

Geology and Soil Mechanics
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Lecture - 62

Earth Pressure Theories and Problems on Earth Pressure on Retaining Wall

Welcome back. So, today we are going to start the new and the last topic of this particular course that is stress in soil. So, stress in soil when you want to understand then you should know why and what situation you basically need this concept. Suppose you are going to find out or you are going to place one foundation or maybe any kind of structure on top of the soil okay.

Now at certain depth of soil at certain distance from the center line of the loaded area, so if you want to find out the stress how much stress is getting developed due to the application of this structure or due to the placement of the structure on top of the soil will be I mean if you want to find out that stress at soil then basically you need to know few say theories as well as the whatever we will be covering in this chapter that will cover that part okay.

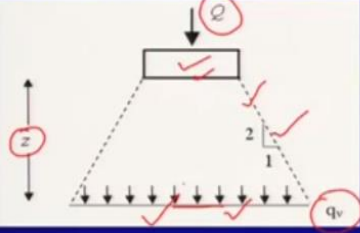
So, it is very essential that means any kind of extra structure or any kind of additional say construction is going on going to happen on top of the soil because of that whatever extra stress you are getting or you are accumulating at the soil at some depth okay below the ground surface so that can be analyzed by this by this process. Now why you need that because suppose you when you are placing the foundation or any kind of additional structure on top of the soil basically you need to know whether your soil is really strong enough to take that much of stress whatever is getting developed due to the placement of the structure right.

So, prior analysis is very much required and then from the soil strength property okay you can think or you can see that okay my soil is good enough or strong enough to take care of the additional stress which is coming on top of the soil. So, this is very essential when you are talking about any kind of foundation or any kind of say any say embankment okay or maybe slope so below that whatever stress will be getting developed you need to know that okay.

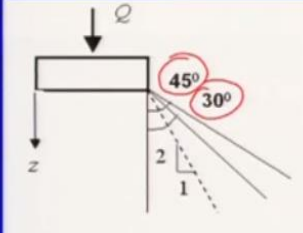
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Stress in soil

- An early method is to use 2:1 slope



- Others have proposed



So, this is an early method so early method is to use 2:1 slope. So, basically the early method it says that you have any say block foundation block or any kind of say structure on top of that you are having say total weight say capital Q and that load is getting transferred to the soil with a 2:1 slope. So, this is your 2:1 slope okay. So, now at base basically at this level say at some depth z below this below the bottom surface of the loaded area okay so you will be getting the stress which is getting developed q_v .

So, q_v is simply is equal to Q capital Q divided by the total base area at this level okay. So, this was the early method when actually no methods or no analysis was there so at that time people used to consider this concept. Still people used to have or used to use this concept and for some say easy competition or maybe quick competition basically people use this method okay. So, now this is this was the earlier method or the very early stage this method was developed.

Now others have proposed that instead of 2:1 slope others have proposed that either this dispersion this is nothing but your dispersion line. This dispersion line either is making 45 degree with the vertical or 30 degree with the vertical right. So, now basically what you are getting you are you are telling instead of 2:1 slope your slope is different so where the slope is defined by this angle the 30 degree angle or 45 degree angle with the vertical so now idea will be remaining same.

The total stress that is small q_v at the base of the say at the base of the loaded area this area will be capital Q divided by the area at that level okay. So, that will be remaining same. Only the dispersion is different. The dispersion people are saying the dispersion could be 2:1 or could be

30 degree dispersion or 45 degree dispersion. So, whatever maybe the case you can find out the stress developed at any depth z below the ground surface by q_v right small q_v .

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Stress in soil

- The pressure increase q_v at a depth z beneath the loaded area due to base load Q

$$q_v = \frac{Q}{(B+z)(L+z)} \quad (7.1)$$

Where, B and L are breadth and length

The Boussinesq Method (1885)

- Boussinesq's equation considers a point load on the surface of a semi-infinite, homogenous, isotropic, weightless, elastic half-space to obtain

Now the pressure increase because this the increase in pressure because soil will be already having some kind of stress right. As you go deep and deep basically at any level the soil will be having the stress vertical stress is equal to γz . That already it is there. So, there is no I mean there is no addition or no kind of alteration in that part okay. So, if you consider any soil deposit at any depth below the ground surface okay you will be getting the vertical stress equal to γz .

Apart from that if you put any structure for which some additional load is coming on top of the soil okay so that load is getting transferred to some depth of the soil and that can be calculated as the additional pressure or the additional stress okay. So, therefore the pressure increase q_v at a depth z beneath the loaded area due to base load Q whatever we have seen just now is equal to q_v is equal to Q divided by $B + z$ into $L + z$.

So, if it is I mean say depending on the depending on the dispersion basically you will be getting this area. So, what is the B, B and L? B and L are the breadth and length at that at that level right. So, now basically the what we are talking about B plus z is the total so I mean if you see from the top so this is the loaded area at any depth z. So, this is your say $B + z$ and $L + z$ okay and on top of that you have sorry this will be your B this will be your L okay.

