Mechanics Of Solids

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Lecture - 12
Tutorial on friction

Welcome back to the course Mechanics of Solids. So, in the last lecture, we solve one truss problem by using method of joints as well as we have given some indication that how you can implement method of section. So, today what we will be taking we will be taking another numerical problem so that is related to friction. So, if you have the frictional force then how your problem can be solved.

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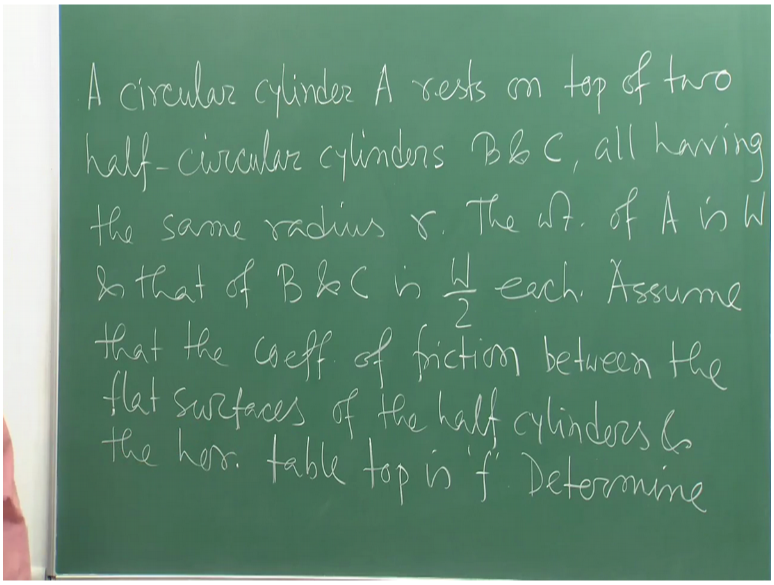

So, first let me write down the problem. A circular cylinder A rests on top of two half circular cylinder B and C, all having the same radius r. The weight of A is W and that of B and C is just half, W by 2 each. Assume that the coefficient of friction between the flat surfaces of the half cylinders and the horizontal table top is f.

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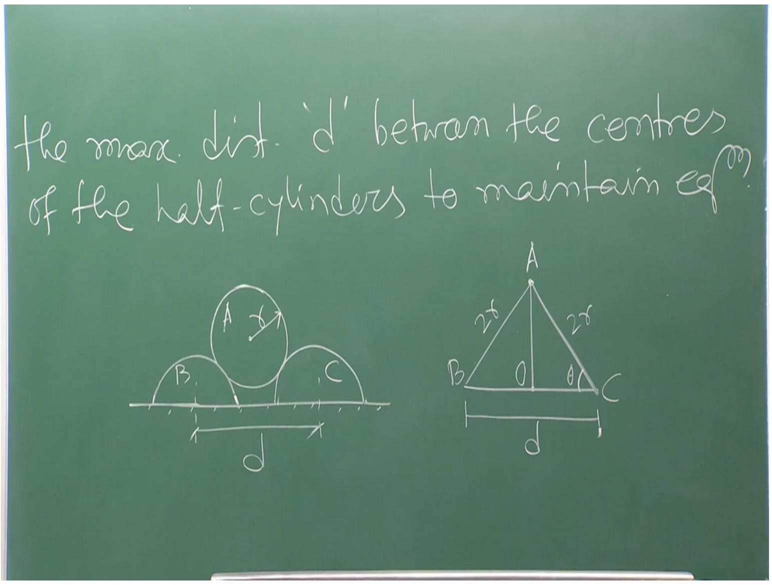

Now, this is your objective. So, determine the maximum distance d between the centers of the half cylinders to maintain equilibrium. So, let me draw it first. This is the problem, the problem says a circular cylinder A as shown in the figure rests on top of two half cylinder half circular cylinders B and C as shown. All having same radius r, the weight of A - cylinder A is W, and that of B and C is W by 2 each assume that the coefficient of friction between the flat surfaces of the half cylinders. So, this is your table top say this is your table top. So, the coefficient of friction between the flat surfaces of the half cylinders and the horizontal table top is f, small f you know coefficient of friction.

So, determine the maximum distance d between the centers of the half cylinders to maintain equilibrium that means, equilibrium means it will not I mean it will not separate out this cylinder B and C will not move in the horizontal direction. So that the cylinder a will fall down or it will not move in the in the inward or outward movement is not permitted for cylinder B and C. So, you have to maintain the equilibrium condition. So, this is the problem. So, now for this problem, I mean if you look at this figure from the geometry basically what we get. So, this is the your centre of A, centre of B and centre of C cylinder, this distance is d as given. So, the centre to centre distance between A and B is twice r, this is also twice r. So, there and this angle if I say theta, and this point if I say O, then what I can write?

