Geology and Soil Mechanics
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Lecture - 54
Tutorial on Permeability - a

Welcome everyone. So, after this is the third tutorial lesson and after compaction so we are going to start today with the permeability. Now I am going to discuss today a few problems on permeability and the first problem says that.

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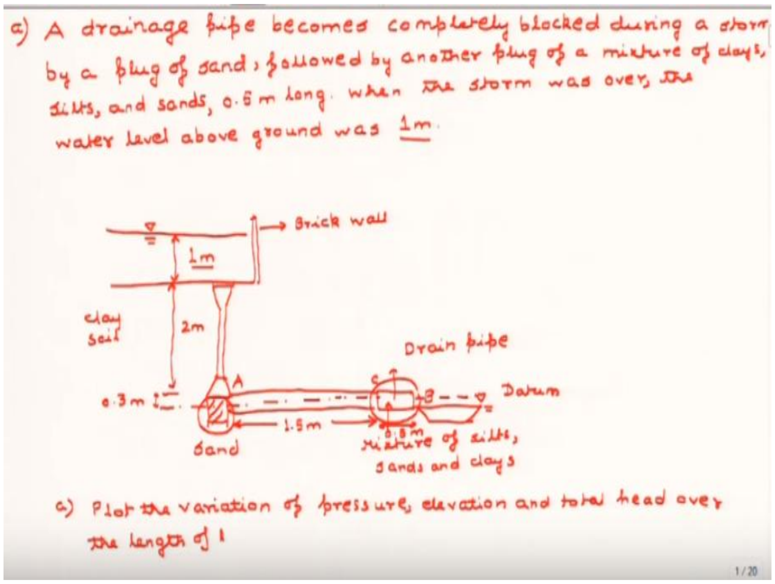

A drainage pipe becomes completely blocked during a storm by a plug of sand followed by another plug of a mixture of clays, silts, and sands 0.5 m long. When the storm was over the water level above ground was 1 m. Now a figure for this is given here. So, this is the water level and this is the brick wall. So, this height is 1 m as said in the question and there is a down pipe connected. This is the layer of sand.

This height is given as 2 m and this is the clay soil while this height from the middle is said to be 0.3 m. Now this is the drain pipe. This point is A this is C and this is the B and the datum is here. This is the clogged mixture of silts, sands, and clays. This is the plug of, this point is the plug of sand while this is the plug of clay, silts, and sands and this is 0.5 m long and this entire length is 1.5 m.

This is the brick wall. So, it is asked in the question to first plot the variation of pressure elevation and total head over the length of the pipe. Now the first strategy that we are going to accomplish during this problem is that we have to select a datum. Now so basically you are going to first select a datum.

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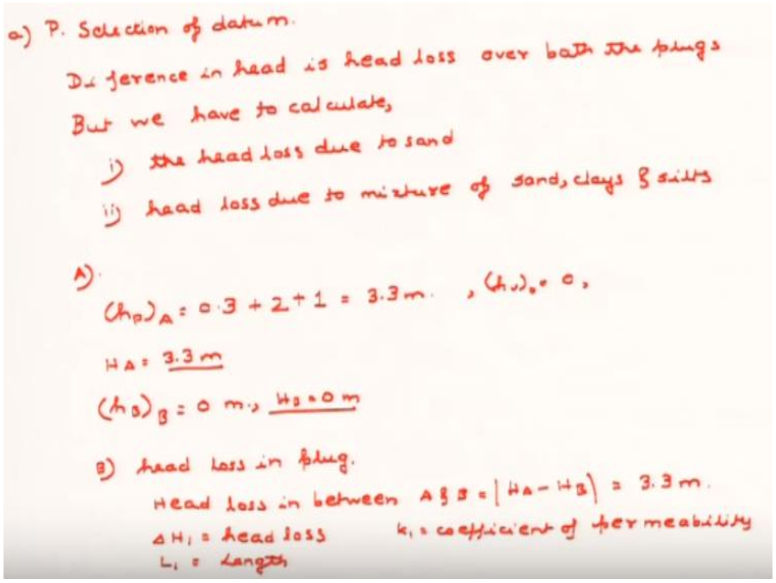

Now as we have said in the question that the datum is selected at this point at the bottom at the bottom of B so basically this point is selected as the datum. Now we have selected the datum. Now from the information given we have to calculate the total head at the points A and B and the difference in the head is head loss over both the plugs but we have to calculate the head loss due to sand first and second head loss due to mixture of sand, clays, and silts.

Now the continuity equation as we know will provide the will provide the opportunity to find out the head loss over the entire pipe. Now let us see how to find out the head loss over the entire pipeline. So, first I have selected a datum. Now basically we will determine the head loss at point A. So, already know that point A is the point where basically the sand is plugged. So, from this point we can say that the head loss means the first is the h p the pressure head loss at the point A is equivalent to 0.3 because this head is 0.3 + 2, 2 is this point 2 is this length + 1 m.

So, this is equivalent to 3.3 m. So, basically the total head loss at the point A and the velocity head loss is the velocity head is 0 so because water flows very slowly through soils as I have said previously as we have already covered in the lecture section that water flows very slowly so always the velocity head loss is 0. Only there is a datum head and there is a pressure head. So, we have found out what the datum head and the pressure head is and this is equivalent to 3.3 m.

