

**Geology and Soil Mechanics**  
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**Lecture - 52**  
**Tutorial on Index Properties of Soil**

Morning everyone. I am Suman Roy a student tutor for this course Geology and Soil Mechanics. Due to some unavoidable circumstances Professor Priyanka Ghosh is not going to take the subsequent lectures and during this period we thought that we will introduce the tutorial section for all these chapters that he has covered including from the index properties of soil to the shear strength of soils. So, today we are going to cover the index properties of soils in which I am going to cover 3 problems. The first problem is basically on the basic soil mechanics relations the basic soil relationships so the problem is.

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A sample of soil with initial degree of saturation of 50% is isotropically compressed in order to achieve a void ratio of 0.55. Calculate the vol change in terms of percentage of initial volume.

$$V = 1$$

$$V_v + V_s = 1$$

$$e = \frac{V_v}{V_s}$$

Moisture content of the specimen = 22.4%

$$G_s = 2.7$$

$$W = \text{Moisture content} = \frac{W_w}{W_s} = 22.4\% = \frac{22.4}{100} = 0.224$$

$$= \frac{W_w}{V_w} \cdot \frac{V_w}{W_s}$$

$$= Y_w \cdot \frac{V_w}{W_s}$$

$$0.224 \times \frac{W_s}{V_w} = Y_w \quad \text{or} \quad 0.224 \times \frac{W_s}{Y_w} = V_w$$

Sample of this a sample of soil with initial degree of saturation of 50% is isotropically compressed. Now take a note on this what that isotropically compressed. Isotropically compressed actually means that the soil mass is actually compressed in all 3 directions with the same pressure. So, it is compressed in order to achieve a void ratio of 0.55. Now we are asked to calculate the volume change in terms of percentage of initial volume.

Let us assume that the entire volume of the soil is equivalent to 1. Now if the entire volume of the soil is equivalent to 1 that means the  $V_v$  the volume of the voids divide by the volume of the solids is also equivalent to 1 is equivalent to 1. Now we also know that the void ratio can be

written in the terms of  $V_v$  by  $V_s$ . Now at first, we have to find out what is the void ratio of the soil.

Now in order to find out the void ratio of the soil we also need some specific informations like what is degree of saturation and the moisture content of the what is the degree of saturation the moisture content of the soil or basically what is the specific gravity of soil solids. Now the question also says that the moisture content of the specimen is 22.4% and  $G_s$  or the specific gravity of soil solids is actually equivalent to 2.71. So, from here we will first plot the void ratio versus saturation in order to know that what is the specific at the degree of saturation of 50% what is the void ratio of the soil. So, for that let us start from the basic relationships.

So, we know that the water content of a soil or the moisture content of a soil is actually equivalent to the weight of water by the weight of solids which in turn is given as 22.4% which means it is equal to  $22.4$  by  $100$  which is equal to  $0.224$ . So, we can write the weight of water by weigh of solids also as the weight of water by volume of water multiplied by volume of water by weight of solids.

So, I multiplied it with the volume of water on the numerator and divided by the volume of water on the denominator. Now we know also that the weight of water by volume of water is nothing but actually equivalent to  $\gamma_w$  that is the specific the unit weight of water and  $V_w$  by  $W_s$ . So, from this relation we get that  $0.224$  into  $W_s$  by  $V_w$  is equal to  $\gamma_w$  or  $0.224$  into  $W_s$  by  $\gamma_w$  is actually equal to the volume of water. So, right now we know what is the volume of water. Now moving to the next part that is the degree of saturation.

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$$\begin{aligned}
 S_r &= \frac{V_w}{V_v} \\
 &= \frac{0.224 \times W_s}{\gamma_w \times V_v} \\
 &= \frac{0.224 \times W_s}{\gamma_w \times V_v} \times \frac{V_s}{V_s} \\
 &= 0.224 \times \frac{W_s}{V_s} \times \frac{V_s}{V_v} \times \frac{1}{\gamma_w} \\
 &= 0.224 \times \left( \frac{W_s}{V_s} \right) \times \frac{V_s}{V_v} \\
 G_s &= \frac{W_s}{V_s} \\
 &= 0.224 \times G_s \times \frac{V_s}{V_v}
 \end{aligned}$$


$$\begin{aligned}
 e &= \\
 e &= \frac{0.224 \times 6.1}{e} \\
 S_r &= \frac{0.224 \times 2.71}{e} \\
 &= \frac{0.607}{e}
 \end{aligned}$$

Now we have already we also know that the degree of saturation can be written in terms of volume of water by volume of voids. Now from our first lesson we already know that the volume of water is 0.224 into W s by gamma w. So, we are going to write that. So, we know that 0.224 into W s by gamma w into V v. Now in the slide just we multiplied the volume of water in the previous case numerator and denominator we are going to multiply the volume of solids numerator and denominator in this case.

We are multiplying volume of solids numerator and denominator in this case because then we will get W s by V s on one hand V s by V v on another hand and 1 by gamma w. Now W s by V s is nothing but the specific unit weight of solids. So, we get gamma s by gamma w into V s by V v. Now we all know again that gamma s by gamma w is nothing but actually the specific gravity of soil solids because G s can be written as rho gamma s by gamma w.

So, 0.224 into G s into V s by V v. Now if we remember correctly e can be written in the first part we wrote that e is nothing but V v by V s. So, e has also come in our calculation. So, now we get that S r is actually is equal to 0.224 into G s into G s by e. From here we get that S r is equal to 0.224 into 2.71 by e which basically gives 0.607 by e. Now coming to the plot so first we have to plot the variation of S r with respect to e.