So, B and L will be the loaded area and below that basically at any depth z if you want to find out the pressure increase the additional pressure Q capital Q v then you can calculate by this formula right. Now these are very crude way I mean whether it is there is no I mean it depends some thumb rule or depends on some experience but more sophisticated elastic analysis is available nowadays okay so based on theory of elasticity.

So, based on theory of elasticity Boussinesq proposed some different expressions by which you can calculate this kind of stress under some loaded area. That load could be concentrated load could be distributed uniformly distributed load over a circular area or could be a stiff line load stiff load right.

So, for any kind of load you can find out the pressure increase at any depth say z okay. Now the Boussinesq method in 1885 Boussinesq proposed this method so Boussinesq equation considers a point load on the surface of a semi-infinite right semi-infinite means it is infinite in 2 directions right. So, semi-infinite homogeneous okay isotropic weightless okay so we are not considering the weight of the soil so the soil medium is weightless elastic half-space okay.

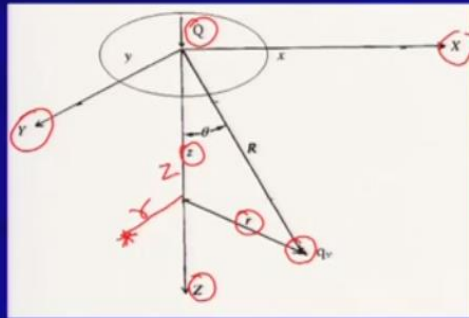
So, we are considering elastic half-space. So, if I mean this Boussinesq equation or the Boussinesq expression was developed based on these I mean parameters like first one is semi-infinite, second one is homogeneous, third one is isotropic, fourth one is weightless, and elastic half-space. So, based on this parameters okay Boussinesq has proposed some equation some expression for a point load expression means so you if you want to find out the pressure increase at any depth z okay below the surface where the point load is acting you can find out from that expression. So, let us see how that expression looks like.

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Stress in soil

The Boussinesq Method (1885)

$$q_v = \frac{3Q}{2\pi z^2} \frac{1}{[1 + (r/z)^2]^{5/2}} \quad (7.2)$$



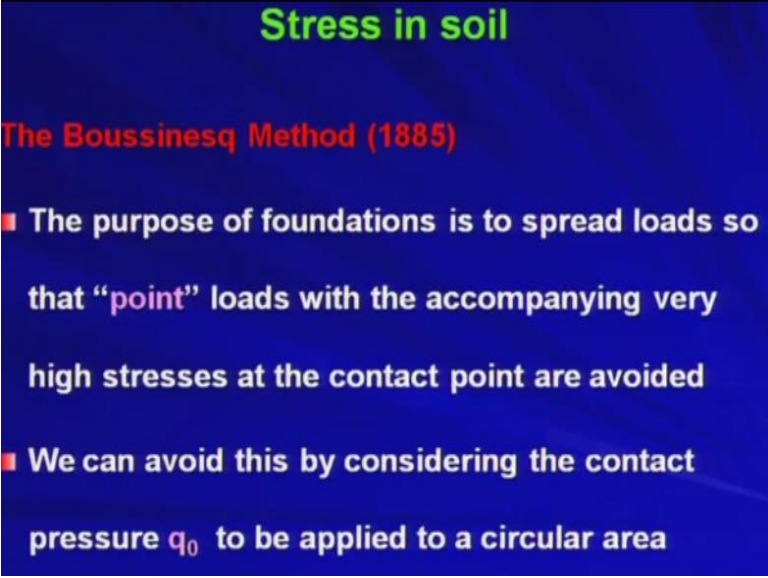
So, basically this q_v is nothing but the pressure increase. Now you are considering one concentrated load Q as I told okay the point load or the concentrated load point load is acting Q . Now along this, this is your z direction, this is your x direction, and this is your y direction okay. So, now along z direction you are going to the deeper strata of the soil. Now for that Q if you want to find out the pressure increase at any point say this point which is defined by 2 coordinates that is small r and small z .

So, if you know small r and small z that means this is the point load I can find out the pressure increase just below the point load when small r equal to 0 if some small r is there, there also I can find out the stress right. So, basically you need r and z to locate at point at which you want to find out the pressure increase and that pressure increase q_v is given by this expression $3Q$ by $2\pi z^2$ into 1 by $1 + r$ by z square to the power 5 by 2 okay.

So, this from this expression so if you know r that means if you know the location of that point at which you want to find out the stress increase or the pressure increase right so that location of the point is defined by small r and z . So, if you know these 2 things and if you know what is the magnitude of the point load you can find out the pressure increase at any depth z at any radial distance small r from the vertical axis okay.

So, I mean anywhere you can find out. So, if you want to find out say state of stress I mean or the pressure increase there so you just measure what is r okay what is r and what is z . So, once you know r and z at any location so you find out so you can find out the stress increase or the pressure increase by this expression okay.