Now similarly the head loss at the point B will be equivalent to 0 m because as we can see that it lies it lies on the datum so basically it is neither above the it is neither there is a pressure head nor there is a datum head so both of them is actually equivalent to 0 in this case. Now the head loss in each plug has to be determined. So, head loss in plug. Now first we have to find out what is the head loss in between A and B. So, this is H A - H B mod of that that is equivalent to 3.3 m. Now let us consider that delta H 1 is the head loss, L 1 is the length, K 1 is the coefficient of permeability.

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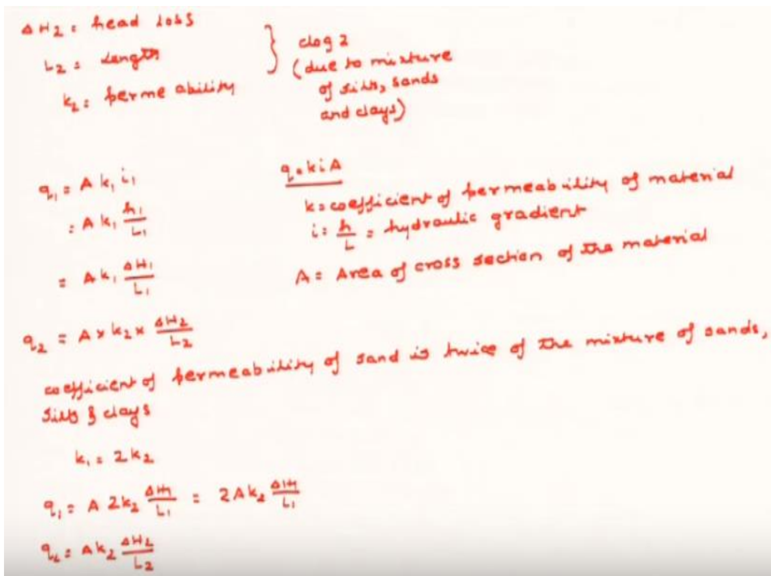

Now if that is the case then delta H 2 be the head loss L 2 be the length and K 2 be the permeability. Now we are going to refer the first clogging as clog 1. This is due to sand entirely and this as clog 2. This is due to mixture of silts, sands, and clays. So, and q 1 and q 2 I am not referring I am not writing here but q 1 and q 2 are basically the quantity of water per unit area through the clog 1 and clog 2 respective.

So, let us start with q 1 that is the quantity of water that is passing through clog 1. Now this is equivalent to A into K 1 into i 1 because from Darcy's law we get q is equal to K into i into A where K is the coefficient of permeability of material i is the hydraulic gradient which is equivalent to h by L where h is the head loss over the length L and A is the area of cross section of the material.

So, here similarly for clog 1 we get q 1 is equal to A K 1 into i 1. Now A is considered to be A because the area of the cross section of the pipe as you can see is almost same in is same in both the cases. So, basically that is why we considered a single area A. Now A into K 1 into i 1 i 1 will be equivalent to h 1 by L 1 or here it will be equivalent to delta H 1 by L 1 where delta H 1 is the head loss over the length L 1.

So, this will be equivalent to A into and for q 2 it will be equivalent to A into K 2 into delta H 2 by L 2. Now let us go back to the question and see that basically it is also given that the coefficient of permeability of sand is twice of the mixture of sands, silts, and clays. So, in other words K 1 is equivalent to 2 times of K 2. So, from here we can we get that q 1 is actually equivalent to A into 2 K 2 into delta H 1 by L 1. So, this is equivalent to 2AK 2 into delta H 1 by L 1 while q 2 is equivalent to A into K 2 into delta H 2 by L 2.

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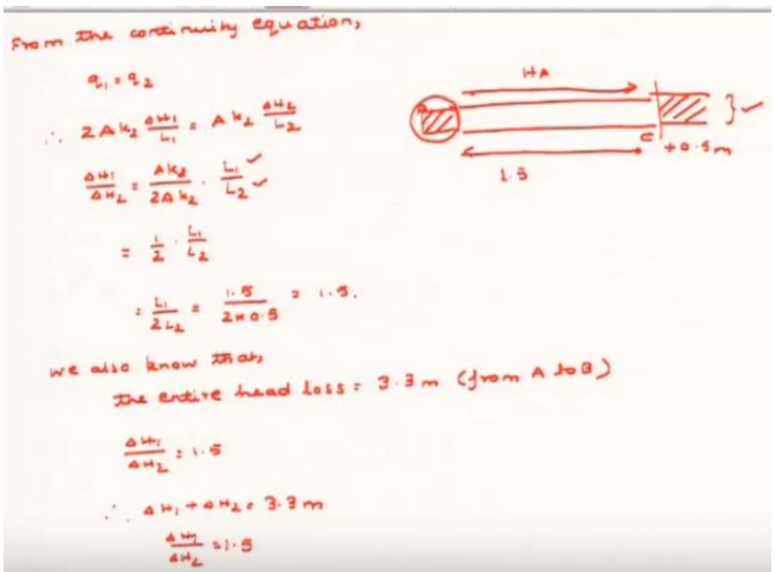

Now as we have said in the beginning that basically in the variation of pressure when we will discuss about this head loss due to mixture of sands, silts, and clays and the head loss due to the sand we will connect both these 2 by the head with the continuity equation. So, basically now we will refer to the continuity equation here. So, from the continuity equation the flow must be sent throughout the entire length of the pipe.