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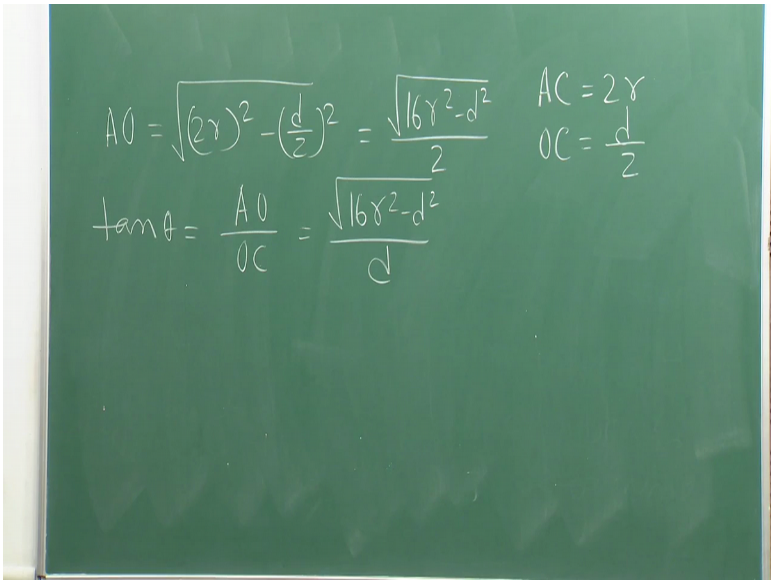

I can simply write AO equal to before that we have seen ac is equal to twice r and OC just you look at the figure this is coming from the geometry nothing to do with the problem. So, how the configuration figure I mean configuration of the cylinders is there. So, depending on that we can find out this. So, this is your d by 2. So, from this AO is nothing but twice r square minus d by 2 square. So, from this, I can simply write root over 16 r square minus d square by 2. Similarly, tan theta which is nothing but AO by OC is nothing but if you put the values 16 r square minus d square by d. So, these are the things these are the geometric configurations we have got from the figure. So, now let us jump on the actual problem. So, what is our next job, our next job is to or next or rather primary job is to draw the free body diagram. So, let us draw the free body diagram of cylinder A.

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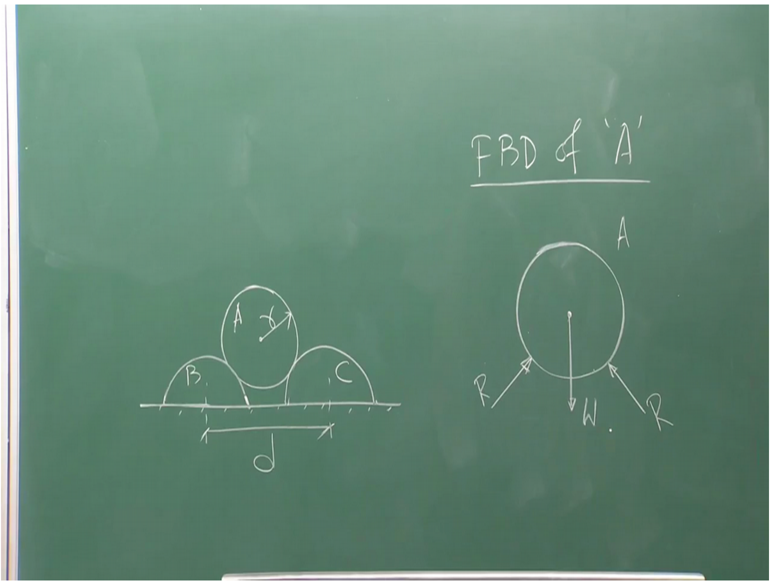

Let us draw the free body diagram of cylinder A; so F BD of cylinder A. So, this is the cylinder. I hope it looks likes cylinder or circle fine anyway well. So, this is your cylinder A. Now, if you look at this cylinder what are the forces are acting if you free body diagram of cylinder a means you are taking out cylinder A from the system. If you want to take out the cylinder A from the system then you have to replace the contact forces or the externally applied forces. So, what are the forces are acting here. The first one is vertically downward that is W that is the weight of the cylinder, and then there are two reactive forces or reactions which are applicable at this interface. So, B will gives some reaction to A, as well as C will give some reaction to A; and that and there will be same magnitude wise they will be same, because everything is symmetric.

So, now you may ask me, how the direction will be there this the reaction force r, the line of action of r, how I can choose. Again the problem is three force problem right three force member this member is three force member. Then the reactions are must pass through the centre that is the common point, because W is if I assume W is acting at the centre of the cylinder that is the CG of the cylinder and then basically this R both R will be coinciding with the centre that is quite obvious from the three force system.

