# Geology and Soil Mechanics
**Prof. P. Ghosh**
**Department of Civil Engineering**
**Indian Institute of Technology Kanpur**
**Lecture - 65**
**Problems on Earth Pressure Theories - 4**

Welcome back. So today we will be taking few more problems on stress distribution in soil. So, in the last lecture we have solved a couple of problems regarding stress distribution in soil. Today we will be taking few more problems.

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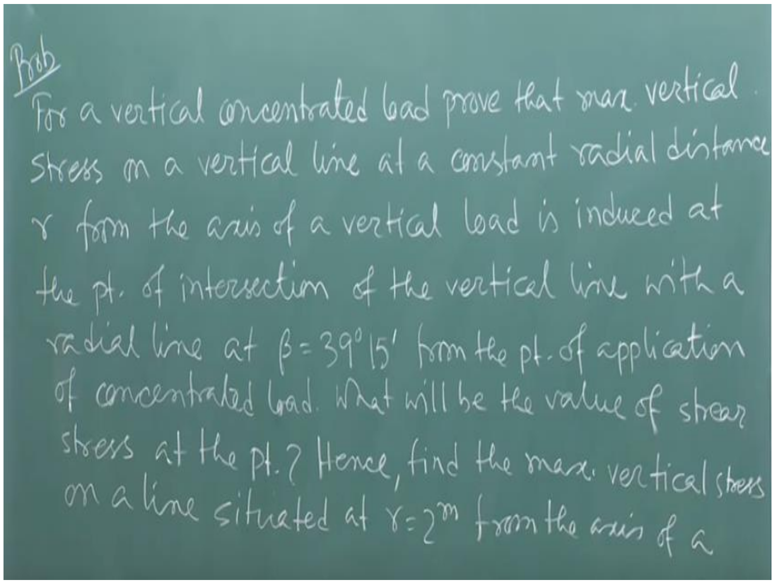


So, first problem is the problem says that for a vertical concentrated load prove that maximum vertical stress on a vertical line at a constant radial distance r from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at beta equal to 39 degrees 15 minute from the point of application of concentrated load. What will be the value of what will be the value of shear stress at the point?

So, this shear stress we have not given the expression when we discussed about the theory however we will be giving that expression here itself and we will be solving that problem okay. So that is also I mean you can find out the normal stress otherwise the shear stress at a particular point due to the application of the concentrated load anyway. So hence find the maximum vertical stress on a line maximum vertical stress on a line situated at r equal to 2 m from the axis of a concentrated load of value 20 kN okay. So, let me draw it.

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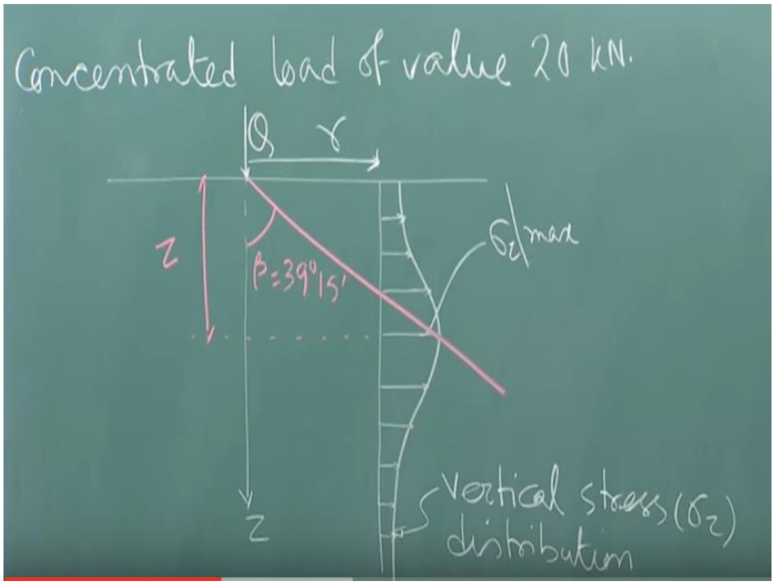


Vertical stress distribution. This is Q. This is radial distance r. This is z. This angle is beta equal to 39 degree 15 minute okay. So, this is the problem. The problem says for a vertical concentrated load such as Q prove that maximum vertical stress on a vertical line at a constant radial distance r okay so this is the vertical stress distribution if you solve this thing by Boussinesq theory whatever we have seen okay for a concentrated load then you will be getting the stress distribution at any radial distance r from the concentrated load axis you will be getting the stress distribution like that.