So, that means basically q 1 must be equivalent to q 2. If this is the case then we get that 2 into A into K 2 into delta H 1 by L 1 is equivalent to A into K 2 into delta H 2 by L 2. So, delta H 1 by delta H 2 is actually equivalent to A into K 2 by 2A into K 2 into L 1 by L 2. So, this is equivalent to half into L 1 by L 2 or L 1 by 2L 2. Now what is the length of L 1 and L 2? Now let us refer back to the figure.

We will see that the length of L 1 is 1.5 m while the length of L 2 is actually 0.5 m. So, L 1 is 1.5 m because L 1 is due to the sand while L 2 is the mixture of sand, silts, and clays. So, 2 into 0.5. So, this comes out to be 1.5. L 1 is this length that is due to the mixture of sand while L 2 is due to the mixture of sand, silts, and clays this mixture while L 1 is the mixture of sand. So, basically it is 1.5 m.

Now we also know that the entire head loss is equivalent to 3.3 m from A to B. As we have found out previously that the entire head loss is equivalent to 3.3 m the head loss at B is 0 m. So, the entire head loss is 3.3 m. Now we know that the ratio of the head losses is equivalent to 1.5. So, from here we get that delta H 1 plus delta H 2 is equivalent to 3.3 m and delta H 1 by delta H 2 is equivalent to 1.5.

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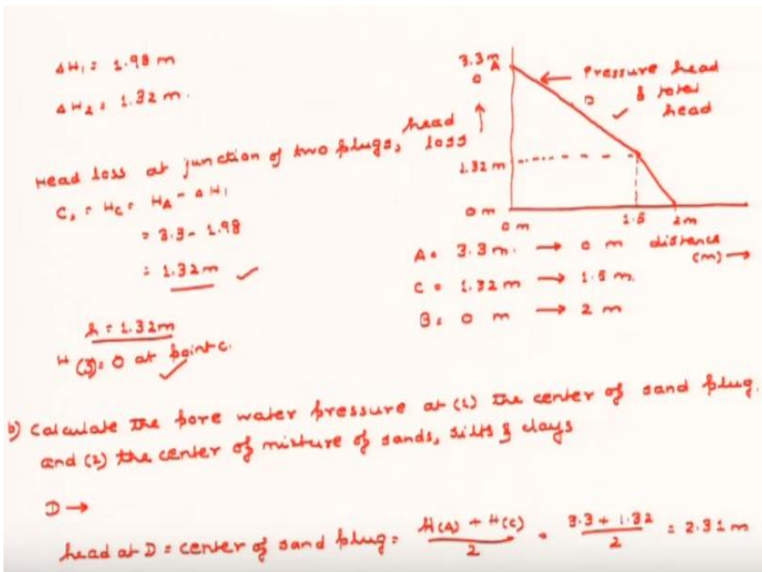

So, solving this 2 we get delta H 1 is actually equivalent to 1.98 m while delta H 2 is equivalent to 1.32 m. Now we have to find out the head loss at the point we have found out what is delta H 1 what is delta H 2. Now we have to find out what is the head loss at the point C. That is the final head loss because we have already known that the head loss at the point A is 3.3 m, the head loss at the point B is 0 m and now we have to find out what is the head loss at the point C.

So, this is where is the point C first let us see the C is at the junction of the 2 plugs. So, basically head loss at junction of 2 plugs is that is at C will be equivalent to H c that is equal to H A because it is the head loss at the point A minus the head loss that is observed across this entire length of the pipe. So, minus del H 1. So, that is equivalent to nothing but 3.3 - 1.98 that is equal to 1.32 m.

Making a little clear this part that delta H 1 is basically the head loss due to the sand due to the mixture of sand or due to the clogging of sand while delta H 2 is due to the clogging of sand, silts, and clays. So, along the entire length of the pipe there will be 2 head losses. One due to the mixture of sand only that is delta H 1 and another due to the mixture of sand, silts, and clays that is delta H 2.

Now the head loss will be along the entire length of the pipe L 1 for delta H 1 and L 2 for delta H 2 because the pipe basically is of length of 1.5 + 0.5 m at the end. So, basically along this length the head loss will be only due to the head loss will be the head loss will be only due to the mixture of sand that is present here while head loss at this point will be due to the mixture of clays that is present here the sand, silts, and clays that is present here.

That is what we have done delta H 1 and delta H 2 and then we have arrived at the conclusion that the head loss at the junction of the 2 plugs that is basically at the point C or at this point is HA - delta H 1 because the plug at the point because the second plug is still absent here. At the end it is due to the second plug and the first plug while at the point C it is only due to the first plug. So, that is what we have found out 1.32.