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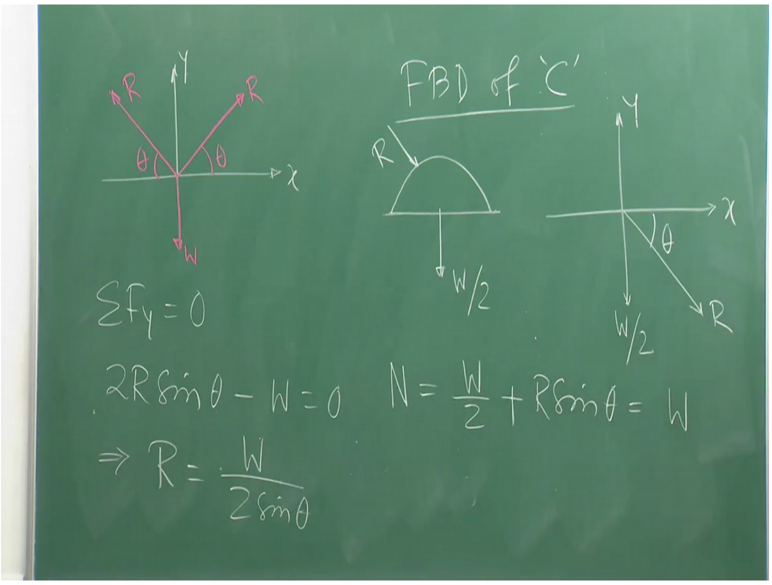

Then what we can write, we can simply write in say x y frame if this is my x y frame. I can simply this is w, this is R which is making an angle theta here; and this is also R which is making an angle theta and theta is already obtained from the geometry. So, now from this if you try to solve because this is in equilibrium. So, I can simply exploit this condition, so that gives me twice R sin theta minus W equal to 0; from which I can simply write R equal to W by twice sin theta. So, the reaction force can be expressed in terms of the weight of the cylinder.

Now, we will consider the free body diagram of cylinder B and cylinder C. So, first we will consider the free body diagram of cylinder C F BD of C. So, that is say this is the table top and your cylinder C is here. Now, the weight is acting W by 2 and you have reaction force R. Now, the cylinder A will give the opposite reaction that is R that is here. So, from this if you transfer this thing to x y frame this is your R which is making an angle theta and this is your W by 2. So, from this therefore, this is the interface right this is the interface between the half cylinder and the table top as shown in the figure.

So, if you want to know how much friction force is getting developed then you need to know the magnitude of N capital N - the normal force. So, add the tabletop this W by @ is any way acting that is the weight of the half cylinder C. Now, due to this R some contribution will be coming towards the normal force N, so that we are going to find out. So, N total normal force will be W by 2 that is any way coming from the cylinder weight of the cylinder half cylinder C plus E sin theta so that gives me, so R sin theta is nothing but W by 2. So, W by 2 plus W by 2 is nothing but w, so that is the total normal force acting at the interface between the half cylinder and the table top fine.

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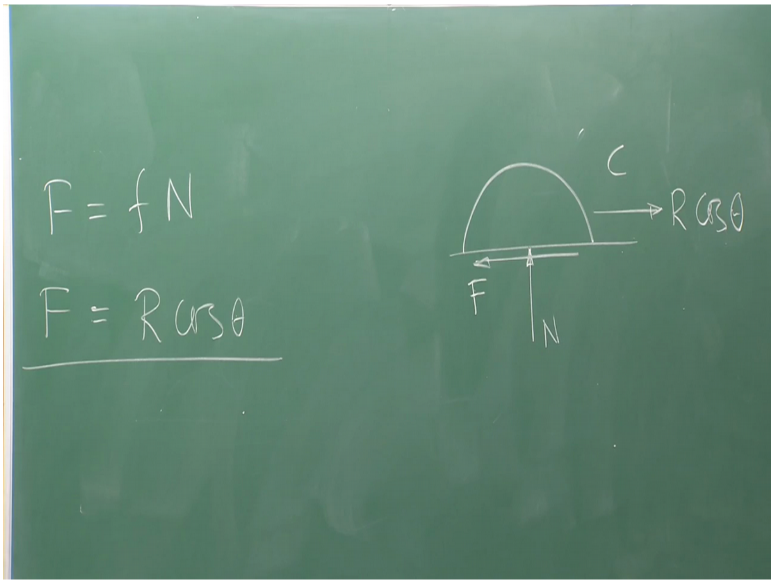

So, now if you look at this is the cylinder, cylinder C say. This is the normal force acting now I am drawing free body diagram, this is normal force acting on the cylinder. And you have frictional force which will be getting developed here. And what will be happening. So, if this C cylinder is moving in the rightward direction, and B is moving in the leftward direction then basically A will be falling down. So, this kind of possible movement this kind of movement is actually possible in this configuration. So, this C block or the C cylinder will try to move from left to right. So, if it moves from left to right then the friction force will be getting developed in this direction and because of that and this movement is possible only due to R if you look at the x component of that force. So, R cos theta will try to push the cylinder half cylinder c from left to right and due to that some friction force will be getting developed that is f and this is the normal force already we have calculated.