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$$S_r = \frac{0.607}{e}$$


$$S_r = 50\%$$

$$e = \frac{0.607}{0.5} = 1.214$$

$$e = \frac{V_v}{V_s} = 1.214$$

$$V_v = 1.214 V_s$$

$$V_v + V_s = 1$$

$$V_v = 0.5483 \text{ m}^3$$

$$V_s = 0.4517 \text{ m}^3$$

$$W = 22.4\%$$

$$S_r = \frac{W}{e} = \frac{0.224 \times 2.71}{e} = \frac{0.607}{e}$$

$$e = 0.55 = \frac{V_v}{0.4517}$$

$$V_v = 0.2487 \times 0.4517 = 0.1122 \text{ m}^3$$

So, we got  $S_r$  equal to 0.607 by  $e$ . Now we are going to plot the curve. So, it is somewhat like this and for our particular void ratio or for particular degree of saturation of 50% we will get our  $e$  as 0.607 by 0.5 which basically is 1.214. So, basically, we are find we are asked to find out the degree of saturation means in the first question the degree of the moisture content of the specimen was 22.4% and then the specific gravity of soil solids was said to be 2.71 along with it it was also said that the degree of saturation of the soil was 50%.

So, now we know the relation and from there we can find out that  $e$  is nothing but 1.214. Now coming back to our original conditions that is 1.214 is now our  $e$ . So,  $e$  is equal to  $V_v$  by  $V_s$  which is nothing but is equal to 1.214. Now we know that  $V_v$  is equal to 1.214 multiplied by  $V_s$  or the volume of the solids. Now we also know that  $V_v$  plus  $V_s$  is nothing but equivalent to 1. So, let us solve for these 2 and we will get that  $V_v$  is actually equivalent to 0.5483 m cube and  $V_s$  is equal to 0.4517 m cube.

Now you can solve this in a more different way because you already know the relation  $WG$  is equal to  $S e$ . Now from this relation you can find out  $e$  is equal to  $WG$  by  $S$  or  $S$  is equal to  $S_r$  is equal to  $WG$  by  $e$ . So, you know  $W$ .  $W$  is nothing but specified as 0.224% or 22.4 where it is specified as 2.71. So, from here also you will get 0.607 by  $e$ . Now I was asked initially to find out that what is the volume change in terms of percentage of the initial volume that means in terms of  $v$  what is the percentage now.

So, coming to  $V_v$  by  $V_s$  that we have found out to be 0.5483 m cube and 4517 m cube. All these are in terms of  $V$  the total volume of the soil being 1, keep this in mind. So, now it is

isotropically compressed. So, when it is isotropically compressed it means basically that the air first which basically has a more compression index or more compressibility than water will be removed while basically the water will be removed second to the air.

So, in that case basically when the isotropically compression is done in that case we find out that  $e$  is nothing but 0.55 is equal to  $V_v$  by the volume of solids is 0.4517 that you already know. So, the total volume of voids is nothing but actually is equal to 0.4517 into 0.55. Just remember this  $e$  is after isotropic compression because it is already been said that after isotropic compression in the first page that is isotropically compressed in order to achieve a void ratio of 0.55.

That means basically the void ratio of 0.55 is after the isotropic compression. So, after isotropic compression your void ratio is 0.55 and now the volume of voids is 0.4517 which basically is which basically is the volume of the volume of voids is 4517 into 0.55 because as I have said that in isotropic compression first the air is going to be removed then the water in any compression actually the air is going to be removed because it is it has more compressibility than water so it will be removed first then the water will be removed and the solids will remain the same because the soil skeleton is generally considered to be incompressible. So, the volume of voids now comes out to be 0.4517 into 0.55 which is nothing but actually equivalent to 0.2484 m cube. So, now you can calculate that what is actually the percentage in the volume change.

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$$\% \text{ volume change} = \frac{(\text{Initial vol.} - \text{Final vol.})}{\text{Initial vol.}} \times 100\%$$

Initially we assumed  $V = 1$   
 Final vol. =

$$V_v = 0.2484 \text{ m}^3$$

$$V_s = 0.7517 \text{ m}^3$$

$$V = 0.7002 \text{ m}^3$$

$$\% \text{ vol. change} = \frac{(1 - 0.7002)}{1} \times 100\%$$

$$= 0.2997 \times 100$$

$$= 29.97\%$$

Now the percentage is what is the percentage in volume change the percentage in volume change is going to be initial volume minus final volume divide by the initial volume into 100. Now

initially we assumed V equal to 1 where V is the total volume of the soil that is written here that V is equal to 1. So, initially we assumed that the total volume of the soil is actually equivalent to 1. Now what is the final volume? Now final volume can also be easily calculated because we know what is the volume of voids now.

The volume of voids now is actually equivalent to 0.2484 and the volume of solids which basically has not been compressed or basically is considered to be incompressible is actually equivalent to 0.4517. So, just add these 2. So, you get it as 0.7001 m cube. So, now your percentage in volume change is initial volume is 1 - 0.7001 divide by 1 into 100 percent which is nothing but 0.2999 equal to 30%. So, the percentage of volume change is actually 30 percent.

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How much of this vol. change is due to outward flow of water from the sample?

$$w = 0.227 = \frac{w_w}{w_s}$$

$$\text{of total vol.} = 1$$

$$w_w = 0.227 w_s$$

$$= (0.227 \times 1.227)$$

$$w_w = 0.2784 \text{ m}^3$$

$$P_d = \frac{G_s}{1+e} P_w$$

$$= \frac{2.71}{1+1.218} \times 1$$

$$= 1.227 \text{ gm/cc}$$

$$\left( \frac{M_s}{V} \right)$$

$$V = 1$$

$$M_s = P_d = 1.227 \text{ gm/cc}$$

$$P_w = 1 \text{ gm/cc}$$

$$Y_d = \frac{G_s}{1+e} \gamma_w$$

$$= \frac{2.71}{1+1.217} \times 10$$

$$= 12.24 \text{ kN/m}^3$$

$$= \frac{w_s}{V}$$

$$w_s = Y_d = 12.24 \text{ kN}$$

$$V_w = w_w \gamma_w$$

$$= 0.2784 \text{ m}^3 \times 10 \text{ kN}$$

$$\therefore w_{w_i} - w_{w_f} = (0.224 \times 1.227) - 0.2784$$

Now it is also said that in the next question it is asked that how much of this volume change is due to outward flow of water from the sample. Now keep in mind that initially when we started with the sample it consisted of it consisted of air, water, and the soil solids. So, this is the soil solids, this is the water, and this is the air. So, basically some amount of as I have said that during isotropic compression it may so happen that all the air is removed and some amount of water is removed or only the air is removed but nothing will happen to the soil solids.