So, it will not be maximum at the top. It will not be maximum at the bottom. It will be maximum somewhere else or some at some depth z okay. So, from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at beta is equal to 39 degree 15 minute. So, you have to prove that this beta angle is 39 degree 15 minute.

That means the point where the maximum vertical stress is coming so that point if you connect that point with the point where the concentrated load is acting so this line will be making an angle beta okay which is equal to 39 degree 15 minute with the vertical axis okay and what will be the value of shear stress at the point. So, you have to calculate the value of shear stress at the point where it is becoming maximum that means where the normal stress that is the vertical stress is becoming maximum at that time you have to find out the shear stress.

We will be seeing the expression for that and hence find the maximum vertical stress on a line situated at r is equal to 2 m from the axis of a concentrated load of value 20 kN okay. So, let us solve that. So, the vertical stress magnitude or the expression is already given in the lecture class okay however shear stress distribution we have not discussed which will be discussed now itself because we are taking this problem.

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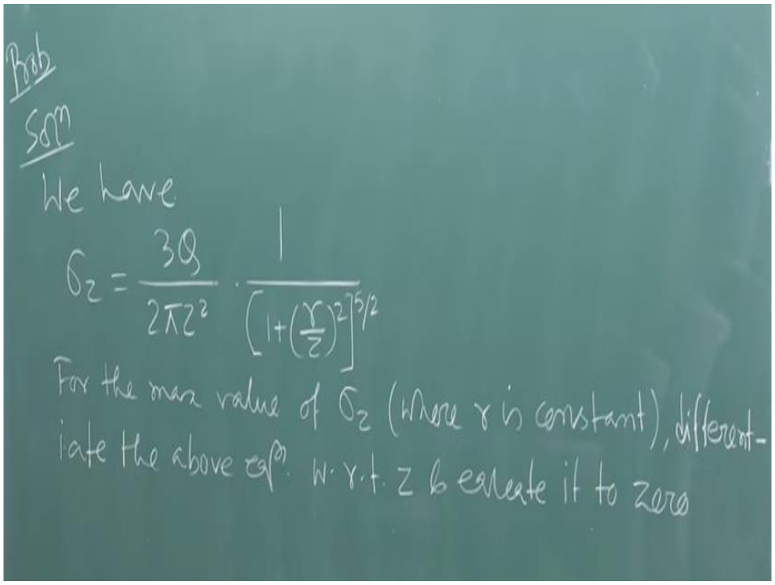


So, we know so solution. We have sigma z is equal to 3Q by 2 pi z square into 1 by 1 + r by z square to the power 5 by 2. This is known to you right. Already we have seen this thing in the lecture class is it not? So, this is already known to you. Now for the maximum value of sigma z where r is constant that means if you consider this vertical section your r is constant. All the times r is same right.

So, r is constant. So, what how I will get the maximum value of sigma z. So, this is the expression of sigma z. So, this sigma z will be differentiated with respect to z. If we do that then we will be getting the maximum value of sigma z right. So, differentiate the above equation with respect to z and equate it to zero.

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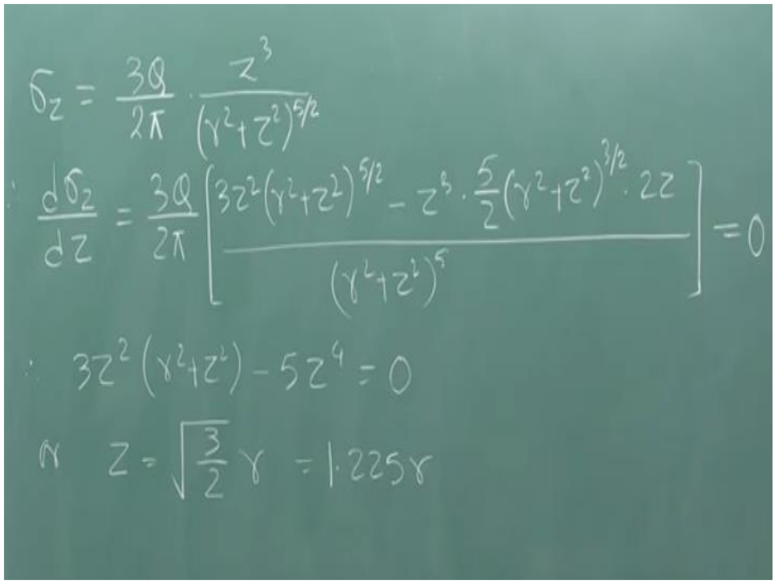