So, from this I hope that you have understood this configuration. So, from this what I can write. Now, this F is nothing but f is given small f is given that is the coefficient of friction and N already we have calculated. So, therefore, to maintain the equilibrium what you should have, if you want to maintain the equilibrium of this configuration whatever configuration I was showing that A is resting on top of two half cylinders then basically the friction force should be equal to diving force this R cos theta. So, F should be equal to for equilibrium F should be equal to R cos theta, so that is the condition for equilibrium. If r cos theta is greater than F then basically the equilibrium will be disturbed and the block or the cylinder A will fall down because there will be separated out this cylinder B and cylinder C.

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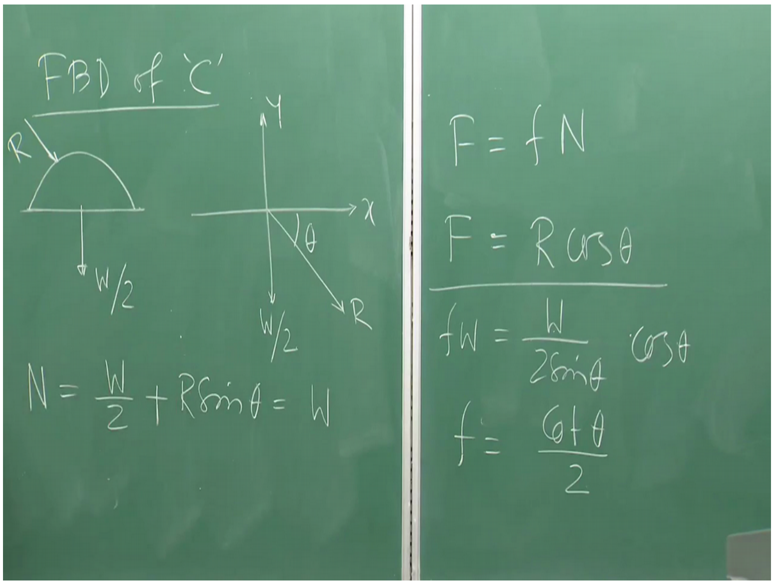

Anyway, so this now we can write down f W because N is nothing but w is equal to R is already obtained W by twice sin theta into cos theta. So, from this, I can simply write f equal to cot theta by 2.

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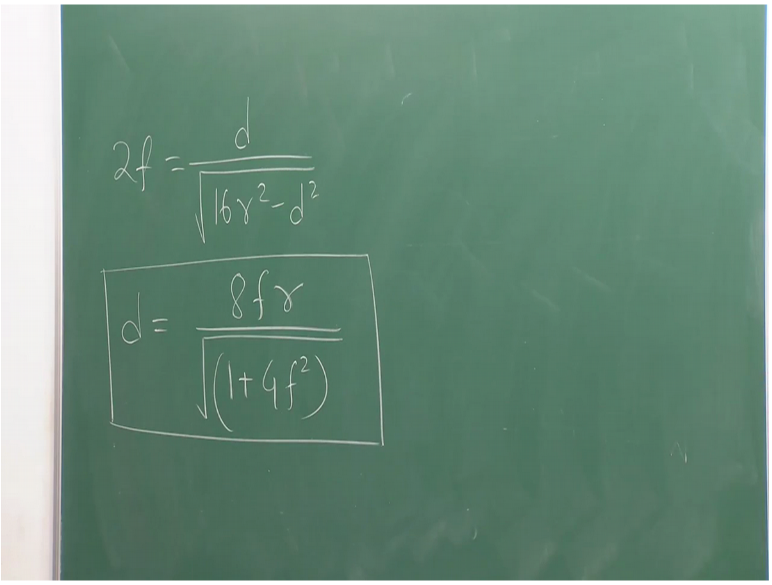

So, therefore, I can write twice f equal to the expression for cot theta already we have got the expression for tan theta 16 r square minus d square. Now, if I do some simplification of this expression, so finally, I will be getting d equal to 8 f r by root over 1 plus 4 f square. So, this is the maximum distance you can maintained between cylinder B and cylinder C, so that the whole configuration or whole system will be under equilibrium. Once this d is more, then basically you will be getting the disturbance or your equilibrium like that you will be getting disturbed.

So, I will stop here today. In the next lecture, we will be looking at the I mean the next part how you can find out the solutions by imposing your geometric compatibility. So, every system will be having some geometric compatibility, I mean everything cannot move by its own choice, so that thing we will see we will start the new chapter from the next class onward. So, I will stop here today.

Thank you very much.