So, in this case basically it is said in the question that basically some amount of water has been removed that is why they have mentioned about the outward flow of water. So, in this case let us try to find out what is the dry density of the soil sample first because when the soil is dry entirely then consist of only air and soil solids. So, first let us find out what is the dry density of the soil

sample. Now we all know that the dry density is given by  $\rho_d$  is equal to  $G_s$  by  $1 + e$  into  $\rho_w$ .

Now  $G_s$  is given as 2.71 while  $e$  is given as initially 1.214. This is before the isotropic compression because initially what is it depends on all the initially what is the void ratio of the soil. In that case it is dry  $\rho_w$  is 1 gram per cc so that is known to all because it is the density of water. This comes out to be 1.224 gram per cc. Now this can be written as the dry density of soil can be written as the mass of the soil solids divide by the total volume because the total volume of the soil is taken in consideration in this case.

Now again in this case we all know what is the mass of the we have an added advantage here because we know what is the mass of the soil solids is. So, what is the mass how do we find out the mass of the soil solids. Now we all know that the water content was initially 0.224 percent which basically is the weight of water by weight of solids. Now if total now basically if the total volume is equal to 1 then the weight of water can be written as 0.224 into weight of solids which is nothing but actually is which basically this which basically is equal to 0.224 into 1.224 because the volume is 1 so that means mass of the solids is actually is equal to  $\rho_d$  which basically is equal to 1.224 gm/cc.

So, basically this is the weight of water that has been removed. The weight of this is the initial weight of the water. Now in order to get the final weight of water. This is the total weight of the water that is consisted of initially. Now what is the final weight of water that basically is already known to us because it is nothing but the volume of voids. So, it is 0.2484 mm cube. Now the volume of water is again equivalent to the weight of water so basically this is also weight of water which basically is equal to 0.2484 whatever unit it is in kilo Newton or whatever.

So, the weight of water initially is known, the weight of water final is known. This is the initial weight of water, this is the final weight of water. So, what is the total weight of water that has been removed?  $W_w i - W_w f$  is equal to 0.224 into 1.224 - 0.2484. This is the final weight of water that has been removed. Now one thing is here that is important that basically I have written that  $W$  is actually written as  $W_w$  by  $W_s$  but here actually it is written in terms of  $M_s$  the mass of soil solids.

It is not a problem because you can write  $\gamma_d$  is equal to  $G_s$  by  $1 + e$  into  $\gamma_w$  as well. So, in that case basically it comes out to be 2.71 by  $1 + 1.214$  into 10 which basically is

nothing but again equivalent to 12.24 kN/m cube then this is equivalent to  $W_s$  by  $V$  but you have taken  $V$  as 1 so basically  $W_s$  is equivalent to  $\gamma_d$  which basically is 12.24 kN.

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The image shows handwritten calculations in red ink on a white background. The equations are as follows:

$$w_c = \frac{0.224}{W_s} = \frac{W_w}{W_s}$$

$$W_{w_i} = 0.224 \times W_s$$

$$= 0.224 \times 12.24 \text{ kN}$$

$$W_{w_f} = 0.2484 \times \gamma_w = \frac{0.2484}{10} \text{ kN}$$

$$\gamma_w = \frac{W_w}{V_w} = \dots$$

$$10 \text{ kN/m}^3 = \frac{W_w}{V_w}$$

$$W_{w_i} - W_{w_f} = \underline{\hspace{2cm}}$$

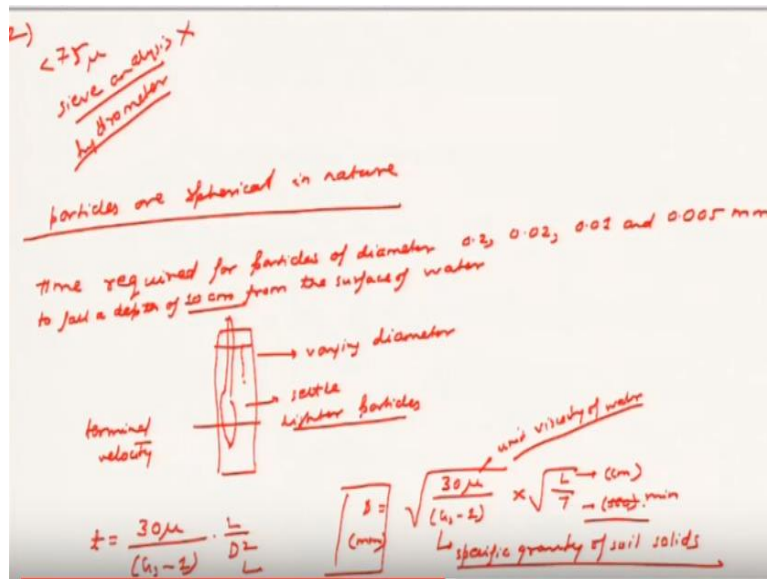
Now we also know again that  $W$  is equal to 0.224 is equal to  $W_w$  by  $W_s$  which basically is the weight of water is equal to 0.224 into  $W_s$ . Remember this 22.4 is again the initial weight of water that means this is the initial water content this is not after isotropic compression but this is before the isotropic compression. So, now the weight of water initial is actually is equal to 0.224 into  $W_s$  which basically found out to be 12.24 kN so 12.24 kN.