So, basically now we have the head loss at the point C is basically just 1.32 m because it is just above the datum and so basically there is no the H z basically equivalent to 0 at the point C. So, now let us find out that how the distribution of the pressure diagram looks like. Now at A we know the head loss is equivalent to 3.3 m and we are starting from this point so the distance is 0 m. At C we know the head loss is 1.32 m.

This is at a distance of 1.5 m this is at a distance of 1.5 m at this point so this is 1.5 m from the point A. While at B the head loss is 0 m and this is at a distance of 2 m that is 1.5 + 0.5 m. So, this is at a distance of 2 m. So, the point A will be here, the point C will be here and the point B will be here. So, this distance is basically equivalent to 1.5 this is 2 m this is 0 m. This is the distance in meter this is the head loss and this height is 3.3 m this is 1.32 m and this is 0 m.

So, this is how the plot looks like the plot of head loss versus that is what we are asked plot the variation of pressure, elevation head, and total head over the entire length of the pipe. Now this is the plot of the elevation head, pressure head, and the total head over the entire length of the pipe but actually this is the plot of the total head. Now as I have said that the elevation head is 0 because you can basically see that the entire pipe lies along the datum.

So, basically the elevation head whether it is A whether it is B whether it is C the elevation head everywhere is 0. The elevation head actually is 0 so it is just a plot of the pressure head and the total head. This is the plot of just the pressure head and total head. The elevation head is 0 again I am telling the elevation head is 0 because the pipe lies along the datum so basically there is no elevation head in this point. There is only pressure head and there is a total head.

As you have seen that basically here also when I said about the pressure at the point A then also I told that basically the elevation head is 0. At B the pressure head is 0 and the elevation head is also 0 while at the point C the elevation head is 0 while the pressure head is 1.32 m. So, that is how we have plotted the variation of pressure head and the total head against the length of the pipe along the distance.

The second question is asked is that calculate the pore water pressure at the center of sand plug and the center of mixture of sands, silts, and clays. Now we have found out that what is the pressure heads at each of these points and the total head at each of these points. Now from there we are going to find out what is the variation of the pore water pressure. So, how we are going to find out the variation of the pore water pressure?

So, let us say that at let us consider a point D first. Now D is basically at the center of point A and point C. That means it is at the center of this sand plug or the center of this point. So, this is the point D. So, since it is the center of the point since D is just at the center and since the variation from A to C is actually linear in nature so we can find out the head D easily by taking a mean of the heads at point A and point C. That means the head at D which basically is the center of sand plug will be equivalent to H a plus H c by 2 because the variation from a to c is linear in nature as we have seen. So, this will be equivalent to 3.3 + 1.32 by 2. This is equivalent to 2.31 m.

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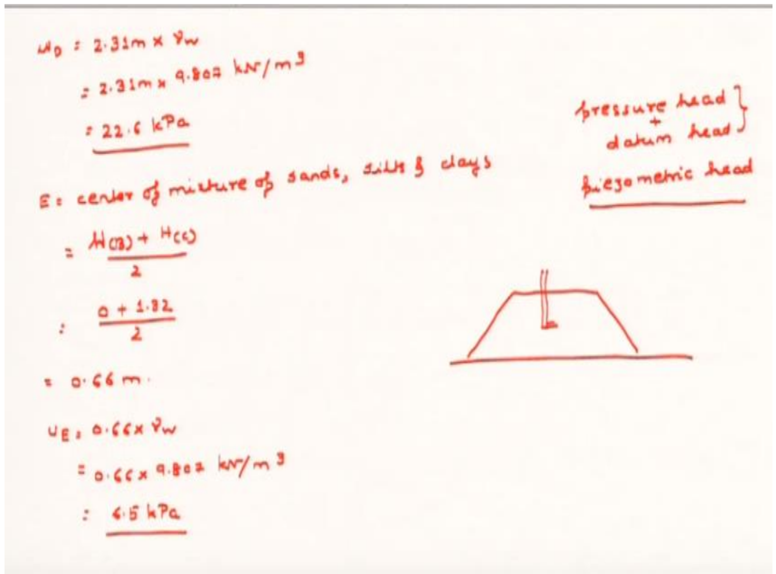

So, if you find out the head at D then the finding out the pore pressure at D is very easy. So, 2.31 m into the unit weight of water. So, this is equivalent to 2.31 m into 9.807 kN/m cube which is equivalent to 22.6 kPa. So, we have found out that what is the pore water pressure at the center of the sand plug. Similarly, the pore water pressure at the center of the mixture of sand, silts, and clay can also be found out because the variation from C to D is again linear in nature.

So, the variation of the pore pressure at the center of the mixture of the center of the sand, silts, and clays will be equivalent to H c + H d by 2 sorry H b + H c by 2 because again from the variation from c to be is linear in nature. So, let us consider another point E which basically is the center of mixture of sands, silts, and clays. So, this will be equivalent to H B plus H C by 2. Now H B will always be 0 and H C is 1.32 so divide by 2 equal to 0.66 m.