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Stress in soil

The Boussinesq Method (1885)

- The purpose of foundations is to spread loads so that “point” loads with the accompanying very high stresses at the contact point are avoided
- We can avoid this by considering the contact pressure q_0 to be applied to a circular area

Now the purpose of foundations okay so foundations means you know that every civil engineering structure is resting on some foundation base right. So, foundation could be square could be rectangular you might have seen that in your say day to day life when some civil construction is going on you might have seen some foundation is let down in the soil. So, foundation what is the purpose of the foundation?

So, if you do not put the foundation basically the column is resting on the soil right if you do not put the foundation base. That means the column is going to the soil. So, the column will try to give the concentrated load. The foundation the function of the foundation is to distribute that load with within a large area so that the pressure increase or the pressure could be sustained by the soil itself okay that is the idea okay.

So, the purpose of foundations is to spread loads so that point loads with the accompanying very high stresses at the contact point are averted. Instead of foundation if you directly put the column in the soil what will happen. So, the load is coming on the column which will be more or less acting as a concentrated load and that concentrated load will be acting at a point on the soil. So, the stress concentration will be more at a certain point in the soil.

The soil will not be able to take care of that high stress concentration okay. So, to distribute or to normalize that high stress okay over a larger area you generally put the foundation okay. So, though we are not talking about the foundation design and all in this particular course because that is beyond our scope of this particular course. However, the foundation purpose is that okay.

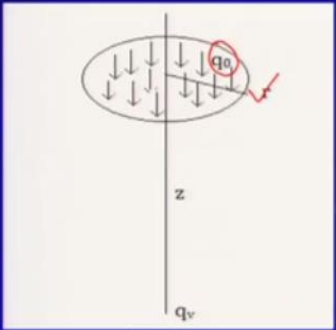
So, we can avoid this by considering so instead of this suppose I mean if I put the concentrated load okay so I know what is the pressure increase. But generally, we avoid that kind of situation by putting the foundation.

So, once we put the foundation we do not have any concentrated load which is giving me the high stress concentration okay at the contact point. So, to avoid that situation we are putting the foundation and we can avoid this by considering the contact pressure q_0 to be applied to a circular area. So, we are considering one circular area over which some q_0 is acting. So, capital Q that is the that was the concentrated load earlier we have seen, the capital Q is uniformly distributed over a circular area okay with some stress intensity or the pressure intensity equal to say q_0 . That means capital Q divided by the circular area is nothing but q_0 .

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Stress in soil

The Boussinesq Method (1885)

$$q_v = q_0 \left\{ 1 - \frac{1}{[1 + (r/z)^2]^{3/2}} \right\} \quad (7.3)$$


So, if we have that then basically we will be getting q_v at any location at any location below the circular loaded area will be obtained by this expression 7.3 where q_0 as already we have discussed q_0 is uniformly distributed loading density okay on over the circular area where r is the radius of the circle okay and z is the depth at which you want to find out the stress okay.

So, if you know r and z so basically you can find out q_v that is the pressure increase below the circular loaded area okay. So, this is also proposed by Boussinesq in 1885. So, all these methods are based on theory of elasticity. So, I am not talking about those complicated calculations or the complicated analysis how to obtain this. This is not empirical please try to remember these equations are not empirical. They can be derived based on theory of elasticity okay.

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Stress in soil

The Boussinesq Method (1885)

- The Eq. (7.3) can be used to obtain the stress q_v directly at depth z for a round footing of radius r

Application to Line Loads

- If we consider a vertical line load q (kN/m^2) of infinite length oriented parallel to the y -axis, then we get,

$$q_v = \frac{2z^3q}{\pi[(x)^2 + (z)^2]^2} \quad (7.4)$$


Now the equation 7.3 can be used to obtain the stress q_v directly at depth z for a round footing of radius r as I told you. So, if you have the circular footing of radius r you can find out the stress at depth z okay by using this equation 7.3. Now application to line loads, suppose if you have the line load. So, line loads means you have a steep kind of load okay. So, in one direction it is infinite okay. So, suppose normal to the board is infinite. So, a continuous steep load continuous line load.

So, if you have that kind of loading which is very common under the steep foundation or under the embankment under the retaining wall whatever is whatever plane strain condition you generally consider so below that you generally consider the line load. So, if we consider a vertical line load q kilo per kilo newton kilo newton per meter square of infinite length oriented parallel to the y axis okay. Then we get so y axis means if you see if you see here so y axis is here right. So, parallel to y axis if you if we have the line load that means along the y axis it is almost infinite.

So, if we consider a vertical line load q kN/m^2 of infinite length oriented parallel to the y axis okay then we get q_v that is the pressure increase at any depth z is equal to 2 into z cube into q divided by π into x square + z square to the power 2 okay the whole square right so where x and z where x and z basically the distance okay along x axis and along z axis. So, if you have the line load along say y axis so at any distance x and at any depth z you can find out the pressure increase by this expression okay.