Now what is the final weight of water? That also we know because after isotropic compression we have found out that the volume of voids is 0.2484 m cube. So, basically it is nothing but 0.2484 into  $\gamma_w$  because  $\gamma_w$  is nothing but weight of water by volume of water which is 10 kN/m cube is equal to weight of water by volume of water. So, by  $\gamma_w$  so basically this is 0.2484 by 10 kN because again the volume of because again the volume is considered to be 1 so now we subtract the initial weight of water and the final weight of water and this is what you get.

Now this is how you find out that what is the water content for the soil sample is in the general case. You can find out in terms of mass as I founded out or basically in terms of unit weight whichever is appropriate. Now in the next question that we are going to discuss in the next question that we are going to discuss we are going to discuss some of the a very basic question about hydrometer analysis, lab testing on the hydrometer analysis.



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Now hydrometer analysis as we have said is a very common phenomenon and mostly it is performed from fine grained soils because you cannot find out below 75 micron because you already know that below 75 micron sieve you cannot perform a sieve analysis. So, in that case basically you have to find out the hydrometer you have to go for a hydrometer. So, when you have to go for a hydrometer analysis then basically it is assumed it is already assumed that the particles are spherical in nature.

If you do not have a particle that is spherical in nature then the equivalent diameter then basically it is considered to be equivalent the particle like clay which basically is flaky in nature then basically in those cases equivalent diameters of the particle are assumed considering the particle to be spherical. In this case let us find out that basically the hydrometer analysis is carried out and here basically it is asked to find out the time required for particles of diameter 0.2, 0.02, 0.01, and 0.005 mm to fall a depth of 10 cm from the surface of water.

Now as you all know basically in a hydrometer analysis the hydrometer is kept immersed here while the particles are allowed to fall. The particles are allowed to fall in this way that the mix is prepared and basically it consists of particles of varying diameter. Now gradually what will happen is that the heavier particles will first settle then the lighter particles will settle and according to Stokes' law basically when it reaches it is assumed that when it reaches the center of the hydrometer center of the bulb of the hydrometer the particle is said to assume terminal velocity and from the Stokes' law equation basically the diameter of the particles are found out.

Now in most of the books basically a general formula for time is given as  $30 \mu$  by  $G s^{-1}$  into  $L$  by  $D$  square or basically you can write  $D$  is equal to a generalized formula root over of  $30 \mu$  by  $G s^{-1}$  into root over of  $L$  by  $T$ . Now this  $L$  is considered to be in centimeter always.  $T$  is considered to be seconds always and  $D$  is considered to be millimeter for this formula  $\mu$  is the unit weight of water is the unit viscosity of water  $G s$  is the specific gravity of soil solids.

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The image shows handwritten mathematical work. At the top left, the formula is written as  $t = \frac{30 \mu}{(G_s - 1)} \times \frac{L}{D^2}$ . Below this, it is substituted with values:  $= \frac{30 \times 8.15 \times 10^{-6}}{(2.65 - 1)} \times \frac{10}{D^2}$ , where  $D$  is in cm. To the right, unit conversions are shown:  $\mu = \text{poise}$ ,  $= 8.15 \times 10^{-3} \text{ poise}$ ,  $1 \text{ poise} = 10^{-3} \text{ gm-sec/cm}^2$ , and  $\mu = 8.15 \times 10^{-6} \text{ gm-sec/cm}^2$ . Below the main formula, a simplified version is shown:  $t = \frac{30 \times 8.15 \times 10^{-6}}{1.65} \times \frac{10}{D^2}$ , with  $t$  in minutes. To the right of this, a list of values for  $D$  and their corresponding times  $t$  is provided:  $0.2 \rightarrow 2.22 \text{ sec}$ ,  $0.02 \rightarrow 3.24 \text{ min}$ ,  $0.01 \rightarrow 14.22 \text{ min}$ , and  $0.005 \rightarrow 57.32 \text{ min}$ . A bracket groups these four lines.

Now according to this formula, you can find out that the you can find out the time required for the settlement of the particles which basically is nothing but  $30 \mu$  by  $G s^{-1}$  into  $L$  by  $D$  square. So, you know like the 30 the viscosity of water which basically is given by in units of poise is 8.15 into 10 to the power - 3 poise. Now 1 poise is equivalent to 10 to the power - 3 gram second centimeter square.

So, in this case because  $L$ ,  $D$ ,  $T$  are all in like centimeters and second so it is advisable that you convert everything in terms of centimeter and second. So,  $\mu$  in terms of second and centimeter will be  $8.15$  into  $10$  to the power - 6 gram second centimeter square. So, this comes out to be  $8.15$  into  $10$  to the power - 6 divide by  $G s$  which basically is given as  $2.65 - 1$  into  $L$  it is written clearly that  $L$  is 10 cm.

So, 10 divide by  $D$ ,  $D$  is the one that you have to find out  $D$  square. Now the times the times you have to find out and  $D$  is given as 0.2, 0.02, 0.01, 0.005 mm. So, as I have said that here the  $D$  is  $D$  comes out in terms if mm this  $D$  comes out in terms of mm. This is what I have said.  $D$  comes out in terms of mm. So,  $D$  is already given in mm. So, 30 into 8.15 into 10 to the power - 6 into

1.65 into 10 divide by D square where D let us say is for 0.2 it comes out to be 2.22 seconds, for 0.02 it comes out to be 3.71 minutes.

Remember this T is in minutes. 0.01 it comes out to be 14.82 minutes this D is in minutes and for 0.005 it comes out to be 59.82 minutes. Now this formula is actually different from the formula that is covered in the slides but I will come that I will come to that that how basically this formula can be derived from the formula that is actually presented in the slides. Now this is the general way in which basically you can find out the diameter of the specific hydrometer for a hydrometer particle.

