material is not linear it will not behave as linear system you may get the non-linear system. So, if you get the non-linear system then the plots will be looking like this P versus delta will be looking like this. But however, for this course as far as this course is concerned will be only considering the linear system, we are not going to talk about the non-linear system that is beyond the scope of this particular course, anyway.

So, now, if you look at this plot this P versus delta plot this plot, now if I plot this thing in normalized say system normalize plot; that means, if I plot this P by A versus delta by L, where L is the length of the member, A is the cross sectional area of the member. So, if I normalize the force P with respect to say or normalize means if I plot P by A versus delta by L; that means, we are basically taking care of difference in cross sectional area and difference in length then basically all the lines you are getting three lines right, three P versus delta curve. So, all these three lines will be converging towards a single line right.

So, this is your linear relation. So, all 1 2 3 lines will be falling or will be merging to a single line that is shown here. So, therefore, if I measure the slope here the slope of this line is having a particular lane that is known as modulus of elasticity the slope of the straight line is known as modulus of elasticity.

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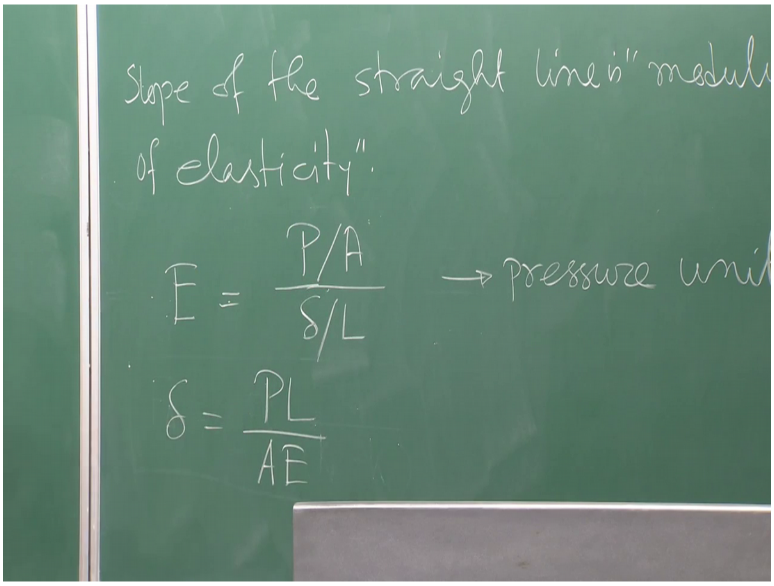

This you might be knowing from your earlier discussion in physics right what is modulus of elasticity. So, what is then if I say modulus of elasticity is E then what I can write from this definition that is the slope of the line; that means, P by A this is your definition. The slope of the line is known as modulus of elasticity and modulus of elasticity can be defined by this expression and if you look at this expression this delta by L basically dimensionless parameter and so what will be the dimension of E or unit of E? That will be in pressure unit; that means, force per unit area will be the unit of your modulus of elasticity.

So, from this I can further write delta equal to P L by A E, I can write that from this expression if you just look at this expression I can simply write the delta that is a deformation is equal to P into L by A into E. Now if you look at this expression basically this is the geometric property of the member L, A is also the geometric property of the member; that means, the cross section area of the member and the length of the member if they are known and if you know how much force you are applying and this is your inherent property of the material that is the material property modulus of elasticity is nothing but the material property, if you know the material then basically you can find out delta; that means, that deformation. So, now, what we have done, we have established a relation between the deformation and the force, right.

Well, so I will stop here today. So, in the next class will be continuing with further discussion on the thin wall pressure vessel and other things.

Thank you very much.