So therefore, I have the expression like this 3Q by 2 pi I am taking z in this term so the same expression z cube by r square + z square to the power 5 by 2 okay. So, this is the expression of sigma z. Now what we will do? We will do the differentiation d sigma z dz equal to 3Q by 2 pi it will be 3 z square into r square + z square to the power 5 by 2 - z cube into 5 by 2 r square + z square 3 by 2 into 2z divided by r square + z square to the power of 5 okay.

So, this is equal to 0 to get the maximum value of sigma z this should be equal to 0. So, if that should be equal to 0 then what we will get? 3 z square into r square + z square - 5z to the power of 4 is equal to 0 okay. So, which gives me z equal to root over 3 by 2 r which is nothing but 1.225r okay.

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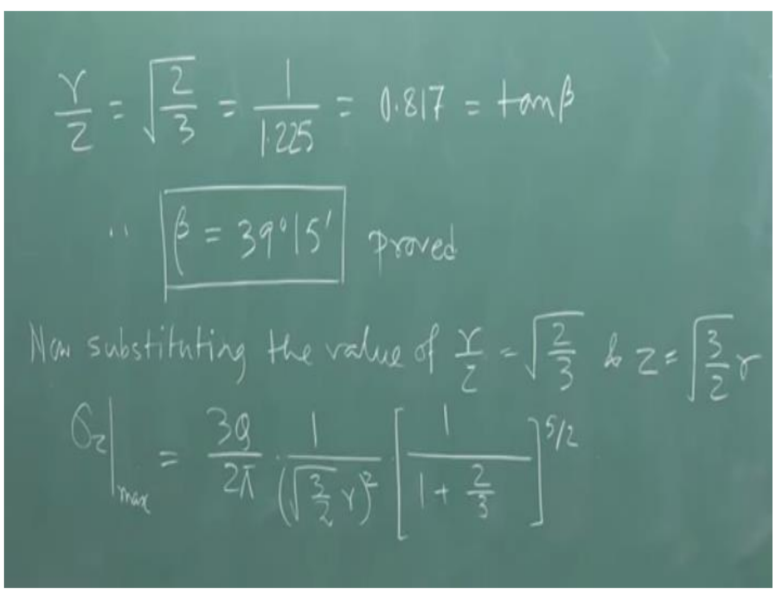

So therefore, r by z r by z is equal to root over 2 by 3 is equal to 1 by 1.225 is equal to 0.817. Now if you look back the figure okay so whatever figure we have given during the problem definition at that time what is r by z that is nothing but tan beta. So, from this I can prove beta is equal to 39 degrees 15 minute. It is proved. Hence proved okay. Now what are the things you need to find out?

You need to find out the maximum normal stress and shear stress at that point okay. So now substituting the value of r by z is equal to root over 2 by 3 and z is equal to root over 3 by 2 r if I put this thing in sigma z expression then I will be getting the maximum sigma z right in that expression that is equal to 3Q by 2 pi into 1 by 3 by 2 r whole square into 1 + 2 by 3 into 5 by 2.

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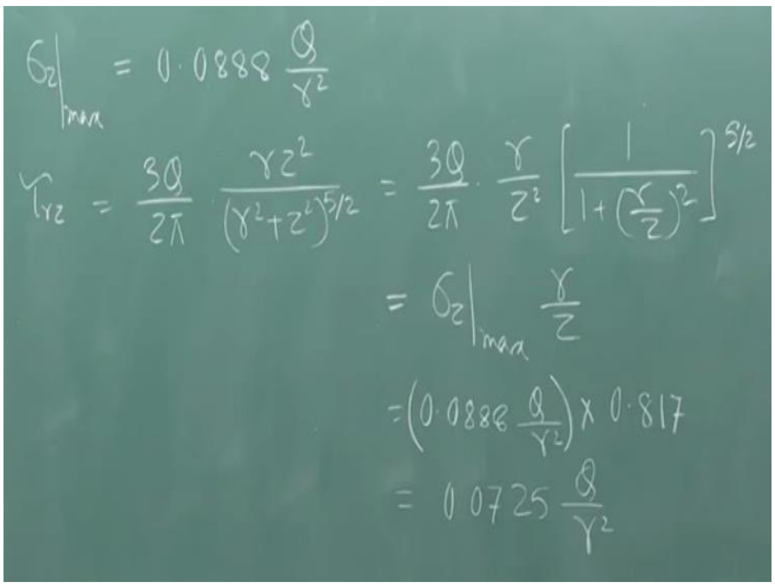