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3) 
$$D = \left( \frac{18\eta L}{(\gamma_s - \gamma_w)Gt} \right)^{0.5}$$

$\eta = \mu = \text{viscosity of water}$

$L = \text{cm}$

$t = \text{min} \quad t \times 60$

$\eta = 0.89 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2 \approx 1 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$

$$D = \left\{ \frac{18 \times 0.89 \times 10^{-3} \times \text{N}\cdot\text{s}/\text{s} \times L}{\text{m}^2 \times \gamma_w \times (\gamma_s - \gamma_w) \times (60 \times t)} \right\}^{0.5} = \left\{ \frac{18 \times 0.89 \times 10^{-3} \times \sqrt{10} \times L}{\text{m}^2 \times \gamma_w \times (\gamma_s - \gamma_w) \times 60 \times t} \right\}^{1.5}$$

Now in general basically what happens is that let us say that you have to the formula that is given in the slides or the formula that is covered in the slides is basically quite different from all these things because it consist of the hydrometer formula  $S \ 18 \ \eta \ L$  divide by  $\gamma \ s - \ \gamma \ w$  into  $60$  into  $T$  whole to the power of  $0.5$ . Now  $\eta$  is the same as  $\mu$  which basically is nothing but the viscosity of water.

$L$  is same as  $H$ .  $L$  is same as this  $L$  that is basically  $(\text{cm})$  (27:00) which basically is in centimeter so  $L$  is in centimeter.  $T$  is specified to be in minutes and you have to convert to seconds it is actually multiplied by  $60$  which you can see here. This  $D$  is in centimeter. Now how do I actually come to that now how do I actually come to that  $30 \ \mu \ 30 \ \mu$  by  $G \ s - 1$  into  $L$  by  $T$ . So, here we will see that how do we find out this.

Now eta is the unit the unit eta is the viscosity of water so the viscosity of water that eta has a unit of Newton second per meter square. So, in this case let us find out that in terms of Newton second per meter square you can write it in the format of  $0.89 \times 10^{-3}$  Newton second per meter square which is more or less given in terms of like  $1 \times 10^{-3}$  Newton second per meter square and this is similar to this Poise unit.

Now Newton second per meter square means it is a force unit so basically, we are going to put it exactly in the same way. So, D is equal to  $18 \times 0.89 \times 10^{-3}$  into newton into second by meter square into now we take gamma w as common and then we will have  $G s^{-1}$  into  $60$  into  $T$  into  $L^{0.5}$ . Now here L is in centimeter T is in minutes but here you see that unit of meter is presented so basically you have to convert either that meter into centimeter or the centimeter into meter.

In order to do that what we will do is that first let us try to convert this  $18 \times 0.89 \times 10^{-3}$  into  $18 \times 0.89 \times 10^{-3}$  into Newton into second. Now in order to now this second is already cancelled out because  $60 T$  is actually in seconds. So, basically that unit is this unit is cancelled out this second unit is cancelled out so now you are left with only Newton and meter square into gamma w into  $G s^{-1}$  into  $L$  by  $60$  into  $T$  whole to the power of  $0.5$ .

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$$\begin{aligned}
 &= \left[ \left\{ \frac{18 \times 0.89 \times 10^{-3} \text{ N}}{\text{m}^2 \times (2.65^{-1}) \times 60 \text{ s}} \right\} \left( \frac{L}{T} \right) \right]^{0.5} \\
 &= \left[ \left\{ \frac{18 \times 0.89 \times 10^{-3} \times \text{m}^3}{10 \times 1000 \times (2.65^{-1}) \times 60 \text{ m}^2} \right\} \left( \frac{L}{T} \right) \right]^{0.5} \\
 \text{D} &= \left[ \left\{ \frac{18 \times 0.89 \times 10^{-3} \times 100}{10 \times 1000 \times (2.65^{-1}) \times 60} \right\} \left( \frac{L}{T} \right) \right]^{0.5} \\
 &= K \sqrt{\frac{L}{T}}
 \end{aligned}$$

Now  $G s$  is also known.  $G s$  like here it is given as  $2.7$ . So, basically  $G s$  like here it was given as  $2.65$  so basically  $18 \times 0.89 \times 10^{-3}$  Newton meter square into  $2.65^{-1}$  into  $60$  into if you remember correctly there is a gamma w term here. So, basically that gamma w is to

be presented here into L by T this entire thing to the power of 0.5. Now this meter square this meter square has to be converted in terms of either this meter square is to be converted in centimeter term or basically this gamma w.

Let us first discuss with the gamma w. So, 18 into 0.89 into 10 to the power - 3 gamma w we can write as 10 now 10 is in 10 kilo Newton per meter cube. So, 10 kilo Newton per meter cube is to be converted into newton. So, basically that is multiplied by 1000. So, this Newton then gets cancelled out into 2.65 - 1 into 60 at the top there is meter cube because it is kilo Newton per meter cube and here there is meter square L by T 0.5 18 into 0.89 into 10 to the power - 3.

Now this meter has to be converted in centimeter because you all know that the diameter that is going to be present is to be in centimeter. So, that is why you multiply 100 here N into 1000 into 2.65 - 1 into 60 L by T power of 0.5. This is often written in the terms of in many books K into root over of L by T. This K is basically obtained from a specific table but this is how you obtain the different values of K. Now let us we will try to do a problem regarding this first.

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3) Sedimentation analysis for a hydrometer, was conducted on 50 gm of oven dried soil. Vol. of suspension = 1000 cc. Hydrometer reading: 19.50 after 60 min.