So, the pore pressure at this point will be equivalent to 0.66 into gamma w which is the unit weight of water so 0.66 into 9.807 you can even take 10 kilo ton per meter cube. So, this is equivalent to 6.5 kPa. So, you have found out the pore water pressure at the center of the sand mixture and the center of the sand, silts, and clays mixture. Now it was easy to found out it was easy to find out because the variation is linear in nature.

If the variation is nonlinear in nature then basically you have to find out the pressure head and datum at every point and as I have already said that the velocity here is always 0 because the velocity of water through sands through soils is very less. So, basically you only consider the pressure head and the datum head. Now since you consider only the pressure head and datum head there is an important point to note here the sum of pressure head and datum head is known together as the piezometric head and this piezometric head is very important because mostly in earth structures when basically you are supposed to find out the phreatic line which we will discuss in the next lecture in that case basically this piezometric head is very important because this piezometric head gives the idea about where the what is the pressure what is the sum of the pressure head at a particular point in the embankment so that basically you can find out that where is the location of the water table.

It is very important because in most of the cases in most of case in embankments you have to find out the seepage laws or basically the flow nets and all those things so basically in all those cases in order to find out the pore water pressures you take help of this piezometric head. You just insert a piezometer at any point in the embankment and basically you find out the head loss at that point.

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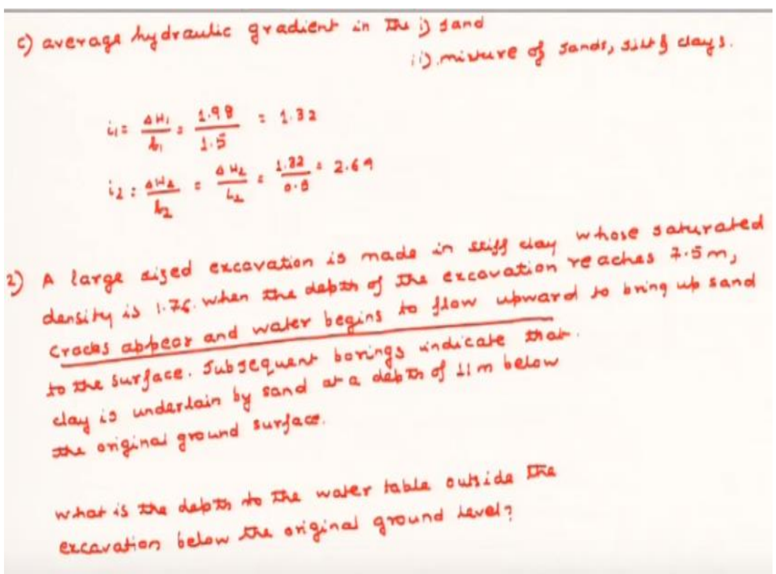

Now in the last question it is asked that what is the average hydraulic gradient in the sand and mixture of sands, silt, and clays. Now the average hydraulic gradient is very easy because we have already found out what is delta H 1 and you also know what is length along which the head loss is occurring. As I have said that in Darcy's law that H 1 by L 1 actually refers to the part that basically that head H 1 is actually the head loss over the entire length L 1.

So, similarly here the head loss delta H 1 due to the clog of sand we have found out to be 1.98 m. So, this is 1.98 and the length over which the head loss is occurring is 1.5 m. So, 1.98 by 1.5. So, this comes out to be 1.32. So, this is the average head loss in average hydraulic gradient in sand

and for the mixture of sand, silt, and clay similarly delta H 1 by L 2 this L 2 delta H 2 by L 2 where L 2 is 0.5 m that is the length of the mixture of silt, clays, and sands.

So, this is by 0.5 and delta H 2 as we have found out previously is 1.32 m. So, this will be 1.32. So, this comes out to be 2.64. So, we find out that the average hydraulic gradient in case of mixture of sand, silt, and clay is more than basically the mixture of sand and this is obvious from the this plot of pressure head versus the distance because we see that it is steeper in the case of sand, silts, and clays while it is less steep in case of sands.

So, we have discussed generally that how to find out a piezometric head in case of in case we basically any question is given that basically how to find out the piezometric head at a certain point. How to use the equation of continuity in order to find out the in order to find out the piezometric heads at certain points and how it is useful in case of soil mechanics. Now let us move to the next part where basically I am going to discuss one important factor of this permeability and of this permeability fact that how to use the how to use the effect of sand boilings and basically mostly that how to find out the effect of sand boiling on the depth of excavations.

So, basically, we will consider a problem that a large sized excavation is made in stiff clay where saturated density is 1.76. When the depth of the excavation reaches 7.5 m cracks appear and water begins to flow upward to bring up sand to the surface. Subsequent borings indicate that clay is underlain by sand at a depth of 11 m below the original ground surface. So, it is asked to find out that what is the depth to the water table outside the excavation below the original ground level. Now here also first before we start doing the problem let us discuss first thing.