So, if you know q if you know x that means location that means x and z so you can find out q_v ultimately okay. So, this is the pressure increase happening in the soil at some depth z due to the line load. Now if you have the square or rectangular loaded area so instead of circular if you have square and rectangular loaded area then what will happen? So, Boussinesq has proposed equations for that also.

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Stress in soil

The Boussinesq Method (1885)

Application to square or rectangular loaded area

- Applicable to square or rectangular bases & round ones converted to equivalent squares
- As per Newmark (1935), stress beneath the corner of an area $B \times L$ is

$$q_v = q_0 \frac{1}{4\pi} \left[\frac{2MN\sqrt{V}}{V + V_1} \cdot \frac{V + 1}{V} + \tan^{-1} \left(\frac{2MN\sqrt{V}}{V - V_1} \right) \right] \quad (7.5)$$

So, applicable to square or rectangular basis okay this formula is applicable this expression is applicable to the square or rectangular basis and round ones of course the circular one can be considered converted provided it is converted to equivalent squares. So, if you have the circular area you can convert that thing with some equivalent square right. So, and you know how to find out equivalent square.

So, if this r okay so the total area of the circular base will be equal to the area of the square base okay. So, based on that you can find out the side of the square the length of the I means sides okay of that particular square. So, it is applicable to square or rectangular basis and if you have the circular base then you can convert that thing to the equivalent square base okay. Now as per Newmark in 1935 stress beneath the corner of an area B into L so please remember it will give you the stress at the corner of any corner.

So, at any corner of the square or rectangular base okay so this will give me the pressure increase. Please try to understand. It will not give me the pressure increase at any location within the square. Rather it will give me the stress below the below the corner of the rectangular or

square base okay. So, if it is rectangular base then B is equal to L. If it is square if it is rectangular base then B is not equal to L sorry. If it is square base then B must be equal to L. Whatever maybe the case you will be getting the stress increase or the pressure increase at the below the corner of the loaded area. So, that q_v that is the load increase or the pressure increase is equal to q_0 into $\frac{1}{4\pi}$ into this big expression.

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Stress in soil

The Boussinesq Method (1885)

Application to square or rectangular loaded area

Where footing size B x L (B is the shorter dimension or width)

$M = B/z, N = L/z$ ($q_v = q_0$ for $z = 0$)

$V = M^2 + N^2 + 1$

$V_1 = (MN)^2$

Where footing size B into L and B is the shorter dimension or width okay so please try to remember B is the shorter dimension or the width, M so in the previous expression we have got M, N, V, and V 1. So, M is nothing but B by z normalized with N is nothing but L by z normalized length normalized with respect to depth and q_v is equal to q_0 at z equal to 0. So, you see you can match or you can get this thing from the previous expressions.

If your z becomes 0 then basically you will be getting q_v equal to q_0 . What is q_0 ? q_0 is the pressure on the loaded area that is the loading density on the loaded area and because of that q_0 you are going to find out q_v at some depth z okay which will be just below the corner of the rectangular or square loaded area. So, V is equal to m square + N square + 1 and V 1 is equal to MN whole square okay. So, we have defined all those things.

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Stress in soil

The Boussinesq Method (1885)

Application to square or rectangular loaded area

■ When $V_1 > V$ then \tan^{-1} term is (-) & it is necessary to add π

■ In general Eq (7.5) can be written as

$$q_v = q_0 I_\sigma \quad (7.5a)$$

■ Where I_σ is the stress influence value as given in Table 7.1

Now when V_1 is greater than V then \tan inverse term is coming negative. So, if you look at that expression if you look at this expression if V_1 is greater than V then what will happen? This \tan inverse term this \tan inverse term will be becoming negative. So, in that situation basically what we do so if V_1 is greater than V then \tan inverse term is negative and it is necessary to add π okay. So, the principle π .

So, we consider the π minus that okay. So, in general equation 7.5 can be written as so whatever equation we have seen just now equation 7.5 in general we can write equation 7.5 as q_v equal to q_0 into I_σ where I_σ is the stress influence value as given in table 7.1. So, we will see that okay.