$L = 14 \text{ cm}$   
 $G_2 = 2.70$   
 $\mu = 0.01 \text{ poise}$   
 $\mu = 0.01 \text{ poise}$

Ans)  $D = \sqrt{\frac{30 \mu}{(G_2 - 1)}} \times \sqrt{\frac{L}{T}}$

$\mu = 0.01 \times 10^{-3} \text{ gm-sec/cm}^2$

$D = \sqrt{\frac{30 \times 0.01 \times 10^{-3}}{(2.7 - 1)}} \times \sqrt{\frac{14}{60}} = 0.0069 \text{ mm}$   
 $= 0.00069 \text{ cm}$

So, in this case basically it is said that you have to find out the sedimentation analysis for a hydrometer. Now it was conducted on 50 gm of oven dried soil. Volume of suspension is equal to 1000 cc. Now the volume of suspension is generally taken to be 1000 cc. It is taken in a 1000 ml graduated cylinder for hydrometers and then the hydrometer is dipped there. Now the hydrometer reading is 19.50 after 60 min.

So, let us first find out they have asked that basically the L effective of the hydrometer is 14 cm. G s is 2.70 and mu is given as 0.01 poise. Now we can take this as 0.089 poise as well to be accurate. So, first we will try to calculate from the formula that is mostly given that is root over of 30 mu by G s - 1 into L by T. Now mu is given in terms of 0.01 poise.

Now this has to be converted into 10 to the power - 3 because you have to convert it into gram second per centimeter square. Now if you do that then D comes out to be 30 into 0.01 into 10 to the power - 3 divide by 2.7 - 1 into root over of 14 by 60. If you solve this it will give D in terms of millimeter which basically is 0.0064 mm or 0.00064 cm. Now accordingly now we will try to do it in terms of the formula that we derived previously K into root over of L by T.

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$$D = \left\{ \frac{18 \times 0.89 \times 10^{-3} \times 45.110}{40 \times (2.7 - 1) \times 1000 \times 10} \right\}^{0.5} \times \left\{ \frac{L}{T} \right\}^{0.5}$$

$$= 0.0061 \text{ mm}$$

$$= 0.00061 \text{ cm}$$

$$\% \text{ finer} = \frac{G_2}{(G_2 - S)} \cdot \frac{R_c}{10W}$$

$$W = \left( \frac{40}{1000} \right)$$

weight of soil retained  
 volume of liquid

Now if you proceed according to that formula so D comes out to be 18 into 0.89 into 10 to the power - 3 into 100 divide by 60 into 2.7 - 1 into 1000 into 10 power of 0.5 into L by T whole to the power of 0.5. So, if you solve this in the same way you will get 0.0061 mm as the answer or 0.00061 cm because here D is in units of cm while in this case the D is in units of mm.

So, you have to remember both of this that in which formula you are going to use depending on the depending on the question or basically what units are given and accordingly you can proceed. Now the equivalent diameter has been found out but that is not the case. In order when you go through a sieve analysis then basically a stack of sieves are given to you and basically what you do is that you find out you know what is the equivalent diameter because you know what is the sieve size opening and from there you find from there you know what is the equivalent diameter

and what you do is that you find out percentage finer that means how much material the sieve has allowed to pass.

In this way also here also you were supposed to find out the percentage finer. Now a percentage finer formula for hydrometer is generally given in this way as  $G_s$  by  $G_s - 1$  into  $R_c$  divide by 10 into  $W$ . Now  $G_s$ ,  $G_s$  is nothing but basically the specific gravity of soil solids that is already known to you.  $W$  is basically a very important part because  $W$  is said to be the percentage of the soil mass that is taken.

That means basically let us say that the weight retained in the pan at the end is let us say 240 gm. Out of the 240 gm suppose you took only 40 gm for hydrometer analysis. That 40 gm is to be immersed in 1000 ml of the liquid. So, basically this  $W$  then would be equivalent to 40 by 1000. This is the weight of the soil taken while 1000 is nothing but the volume of liquid. So, this is to be kept in mind. Now  $R_c$  is the most important part of all this.  $R_c$  is the corrected reading of hydrometer.

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$R_c = \text{corrected reading of hydrometer}$   
 zero correctn = 2.50 (positive) → negative  
 (negative) → positive

$R_H - C_0 + C_T$   
 $R_H + C_0 + C_T \left\} C_m$

$C_0 = 2.50$   
 $C_m = 0.52$   
 $C_T = 1.3$   
 $R_H = 19.5 - 2.50 + 0.52 + 1.3$   
 $R_c = 18.82$

Diagram 1: Hydrometer with 1000 ml scale. A correction of 2 is shown, with 0.96 and 0.04 below it.

Diagram 2: Hydrometer with a note "upper meniscus" and arrows pointing to the top of the liquid.

Now how do you find out a corrected reading of hydrometer? A corrected reading of hydrometer is generally obtained from the temperature from the suspension from the suspension slip field or basically the chemicals and basically from the zero correction. Now I am going to discuss them one by one. So, the first thing is the zero correction. This is supposed to be given to you in the question. For example, in this question the zero correction is given to be as 2.50.

Now a zero correction means that basically when basically the hydrometer when it is immersed in water it should show a value equivalent to 1 because that is what is the specific unit weight of water but if somehow it shows some values above and below then according to it the zero correction is to be adjusted. That means it should be added or it should be subtracted. If basically the zero correction is positive in nature where basically it is here then basically the correction should be negative.

That means from the original hydrometer reading if the original hydrometer reading is say let us say  $R_H$  this should be subtracted from the original hydrometer reading. Again I am telling this if the zero correction is positive in nature then the correction should be negative. That means basically it should be subtracted from the original hydrometer reading because it means that it is above the it is above that mark that means that mark it is not able to correct so basically that is why you have to subtract it from the original reading.

If on the other hand the zero correction is negative in nature. Negative in nature means basically that this correction is actually below this mark that means is actually above this mark. That means instead of showing a specific unit of 1 it is showing actually a specific unit weight of let us say 0.96. Then basically the zero correction is 0.04. So, basically if it is negative in nature then basically you have to add it.