So that is giving me sigma z max is equal to 0.0888 Q by r square okay. So that is your sigma z max and the expression for tau r z is equal to 3Q by 2 pi please make a note of this expression into rz square again from theory of elasticity I can find out this r square + z square whole to the power 5 by 2 so which is nothing but 3Q by 2 pi into r by z square into 1 by 1 + r by z square to the power 5 by 2 okay.

So now from this expression if you compare this expression with sigma z max okay so basically 3Q by 2 pi that is there and 1 + r by z whole square all those things are there so I can write this thing if you compare the tau rz is equal to sigma z max into r by z. So that is nothing but 0.0888 Q by r square into r by z is what value? r by z already root over 2 by 3 that is nothing but 0.817. So, from this I can write 0.0725 Q by r square. So, this is your tau rz at that point okay.

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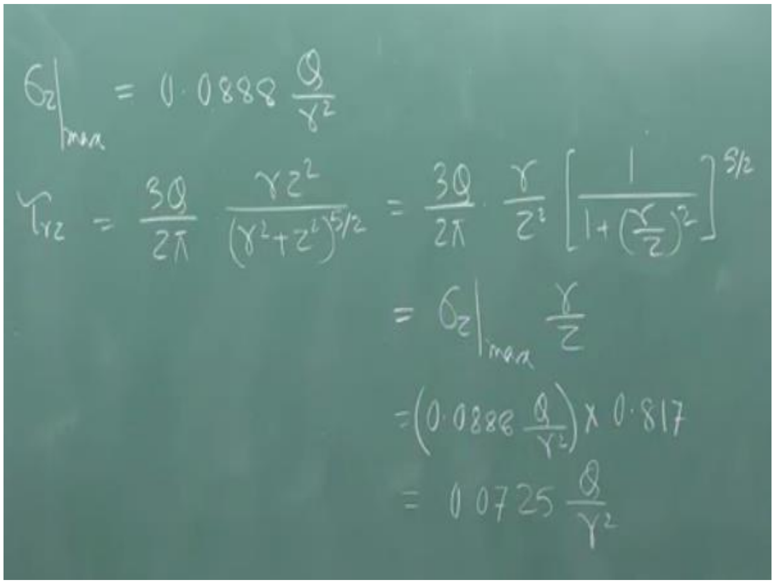


So as given in the problem for r equal to 2 m and q equal to 20 kN sigma z max will be 0.0888 into 20 by 2 square equal to 0.444 kN/m square and tau rz equal to 0.0725 into 20 by 2 square that is equal to 0.362 kN/m square okay. So, we have got the maximum sigma z at that location and also shear stress. I hope that you have understood the problem. Only new thing is that the expression of tau rz that is the new thing we are introducing and that is coming from theory of elasticity as we have got the expression for sigma z similarly we can find out tau rz also okay.

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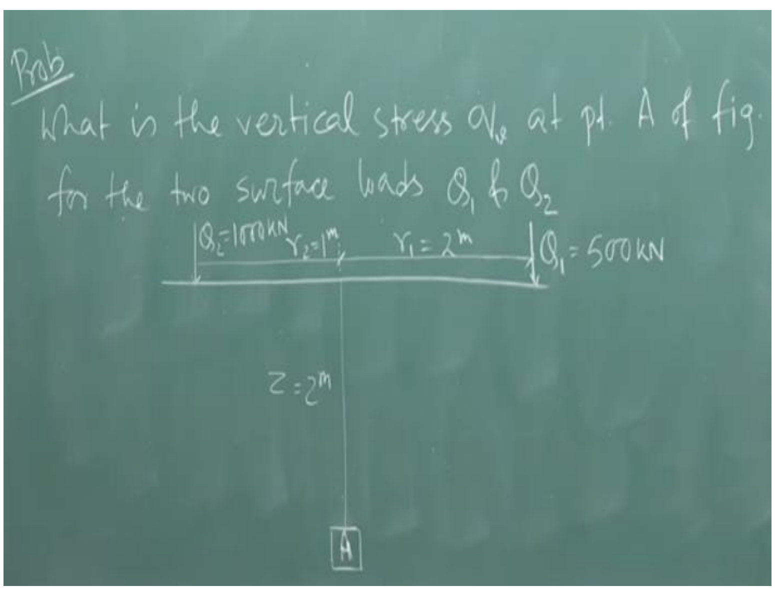