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Stress in soil

Table 7.1: Stress influence values I_σ , $M = B/z$; $N = L/z$

N	M									
	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
0.100	0.005	0.009	0.013	0.017	0.020	0.022	0.024	0.026	0.027	0.028
0.200	0.009	0.018	0.026	0.033	0.039	0.043	0.047	0.050	0.053	0.055
0.300	0.013	0.026	0.037	0.047	0.056	0.063	0.069	0.073	0.077	0.079
0.400	0.017	0.033	0.047	0.060	0.071	0.080	0.087	0.093	0.098	0.102
0.500	0.020	0.039	0.056	0.071	0.084	0.095	0.103	0.110	0.116	0.120
0.600	0.022	0.043	0.063	0.080	0.095	0.107	0.117	0.125	0.131	0.136
0.700	0.024	0.047	0.069	0.087	0.103	0.117	0.128	0.137	0.144	0.149
0.800	0.026	0.050	0.073	0.093	0.110	0.125	0.137	0.146	0.154	0.160
0.900	0.027	0.053	0.077	0.098	0.116	0.131	0.144	0.154	0.162	0.168
1.000	0.028	0.055	0.079	0.101	0.120	0.136	0.149	0.160	0.168	0.173
1.100	0.029	0.056	0.082	0.104	0.124	0.140	0.154	0.165	0.174	0.181
1.200	0.029	0.057	0.083	0.106	0.126	0.143	0.157	0.168	0.178	0.185
1.300	0.030	0.058	0.085	0.108	0.128	0.146	0.160	0.171	0.181	0.189
1.400	0.030	0.059	0.086	0.109	0.130	0.147	0.162	0.174	0.184	0.191
1.500	0.030	0.059	0.086	0.110	0.131	0.149	0.164	0.176	0.186	0.194
2.000	0.031	0.061	0.089	0.113	0.135	0.153	0.169	0.181	0.192	0.200
2.500	0.031	0.062	0.089	0.114	0.136	0.155	0.170	0.183	0.194	0.202
3.000	0.031	0.062	0.090	0.115	0.137	0.155	0.171	0.184	0.195	0.203
5.000	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.204
10.000	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205

So, this is the stress influence value I_σ for M and N. So, along this you have M along this you have N. So, for different values also whenever you are having so the try to understand the process or the steps involved in calculation of this stress or the pressure increase. Suppose you have the loaded area B by L and you know the total pressure or the load intensity on the loaded area say q_0 . So, if you know B and L you can find out M and N how?

Already we have decided M is nothing but what M is B by z and N is L by z. So, if you know B and L and which location you want to find out that means at what depth you want to find out the pressure increase that means if you know the z you can find out or you will be knowing the magnitude of M and N. Once you know M and N you can find out I_σ from this table okay.

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Stress in soil

The Boussinesq Method (1885)

Application to square or rectangular loaded area

- When $V_1 > V$ then \tan^{-1} term is (-) & it is necessary to add π
- In general Eq (7.5) can be written as

$$q_v = q_0 I_\sigma \quad (7.5a)$$

- Where I_σ is the stress influence value as given in Table 7.1

And once you know I_σ from this expression equation 7.5a you can find out q_v , very simple okay. Instead of going for different calculations and all those things from the equation 7.5 that will be very big equation right. So, everything is done and everything is given in this table in terms of stress influence value okay. So, this is for M equal to 0.1 to 1 and N equal to 0.1 to say 10.

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Stress in soil

Table 7.1: Stress influence values I_σ , $M = B/z$; $N = L/z$

N	M									
	1.100	1.200	1.300	1.400	1.500	2.000	2.500	3.000	5.000	10.000
0.100	0.029	0.029	0.030	0.030	0.030	0.031	0.031	0.031	0.032	0.032
0.200	0.056	0.057	0.058	0.059	0.059	0.061	0.062	0.062	0.062	0.062
0.300	0.082	0.083	0.085	0.086	0.086	0.089	0.089	0.090	0.090	0.090
0.400	0.104	0.106	0.108	0.109	0.110	0.113	0.114	0.115	0.115	0.115
0.500	0.124	0.126	0.128	0.130	0.131	0.135	0.136	0.137	0.137	0.137
0.600	0.140	0.143	0.146	0.147	0.149	0.153	0.155	0.155	0.156	0.156
0.700	0.154	0.157	0.160	0.162	0.164	0.169	0.170	0.171	0.172	0.172
0.800	0.165	0.168	0.171	0.174	0.176	0.181	0.183	0.184	0.185	0.185
0.900	0.174	0.178	0.181	0.184	0.186	0.192	0.194	0.195	0.196	0.196
1.000	0.181	0.185	0.189	0.191	0.194	0.200	0.202	0.203	0.204	0.205
1.100	0.186	0.191	0.195	0.198	0.200	0.207	0.209	0.211	0.212	0.212
1.200	0.191	0.196	0.200	0.203	0.205	0.212	0.215	0.216	0.217	0.218
1.300	0.195	0.200	0.204	0.207	0.209	0.217	0.220	0.221	0.222	0.223
1.400	0.198	0.203	0.207	0.210	0.213	0.221	0.224	0.225	0.226	0.227
1.500	0.200	0.205	0.209	0.213	0.216	0.224	0.227	0.228	0.230	0.230
2.000	0.207	0.212	0.217	0.221	0.224	0.232	0.236	0.238	0.240	0.240
2.500	0.209	0.215	0.220	0.224	0.227	0.236	0.240	0.242	0.244	0.244
3.000	0.211	0.216	0.221	0.225	0.228	0.238	0.242	0.244	0.246	0.247
5.000	0.212	0.217	0.222	0.226	0.230	0.240	0.244	0.246	0.249	0.249
10.000	0.212	0.218	0.223	0.227	0.230	0.240	0.244	0.247	0.249	0.250

Similarly, this is for N equal to 0.1 to 10 and M equal to 1.1 to 10 okay. So, for any value of M and N combination okay you will be getting the stress influence value I_σ and once you know I_σ you multiply I_σ with your q_0 you will be getting q_v which is nothing but the pressure increase at just below the corner at some depth z okay. I hope that you have understood that thing.