That means basically you have to add 0.04 to this number in order to get the actual hydrometer reading. That is what happens. So, basically in that case it should be  $R_H + C_0$ . So, even if the actual formula is not given to you, you should know that basically how the hydrometer readings are corrected and they shall be adjusted. So, here basically these are the 2 parts that  $R_H + C_0$  and  $R_H - C_0$ .

Once the hydrometer reading correction is done then basically the next one is the temperature correction. The temperature correction is already given and you have to just add it. The temperature correction is always negative in nature and you just add it with the original hydrometer reading. The third important correction that basically comes is basically correction due to the dispersing liquid but this is generally neglected right now or the meniscus correction there is also a term called meniscus correction because in hydrometer what happens is that you are if it is a if it is like a if it like clay and all this in water basically what happens is that you are able to read the lower meniscus of the hydrometer of the fluid and basically from the lower meniscus you are able to identify that what is actually the hydrometer reading but in certain cases



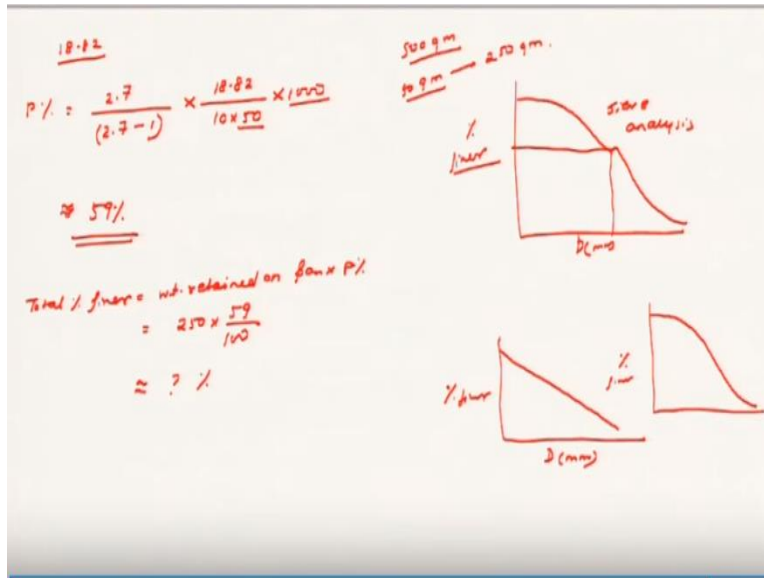
like let us say if it is a clayey water then basically you cannot read the lower meniscus because it is so dark in there.

So, basically what you do is that you read the upper meniscus and when you read the upper meniscus then basically a meniscus correction comes in consideration because you are supposed to read the lower meniscus but you actually read the upper meniscus. So, in that case basically what happens is that the meniscus correction has to be added or subtracted that is minus cm or plus minus cm depending on whatever but if the but since you are supposed to read the upper meniscus therefore basically the meniscus correction should be adjusted and it should be plus cm because you are supposed to read the lower meniscus that means you are supposed to read a greater value than the actual value.

So, in this way basically all the corrections have to be adjusted. Now here in our question we are given that  $C_0$  is actually 2.50 while  $C_m$  is actually 0.52 the meniscus correction and it is also given that basically the temperature correction  $C_T$  is actually 1.3. So, basically the original hydrometer reading is 19.5. Now as I have said if  $C_0$  is positive in nature then the correction should be negative so basically this is - 2.50.

The meniscus correction is always positive so it is 0.52 and the  $C_T$  is always as I have said is positive so it is 1.3. So, adding all these 3 things come out to be 18.82 that is the hydrometer reading. So, initially your hydrometer reading was actually the initially your hydrometer reading was 19.5 but now your corrected hydrometer reading has turned out to be 18.82. This 18.82 you are supposed to use in this percentage finer.

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So, now you know that 18.82 is actually the percentage finer part. So, basically, we will go back and we will see that how to find out the what is the percentage finer for a hydrometer. So, now you know that it is 2.7 by 2.7 - 1 into 18.82 divide by 10 into 50 into 1000. Now where from this 50 came? If you remember I have already said that 50 gm of oven dried soil is taken. 50 gm of oven dried soil is taken means this 50 the suspension is of 1000 cc.

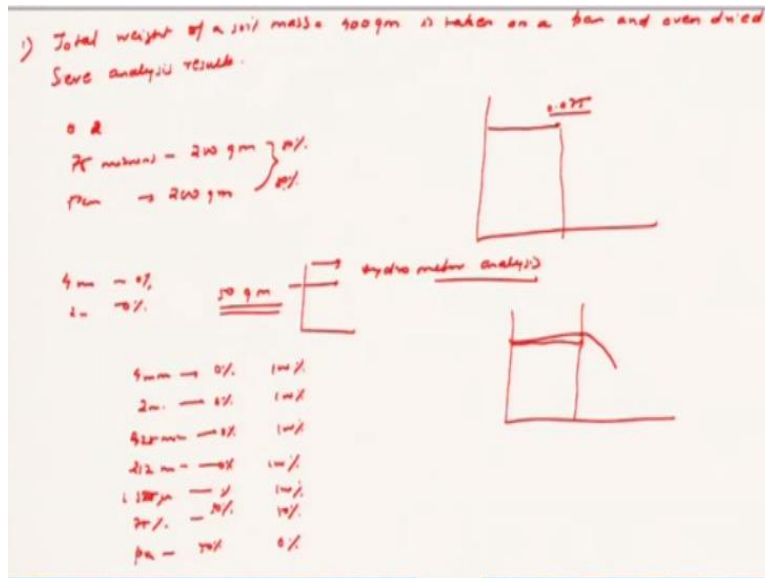
So, basically the suspension is 1000 so basically that is W s. So, if you solve this it comes out to be something about 59% approximately equal to 59%. So, this is the percentage finer for the hydrometer. Now remember this that this is the this is only the percentage finer for the hydrometer but basically you have to find out that what is the actual percentage finer.