So now we will go to the next problem. Next problem says what is the vertical stress q v at point A of figure given below for the 2 surface loads Q 1 and Q 2 okay. So now it looks like this. This is the surface. Here Q 2 is acting which is equal to 1000 kN and we are considering one vertical section below which you have the point A okay at that location you have to find out the vertical stress and you have another force surface force say Q 1 which is equal to 500 kN. The distance between Q 2 and this vertical axis is r 2 is equal to 1 m and this r 1 is equal to 2 m and z is equal to 2 m.

That means point A is 2 m below the surface on which Q 1 and Q 2 are acting okay. So, this is the problem. So, we will be considering the point A and we will be finding out the vertical stress developed for the application of Q 1 separately and for Q 2 separately and then we will combine them together to get the resultant vertical stress right. This is the way we will solve this problem okay.

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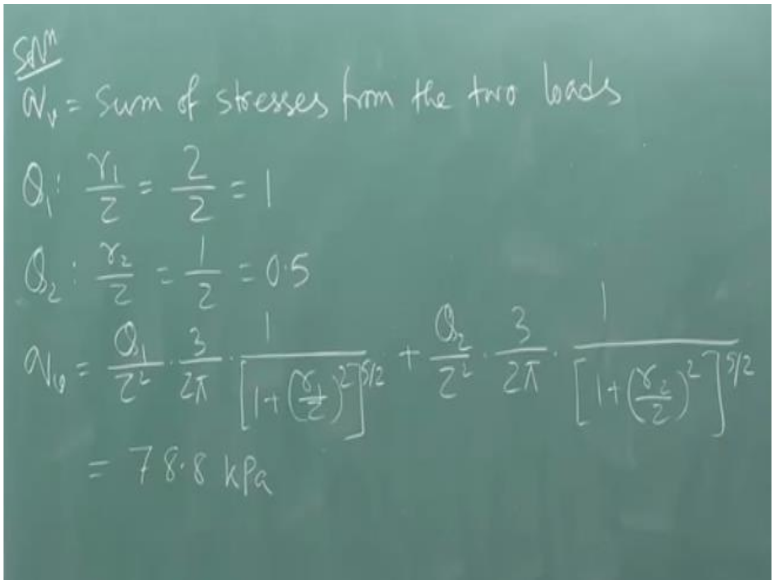


So, solution q v is equal to sum of stresses from the two loads okay. Now for Q 1 r 1 by z is equal to 2 by 2 is equal to 1 just look at the figure r 1 by z that ratio of radius to depth that is r 1 by z is equal to 2 by 2 is equal to 1 and for Q 2 r 2 by z is equal to 1 by 2 that is 0.5 okay. Therefore, q v is equal to Q 1 by z square into 3 by 2 pi into 1 by 1 + r 1 by z whole square to the power 5 by 2 + Q 2 by z square into 3 by 2 pi into 1 + r 2 by z whole square to the power 5 by 2.

Now if we put the values of r 1 by z and r 2 by z that ratio as well as Q 1 and Q 2 as well as z if we put all the values we will be getting this is equal to 78.8 kPa okay. I hope you have understood. So, if you if you have this kind of situation that means when you have several concentrated load and if you want to find out the vertical stress at certain point okay so you have to take the contribution of each individual load at that point and then you add them together to get the total vertical stress increment or the stress developed at that point due to the application of the concentrated load okay. Now we will go to the next problem.