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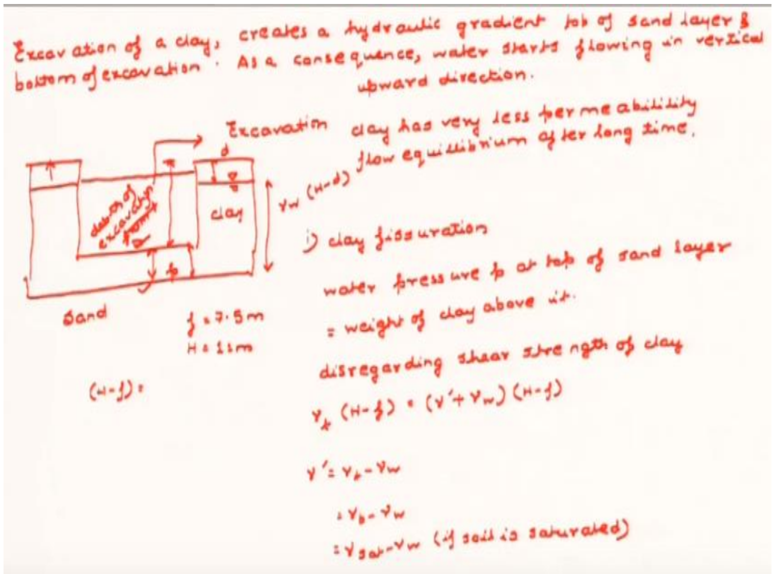

How to how making a excavation of clay affects the entire soil profile. Now when you make an excavation of a clay then this creates a hydraulic gradient from the top of the sand layer to the bottom of the excavation because if you see the because if you see the diagram means how basically it was made then in this case let us say that this is the excavation that is made and this is the sand layer.

So, this is the clay this is the excavation and this is the water table. So, you see channel has been made from the water table to the sand layer. So, therefore basically hydraulic gradient established between the sand layer and the top water layer. So, it creates a hydraulic gradient on the top of sand layer and bottom of excavation. That is what I said that between this water layer the water layer here and basically the sand layer at the top.

So, there is a hydraulic gradient in this between these 2 parts. Now what will happen as a consequence? So, as a consequence water will now start flowing in vertical upward direction and because clay has a very less permeability so this flow equilibrium will reach after a long time. Now the problem should be addressed only for a short time interval and in this case we will consider 2 phases.

The first is basically clay fissuration because we have already said that cracks that when basically the excavation is made then cracks are appearing in the clay layer that I have already said cracks appear. So, now we have to first consider physically that when the floor of the excavation is going to be stable. Now the floor of the excavation is going to be stable only if the water pressure p at top of sand layer that means at this point p if p is the water pressure is equivalent to weight of clay above it disregarding the shear strength of clay.

So, now this equivalency condition means basically until and unless this equivalency condition is held the clay is not stable. If the water pressure at the point p above the sand layer is more then basically what will happen is that the water will rush out outside. So, in order to achieve this what we have to say is that gamma total into H - f. Now what is this H and f and what is this f? This f is basically from the top to this layer. So, H - f is actually equivalent to this length. H is basically the depth of excavation plus from depth of excavation just the depth of excavation because we do not want to consider any water table.

So, basically f is just the depth of the excavation. So, H - f. Now the depth of excavation is said to be 7.5 m. So, basically f is given as 7.5 m and the value of H is given as 11 m because it is said the subsequent borings indicate that the clay is underlain by sand at a depth of 11 m below the original ground surface. So, basically the H is 11 m. H is the length of the clay layer from the top of the sand layer.

So, H - f is just the thickness of this or basically it is just the length where basically the hydraulic gradient is created from the top of the sand layer to the bottom of excavation now so H we have found out the length of the H if H is the height H basically says the depth of the clay layer by f is the depth of the excavation. So, H - f is nothing but the hydraulic gradient from the top of the sand layer to the bottom of the excavation.

Now gamma total is basically the total unit weight of the clay during that layer so in that layer. So, basically gamma total is nothing but gamma dash that is the buoyant unit weight plus gamma w because we all know that gamma dash is nothing but gamma total - gamma w or gamma total can also be referred to as gamma bulk gamma bulk - gamma w or you can also refer to that as gamma sat - gamma w if it is entirely saturated if soil is saturated.

So, gamma dash + gamma w into H - f. So, this is the water pressure p at the top of the sand layer. This is the water pressure now we have to find out the water pressure p at the top of the sand layer. Now this is the weight of the clay above the sand layer because as you can see that the this is the total weight of the clay of the sand layer. Now what is the water pressure p at the top of the sand layer? That is very easy.

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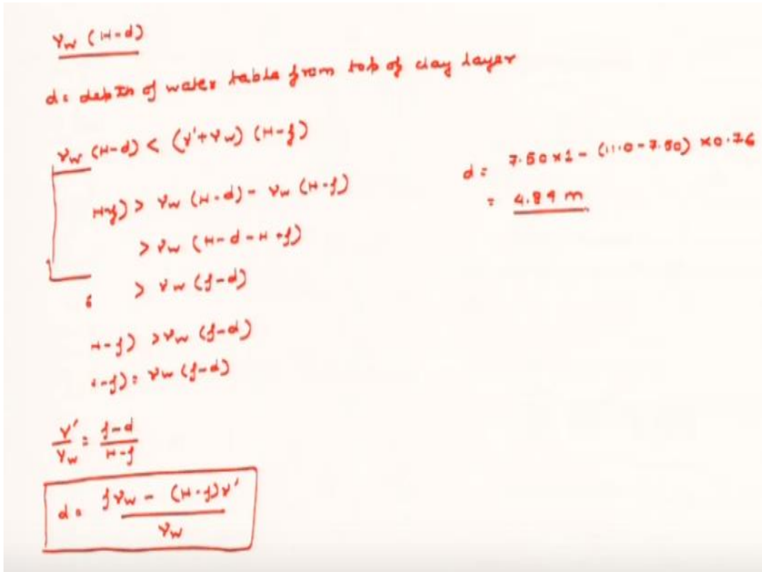