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Stress in soil

Westergaard's Method

- When the soil mass consists of layered strata of fine & coarse materials, as beneath a road pavement, or alternating layers of clay & sand Westergaard equations (1938) give better estimate of the stress q_v
- The Westergaard equations, unlike those of Boussinesq, include Poisson's ratio μ

So, now we are coming to the another method that is the Westergaard method. So, when the soil mass consist of layered strata of fine and coarse materials as beneath a road pavement or alternating layers of clay and sand, Westergaard equations given better estimate of the stress q_v . Now what does it mean? So, if you consider a layered strata. So, so far, the Boussinesq theory is

considering homogeneous soil media as we have decided homogeneous, isotropic, semi-infinite, weightless all those things should be there then only you will be getting the Boussinesq solution.

Now if the soil is layered strata that means you have which is very common right. You will not be getting very homogeneous kind of strata so if the soil is having layers okay just something like you if you consider the road pavement that means you have the base coats, sub-base coats, voiding coat all those things will be coming right. So, layer wise it will be laid down. So, at any depth suppose due to the vehicle movement what will be the stress increase at the base coats or the sub-base coats?

If you want to find out then Westergaard method will be pretty applicable or pretty suitable okay. So, this will give you the better estimate for q_v than the Boussinesq soil because Boussinesq considers homogeneous strata whereas Westergaard is good for layered strata. The Westergaard equations unlike those of Boussinesq include Poisson ratio that is the biggest advantage of Westergaard method. Westergaard method considers Poisson ratio μ .

That means once you consider Poisson ratio that means you are considering the lateral strain right or the that kind of because Poisson ratio means because of this vertical application of stress you will be getting some lateral contraction or expansion right so that can be taken care of by this Westergaard method. So, that is the beauty that is the basic difference between Westergaard and Boussinesq method.

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Stress in soil

Westergaard's Method

■ As per Westergaard, for a point load Q

$$q_v = \frac{Q}{2\pi z^2} \frac{\sqrt{a}}{[a + (r/z)^2]^{3/2}} \quad (7.6)$$

Where, $a = \frac{1-2\mu}{2-2\mu}$

Now as per Westergaard, for a point load Q the q_v that is the pressure increase can be obtained by this expression where a small a is nothing but $1 - 2\mu$ by $2 - 2\mu$ where μ is the Poisson's ratio okay, this is the difference. So, if you know the location of that point r and z as I told you I mean this is pretty similar, the diagram will be pretty similar than whatever we have seen for Boussinesq method okay. If you know r and z so you can find out q_v provided you know the Poisson's ratio of the soil. So, that is the advantageous thing so layer wise and all those things you can calculate.

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Stress in soil

Westergaard's Method

■ As per Westergaard, under circular loading area

$$q_v = q_0 \left(1 - \frac{a}{\sqrt{a + (r/z)^2}} \right) \quad (7.7)$$

As per Westergaard, under circular loading area I mean like we are trying to map whatever we have learnt for Boussinesq the same thing from Westergaard how we can find out. So, for the under the circular loading area that q_v is equal to q_0 into $1 - \frac{a}{\sqrt{a + (r/z)^2}}$ where a is nothing but same whatever we have seen in the previous expression $1 - 2\mu$ by $2 - 2\mu$.

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Stress in soil

Westergaard's Method

- The stresses for a rectangle of B x L can be obtained for the corner of the rectangular area

$$q_v = \frac{q_0}{2\pi} \tan^{-1} \left\{ \frac{MN}{\sqrt{a} (M^2 + N^2 + a)^{1/2}} \right\} \quad (7.8)$$

Where M & N are defined earlier for Boussinesq & \tan^{-1} is in radian

The stresses for a rectangular rectangle of B into L can be obtained for the corner of the rectangular area so if I consider one rectangular area just below the corner of the rectangular area we can find out q_v as before whatever we have done for Boussinesq the same thing same logic will be applicable here also. Only thing is that the analysis procedure will be different okay. So, q_v is given by this expression where a is already defined and M and N are defined earlier for Boussinesq and \tan inverse is in radian okay.

So, this \tan inverse part is in radian. Now M and N will be calculated or will be can be found out in a very similar way right. What is M and N ? M is nothing but B by z and N is L by z . So, those things will be remaining as it is. Only thing is that \tan inverse will be in radian. So, if you know M and N value you can find out this q_v okay.

This is all about your stress in soil. So, as you have seen that how you can calculate the stresses pressure increase at different level at different location below the below the ground surface due to some additional load on top of the soil right. So, now we will take some numerical examples on retaining structure where you will be getting earth pressure theory that means the lateral pressure and we will be taking few numerical examples on the stress in soil.

So, this with this basically your subject that whole subject that is Geology and Soil Mechanics that subject is concluded. Now we will be taking few more numerical problems so that you can understand whatever concept we have covered in earth pressure theory as well as stress in soil in separately okay. So, now we will take some numerical problem on earth pressure theory okay.