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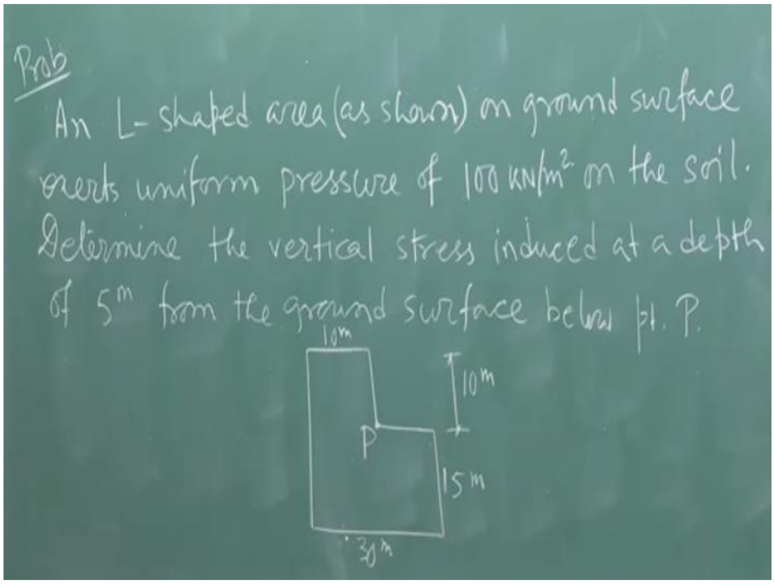


Next problem says an L-shaped area as shown on ground surface exerts uniform pressure of 100 kN/m square on the soil. Determine the vertical stress induced at a depth of 5 m from the ground surface below point P as shown in the figure. So, this is the L-shaped area. This is 10 m. This is also 10 m. This side is 15 m. This is 30 m and this is the point P okay you need to. So, an L-shaped area as shown in the figure on ground surface exerts uniform pressure of 100 kN/m square that is nothing but Q 0 on the soil. Determine the vertical stress induced at a depth of 5 m. So, z equal to 5 m from the ground surface below point P.

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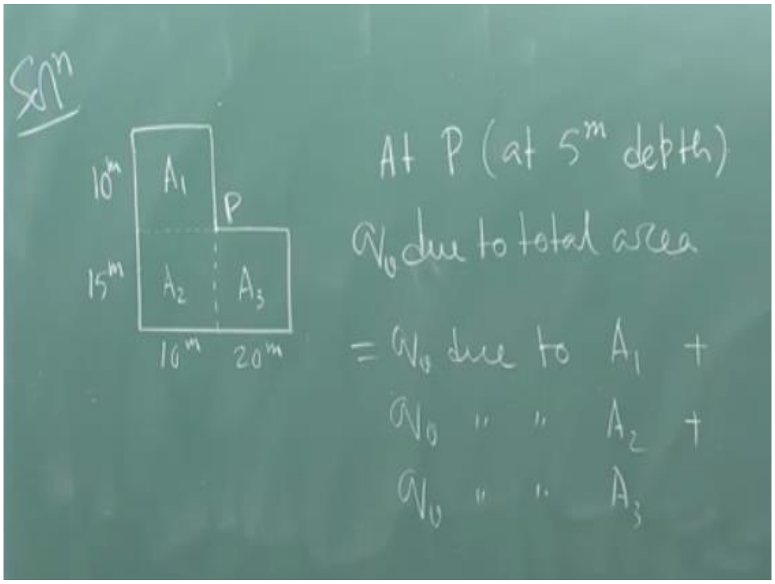


So now if you solve this so basically this area you have to discretize. So, this so P point is not coming as the corner of any rectangular area right. So, as I as we have concluded or we have decided that any point at any point you can find out the stress provided that point belong to some corner of the rectangular area.

So, P does not come as the corner of some rectangular area. So, we have to make the area or we have to discretize the area in such a way that P should come as the corner of the rectangular area. So, let us discretize that area. So, this is the original area. This is the point P. So, we are making 3 different area. This is A 1, this is A 2, and this is A 3. So, this is your 10 m, this is 15 m, this is also 10 m, this is 20 m.

Now we are going to find so at P at 5 m depth q v is equal to q v due to total area is equal to q v due to area A 1 + q v due to area A 2 + q v due to area A 3 agreed. So now P point is the corner of area A 1, corner of area A 2, and corner of area A 3. So now if we try to find out the vertical stress at point P due to area A 1, due to area A 2, and due to area A 3 then if we add them together we will be getting the total stress that is the vertical stress q v for the whole loaded area okay. So that is the idea.