That is equivalent to gamma w into H - d. Now what is this d? d is basically the depth of the water table from the top of the clay layer. So, d is depth of water table from top of clay layer. So, this is the pressure at that point. This is the pressure at this point. So, this length pressure is gamma w into H - d where d is this length and H is the total height of the clay layer that I have already said.

So, in order to achieve the equilibrium condition gamma w into H - d must be less than the total weight of the clay layer that is gamma dash + gamma w into H - f. So, if you solve this then it comes out to be gamma dash into H - f must be greater than gamma w into H - d - gamma w into H - f. So, this is equivalent to gamma w into H - d - H + f.

So, this is equivalent to gamma w into f - d. So, gamma dash into H - f must be greater than gamma w into f - d. So, when failure will occur if basically gamma dash into H - f will be equivalent to gamma w into f - d. So, then from here you find we can find out that gamma dash by gamma w is actually equivalent to f - d by H - f or d is actually equivalent to f into gamma w - H - f into gamma dash by gamma w.

We can again say that that again it may repeat that what I have done that during this clay fissuration the first and important portion is that basically you have to consider that when that boiling condition is not going to occur. The boiling condition is not going to occur only if disregarding the shear strength only if the water pressure at the top of the sand layer is equivalent to the weight of the clay above it.

So, we have found out that what is the weight of the clay above it? We considered that the height H is basically the height of the total clay layer, d is the depth of the excavation, and f is the depth of the excavation, and d small d is basically the height of the water table from the top of the clay layer. So, regarding that we have found out the weight of the clay just above the layer of the sand and just below the excavation which is equivalent to nothing but gamma dash gamma or basically you can say gamma sat into H - f because it is just under the water table so we took gamma sat.

Now gamma sat we can write it in the form of gamma dash + gamma w into H - f while the weight water pressure at the top of the sand layer is taken to be as gamma w into H - d where d as we have said is nothing but the weight of the distance of the water table from the top of the clay layer. Now in order to the for the sand for the entire thing to be stable or the excavation to be stable basically we will consider that gamma w into H - d or the water pressure from the top must be less than the weight of the clay layer above.

So, that is what we have considered and we have come to the conclusion that only failure conditions will occur if d is equal to f into gamma w - H - f into gamma dash by gamma w. Now all these units are given if you can see in the question that basically the unit weight of clay the saturated density is given to be 1.76 and f is given to be f or basically the depth of the excavation is given to be 7.5 m while the height of the clay layer is given to be as 11 m. So, if you put all these then you get d is equal to 7.50 into 1 - 11 - 7.5 into 0.76 which is equivalent to 4.84 m. Now in the second phase we are going to consider the quick sand condition.

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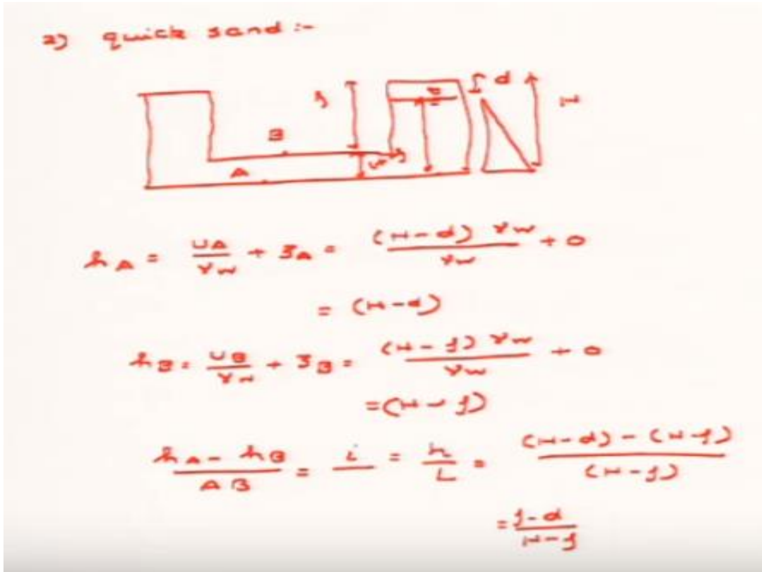

Now in quick sand condition we have to first consider that what is the we have to consider the heads of the water at different points so basically, we will draw this diagram again and we will find out that what are the heads at the different points. So, let us consider the head at this point is A at this point it is B. So, what is the head at the point A. Now as I have said that is the piezometric head just in the previous question we have said that it is just the piezometric head so it is just the sum of the pressure head and the datum head nothing else.