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P.25
A 6 m high retaining wall is to support a soil with unit wt, $\gamma = 17.4 \text{ kN/m}^3$, Soil friction angle $\phi' = 26^\circ$ & cohesion $c' = 14.36 \text{ kN/m}^2$. Determine the Rankine active force per unit length of the wall both before & after the tensile crack occurs & determine the line of action of the

So, the first numerical problem that is your point problem 25 in continuation with the previous problems. So, problem says a 6 m high retaining wall is to support a soil with unit weight gamma equal to 17.4 kN/m cube, soil friction angle phi prime is equal to 26 degree and cohesion c prime is equal to 14.36 kN/m square. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs and determine the line of action of the resultant after tension crack develops okay. So, this is the problem.

The problem says a 6 m high retaining wall is to support a soil with unit weight gamma is equal to 17.4 kN/m cube. Soil friction angle phi prime is equal to 26 degree and cohesion c prime is equal to 14.36 kN/m square okay. So, these are all effective parameters okay. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs.

So, that means you know from the theory that in case of c phi soil that is the cohesive soil if you want to find out the active pressure then basically you will be getting some tension which will be developed at the top part of the wall and due to the tension you may get the tensile crack. So, you need to find out the active pressure before and after the tensile crack occurs and determine the line of action of the resultant after tension crack develops okay.

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Resultant after tension crack develops.

Ans: For $\phi' = 26^\circ$, $K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = 0.39$

$\sqrt{K_a} = 0.625$

$p'_a = \gamma z K_a - 2c' \sqrt{K_a}$

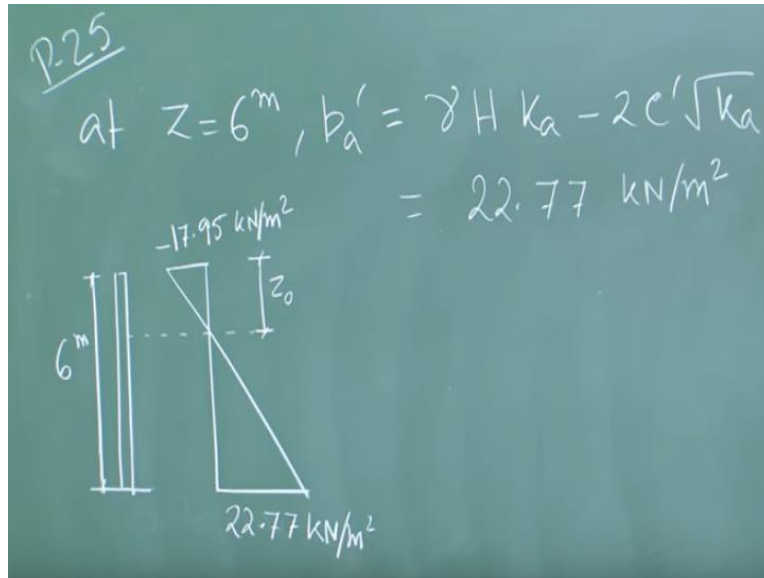
at $z = 0$, $p'_a = -2c' \sqrt{K_a} = -17.95 \text{ kN/m}^2$

So, that means first so answer so for first our job will be to calculate the active earth pressure coefficient. So, phi prime is equal to 26 degree. For phi prime, equal to 26 degree your active earth pressure coefficient K_a is nothing but $1 - \sin \phi'$ by $1 + \sin \phi'$ which gives me 0.39 and for our calculation we find out root over K_a is nothing but 0.625 okay. It will be required later on. So, these things we have got at the initial stage.

Now active earth pressure P_a in case of cohesive soil you know from this expression γz into $K_a - 2c'$ prime into root over K_a agreed. So, if you do not remember please recall whatever we have covered when we talked about the active earth pressure distribution okay for cohesive soil. So, now at z equal to 0 at z equal to 0 what is your p_a prime that means effective earth pressure I mean earth pressure p_a prime is equal to $-2c'$ prime into root over K_a which gives me -17.95 kilo newton per meter square okay.

So, 17.5 kN/m^2 square. So, that that negative sign is coming due to your tension developed at the top of the wall which is pretty obvious right. In case of active pressure already we have seen that you will be getting tension which will be developed at the top of the wall. So, at z equal to 0 that means at top of the wall you are getting active earth pressure p_a prime is equal to -17.95 . That means 17.95 kN/m^2 square tension okay.

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Now at z equal to 6 m that means at the bottom of the wall what will be your active earth pressure p_a' is equal to $\gamma H K_a - 2c'$ into root over K_a . So, if I put all the values I will be getting 22.77 kN/m square okay. So, that is at the bottom of the wall. So, if I have the wall like this, this is my say wall. This is your 6 m. So, you are getting the pressure distribution like this. So, this is - 17.95 kN/m square. What is this depth? This depth is nothing but z_0 if you recall okay and this is your 22.77 kN/m square okay.