Now suppose let us consider a grain size distribution. Now the initial part of the grain size distribution is obtained from the sieve analysis. A part is obtained from the hydrometer readings. This is also like percentage finer and this is the diameter in millimeter. Here also it is diameter in millimeter and it is the percentage finer. Now you have to connect these 2 parts in order to get the entire grain size distribution.

Now that is the main objective. So, basically in this case how do you connect the 2 parts? The total percentage finer is obtained as weight retained on pan into P percentage. Now what is the weight retained on pan? Now if let us say that out of the out of the entire volume let us say that you took 500 gm of sample. Now out of that 500 gm of sample you took only 50 grams. The weight retained on pan is say suppose 250 gm.

So, then the weight retained on pan is actually 250 gm into 59 by 100. So, the number that will come out that is your the total percentage finer and that percentage finer should start from here. So, in this case your entire grain size distribution comes out and then you can obtain that what is the percentage finer versus D millimeter. So, basically today so basically the first important so basically let us go back to the so basically now let us take another problem.

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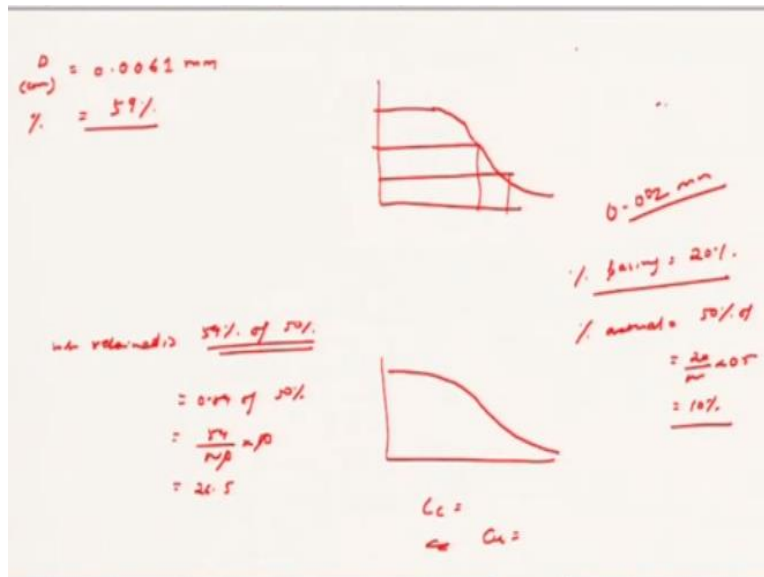
Suppose let us say that the total weight of a soil mass equal to 400 gm is taken on a pan and oven dried. Sieve analysis results. Now let us say that only through let us say 75 microns only 200 gm has passed 200 is retained and basically in pan the rest 200 gm is retained. So, for the initial like 4 mm, 2 mm all these percentage will be 0. So, your grain size distribution would start immediately from 0.075.

Now we know that here basically 50% passing has already occurred while the rest 50% passing has occurred will occur through the pan while the rest 50% is retained on the pan. Now out of 200 gm you took 50 gm and from the 50 gm we actually found out the hydrometer analysis.

So, now our the entire grain size distribution would like 4 mm 0%, 2 mm 0%, then 425 microns 0% because in all these cases the weight retained on the pan is 0 then 212 micron 0%, 150 micron 0%, 75 microns 50% has passed. So, that means 100% passing, 100% passing, 100% passing, 100% passing, 100% passing, and 50% is the final passing.

For pan retained is also 50% so that is it 0 percentage passing. So, basically from 0.075 that means from this line we start the we start our grain size distribution. So, it will be like this in the beginning and then we do not know what is going to be.

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So, out of the hydrometer analysis that we performed now let us see if the hydrometer analysis that we performed has given us in the same way in the same manner D in cm if it belong to the same hydrometer analysis. Suppose it is given as 0.0061 mm for a percentage passing of 59% as given. Now 0.0061 is my percentage passing of 59%. So, basically, we know that it has already started from here then 59% is passing so now we are going to convert that to the percentage passing.

Let us state that the weight retained on the pan is whatever we have found out the weight retained on the pan is like 50 gm so basically 59% of 50% you have to find out. 59% of 50% that is what is the actual weight that means basically 0.59 of 50% because 50% is the weight that is retained on the pan. So, 59 by 100 into 50 whatever the number comes out to be 26.5.

So, basically it will start from 26.5. So, depending on that in the consecutive steps basically you will find out that what are the in the consecutive steps what are the weight what are the weights for the other percentage of soils. So, let us say basically there is another like 0.02 like there is another soil mass which basically has 0.02 mm as one of its points. Then in that 0.002 mm then in that case again you have to find out in the similar way the percentage passing.

If the percentage passing is let us say in this case let us say 20% from the hydrometer. Then in that case the percentage actual will be how much? The percent that is retained on the sieve retained on the pan so basically the percentage retained on the pan is 50%. So, 20% of 50% that means 20 by 100 into 0.5 so 10%. So, in that way you have to proceed and you will find out the entire grain size distribution. Now from there you can find out what is the  $C_c$  the coefficient of compression.

Then basically you can also find out what is the coefficient of uniformity  $C_u$  on depending on all these basically your grain size distribution. So, in the next class basically we are going to discuss about in the next class we are basically going to discuss about the today we have discussed about the general  $W_g$  the general  $W_g$  is equal to  $S_e$  the general soil index property relationships the hydrometer analysis and how to incorporate a hydrometer analysis in a modified in a grain size distribution so that basically you can get the entire grain size curve and how basically you can find out the diameter of the spherical particles from the Stokes' law. Now in the next class we are going to use this, incorporate this in a more modified way in compaction and basically all the other index property relationships.