So, if you consider the pressure head that is nothing but UA by gamma w + za. Now of course the z of course the za is 0 so basically, we will consider this as of course the za at this point at the A we will consider this as H - d because d is the depth of the water table from the top of the clay layer and H is the entire length of the clay layer where H - d into gamma w is the pressure at this point obviously this is divide by gamma w and z a is 0 because this is the datum.

So, this is equivalent to H - d. Similarly, head at point B will be equivalent to UB by gamma w + z b. Now as you have known that the depth of the excavation is f so this will be equivalent to H - f into gamma w by gamma w + 0 into H - f the pressure at the point d. Now so the hydraulic gradient so basically the hydraulic gradient is constant now the hydraulic gradient is constant from A to B because as we can see that this is just a water table so it is linear in nature.

So, basically the hydraulic gradient will be constant from A to B and this can be given as h a - h b that is the head loss divide by AB is equal to nothing but equivalent to i by is equal to i which is equivalent to h by L. So, this will be equivalent to H - d - H - f divide by L L means this entire length so it is nothing but H - f. This entire length of the this entire length so basically this is H - f. So, this is d f - d by H - f.

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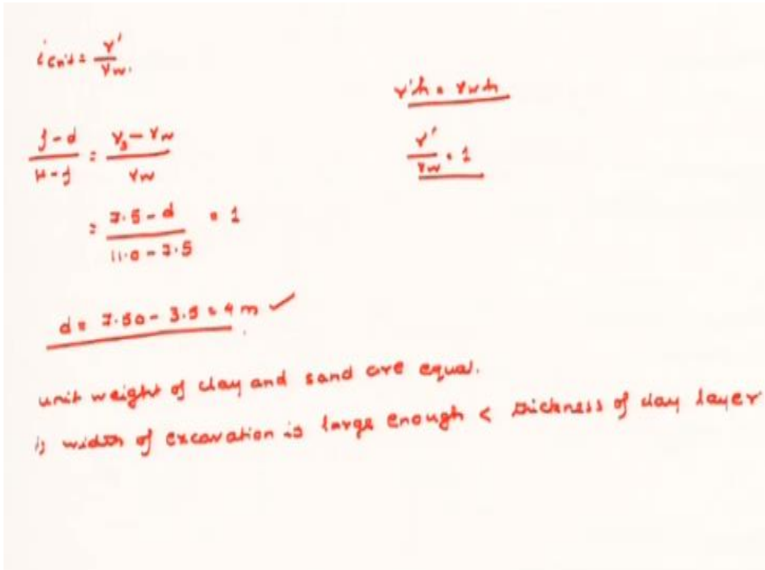

Now considering critical conditions. If we consider i critical gamma dash by gamma w then this head loss will be equivalent to gamma - gamma w that is gamma sat - gamma w by gamma w. So, if we put these values then we will get 7.5 - d by 11 - 7.5 will be equivalent to 1 because in the critical conditions gamma dash will be equivalent to gamma w that we have already said because that is what I said water pressure at the top of the sand layer must be equal to the weight of the clay.

That means that is what the critical condition is. That is gamma dash the water pressure at the top must be equivalent to the weight of the clay if the height is same. So, that means gamma dash into h must be equal to gamma w into h. So, basically then gamma dash by gamma w will be equivalent to 1. Then only critical conditions will reach. So, from here we found out that d is equivalent to 7.5 - 3.5 equivalent to 4 m.

So, considering the weight of the clay considering the clay fissuration we have considered one aspect and considering the considering the quick sand condition we have considering another aspect. Now considering both the aspects we find out that the we will take the least height so basically 4 m is the actual appropriate answer. Now there are certain things that we have to remember in this case.

The results will be same if basically the unit weight of clay and sand are equal and remember that I have said as just in the first case that the shear strength this is all regarding this regarding the shear strength of clay in the clay fissuration aspect disregarding the shear strength of clay. So, basically the shear strength of clay is so basically the so basically when you considered the first condition then it was basically disregarding the shear strength of clay.

In the second phase the shear strength was also neglected and also it was said in the second case the shear strength was also neglected it was said that basically the water would rise the cracks because that is what the question was given as. Now the shear strength of clay you can neglect if basically the width of the excavation is if the width of the excavation in real life problems actually if the width of the excavation is large enough.

You can neglect the shear strength of clay you can neglect the shear strength of clay then basically it acts as a uniform load layer okay but if the width of the excavation is small enough compared to the thickness of the clay layer this is less than thickness of clay layer then you have to consider the shear strength of clay. In that respect, you have to consider both the critical conditions as well as the shear strength of the clay layer.

So, basically today we discussed about the aspects of the quick sand conditions and basically about how you can find out the factor of safety in excavation and basically how you find out the critical depth for an excavation. We also considered that what is concerned by the piezometric head and how you find out the head losses at the different lengths along the pipe. In the next tutorial section that you are going to cover we are going to consider about the embankments and how you are going to employ both these excavation both these quick sand conditions, the effect of effective stress and the piezometric heads at different point in the clay layer at different points for an embankment and basically how to find out the phreatic line for an embankment using the parabolic theory that is already discussed in the lectures. Thank you.