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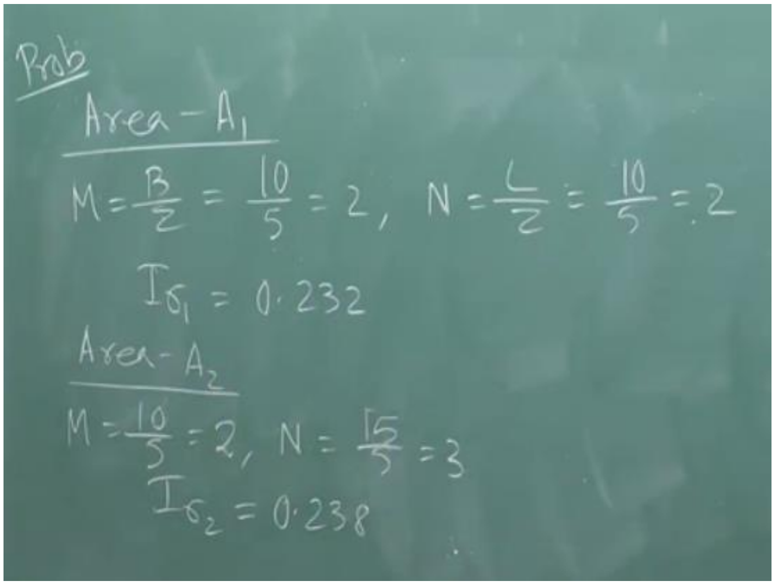


So, for area A 1 your M is B by z what is the shortest dimension for area A 1? Area A 1 is nothing but the square area. So, it does not matter 10 by 5 is equal to 2. N is L by z is equal to 10 by 5 equal to 2. So, for M equal to 2 and N equal to 2 your I sigma 1 is equal to 0.232 that you will get from the table. Now for area A 2 that is your rectangular area. So, M is 15 by 5 that is 3 and N is 20 by area A 2 is something different sorry. Area A 2 is 10 by 5 sorry 10 by 5 so that is your 2 and N is 15 by 5 so 15 by 5 that is 3. So, M equal to 2 and N equal to 3 so with for that combination your I sigma 2 is equal to 0.238 that you will get from the table.

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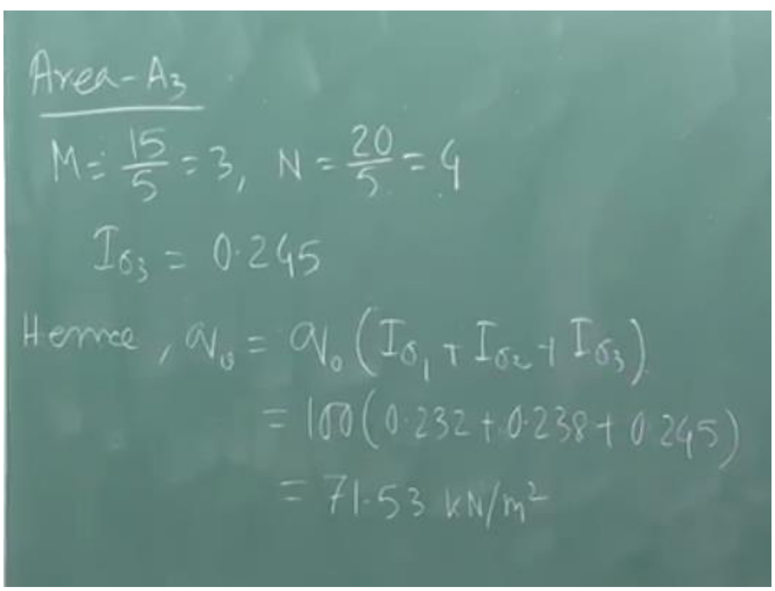


Similarly, for area A 3 your M is 15 by 5 that is 3 and N is 20 by 5 that is 4. So, for M equal to 3 and N equal to 4 you will be getting I sigma 3 is equal to 0.245. Therefore, hence your q v is equal to q 0 into I sigma 1 + I sigma 2 + I sigma 3. What is the value of q 0? That is given in the problem 100, 0.232 + 0.238 + 0.245. So, total is coming 71.53 kN/m square okay. So, this is the problem. So, this is the answer for that problem. So, at point P at some depth say 5 m below point P you will be getting vertical stress as 71.53 kN/m square due to the application of uniform distributor load q 0 that is 100 kN/m square on the L-shaped area okay.

So, with that I will conclude this course. So, I hope that you have enjoyed this course and you have learnt this course. The most important thing is learning the course okay because you may or may not get the opportunity to learn this kind of say very basic and fundamental course. I hope that you have enjoyed each and everything every chapter and feel free to contact me if you have any kind of doubt okay through the forum. Thank you very much and all the best for the exams. Thank you.