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Now active force before the tensile crack appeared

$$P_a = \frac{1}{2} \gamma H^2 K_a - 2c' H \sqrt{K_a}$$

$$= \frac{1}{2} (17.4) (6)^2 (0.39) - 2 (14.36) (6) \sqrt{0.39}$$

$$= 14.46 \text{ kN/m}$$

Now active force before the tensile crack appeared will be how much? P_a is equal to half γH^2 into $K_a - 2c'$ prime right. So, half γH^2 into $K_a - 2c'$ prime H into

root over K_a that means 0 to H P_a prime into $d z$ right so in that way basically you can find out capital P_a that is the total active thrust that is active force before the tensile crack appeared.

Once the tensile crack appeared then basically we have to consider you know you have to consider the only the bottom part only the bottom part of the pressure distribution because at that time once tension cracks is developed at that time this tension is not getting considered, already we have seen okay. So, if you put the values half into 17.4 into 6 square into $0.39 - 2$ into 14.36 into 6 into root over 0.39 okay. From this I will get 14.46 kN/m. So, this is the first part of the problem. That means the active force before the tensile crack occurs tensile crack happens okay.

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P-25
 And active force after the tensile crack appeared

$$z_0 = \frac{2c'}{\gamma \sqrt{K_a}} = 2.64 \text{ m}$$

$$P_a = \frac{1}{2}(H - z_0) [\gamma H K_a - 2c' \sqrt{K_a}]$$

$$= \frac{1}{2}(6 - 2.64) [(17.4)(6)(0.39) - 2(14.36)\sqrt{0.39}]$$

$$= 38.25 \text{ kN/m}$$

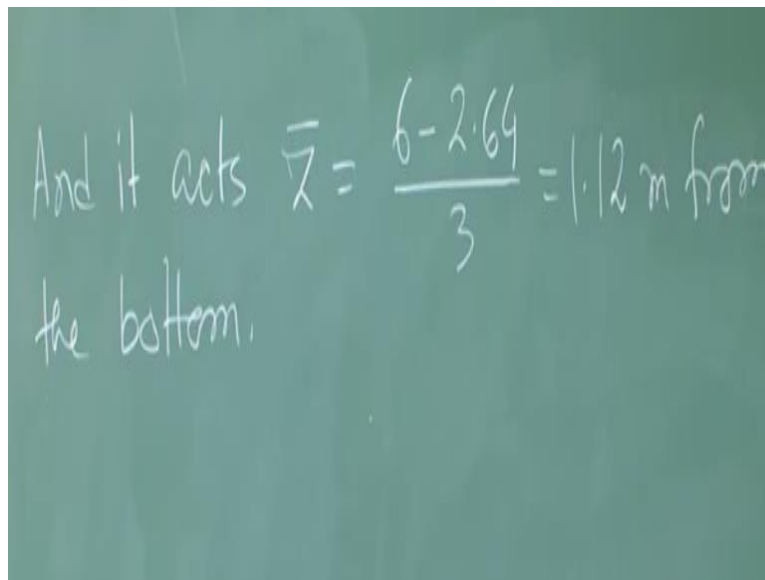
And active force after the tensile crack appeared will be how much? So, for that first you need to calculate z_0 you have to see that what will be what will be the portion which will be left out right. So, z_0 has to be calculated, z_0 is you know from our previous discussion z_0 is equal to $2c'$ by gamma root over K_a . By putting all the values I will be getting 2.64 m okay. So, therefore your P_a that is the total force is equal to half into $H - z_0$, I hope you remember this expression you just refer to your class note okay.

So, from here I can write $6 - 2.64$ gamma 17.4 H is 6 $0.39 - 2$ into 14.36 into root over 0.39 . So, from this I will get 38.25 kN/m. Now you just compare. So, before tension crack happens you are getting this much of thrust on the wall 14.46 right. Now after tension crack happens okay you are getting the total thrust on the wall is 38.25 . Why it has been increased? Because due to the

tension crack you are neglecting the tension of the top part that means up to z_0 whatever tension was getting developed you are just neglecting.

So, from z equal to z_0 to z equal to H so whatever pressure distribution is there that is the total compressive pressure is now coming into the picture. So, earlier case when the tension crack did not appear at that time the tension from 0 to z_0 and the compression from z_0 to $2z_0$ so this was nullifying each other. So, therefore whatever was left out that was here. That is why you are getting increase or enhancement in the active pressure on the wall after the tension crack happens okay.

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And it acts $\bar{z} = \frac{6 - 2 \cdot 64}{3} = 1.12$ m from the bottom.

And it acts at \bar{z} which is nothing but $6 - 2.64$ divide by 3 . So, $H - z_0$ by 3 okay. So, at that location the point of application of this P_a will be $H - z_0 - 3$ which is equal to 1.12 m from the bottom okay. So, I will stop here today. So, in the next class we will be solving few more numerical problems on the retaining wall and then we will be solving few more problems on the stress distribution or the stress below the soil. Thank you very much.