**Geology and Soil Mechanics**
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**Lecture - 51**
**Earth Pressure on Retaining wall - A**

Welcome back. So, in the last lecture we just started discussion on the earth pressure on retaining wall. So, there we have seen that how you will be getting or how the earth pressure at rest at passive at active condition are going to develop. So, those things we have seen in the last lecture.

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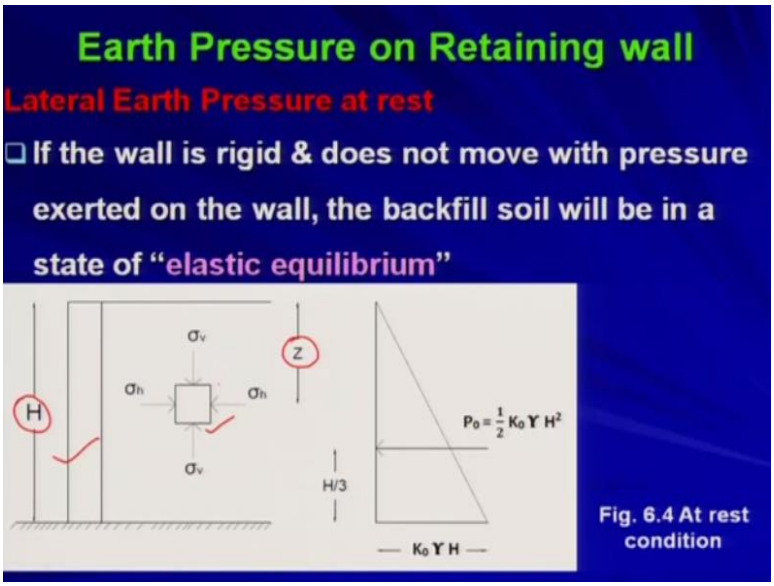


So, now in this lecture we will be talking about the elastic equilibrium and we will be trying to establish the relation or trying to establish the expression for earth pressure at rest and earth pressure at active and passive condition. So, if the wall is rigid and does not move with pressure exerted on the wall the backfill soil will be in state of elastic equilibrium.

That means when the wall is not moving at all when the wall is at rest condition when the backfill is at rest condition at that time basically the soil will be under elastic equilibrium. So, what does this mean? So, if I look at this figure so this is the wall which is having the height say capital H we are considering at some depth z from the top surface of the backfill we are considering one soil element. So, this soil element will be under this kind of state of stress.

On the horizontal plane, you will be having sigma v and on the vertical plane you will be having sigma h. So, horizontal and vertical plane both are will be both will be your principle planes right because there is no shear stress on this plane so they must be the principle planes.

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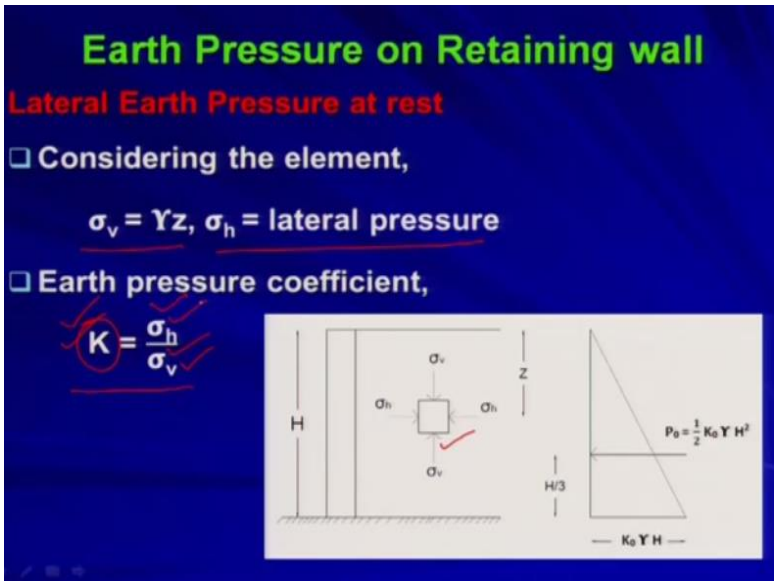


Now if we consider this soil element whatever is shown at depth z below the ground surface so considering this element sigma v is nothing but gamma z as I told you in the last lecture and sigma h it is nothing but the lateral pressure am I right so which is nothing but your objective function which is which needs to be determined okay. So, sigma v is known to you because if you know at what depth you are considering okay so you know the depth you know the unit weight of the soil depending on the situation of the water table and all we will come to that point later on. So, based on that you can find out sigma v.

So, once you know sigma v you can find out sigma h if you know the relation between sigma v and sigma h am I right. So, sigma h is nothing but your lateral earth pressure which is I mean in case of earth pressure at rest condition this is the earth pressure at rest. In active situation, it will be the active pressure and in passive situation it will be the passive pressure right. This lateral pressure is nothing but the earth pressure okay.

Now earth pressure coefficient is defined by this expression where earth pressure coefficient K is nothing but equal to sigma h by sigma v. So, in different situations in different conditions whether it is under rest condition whether it is under active or passive condition depending on the situation you can find out this earth pressure coefficient. So, earth pressure coefficient at rest condition, earth pressure coefficient at active condition, earth pressure coefficient at passive condition.

So, once you know the earth pressure at different conditions right earth pressure rest. If you know this I mean earth pressure coefficient then you can find out sigma v provided if you know sigma v sorry. So, basically this earth pressure coefficient at different conditions we are going to find out, earth pressure at rest, earth pressure at active, earth pressure at passive. So, this earth pressure coefficient we are going to obtain or we are going to formula is this thing.

So, once you know this earth pressure coefficient and if you know sigma v that is obviously at some depth you are considering so sigma v is known to you so if you know sigma v if you know the earth pressure coefficient you can find out sigma h that is nothing but your earth pressure that is the lateral earth pressure at different conditions. If you are using earth pressure coefficient at rest condition then you will be getting earth pressure at rest condition. If you are using earth pressure at active condition you will be getting earth pressure at active condition. So, in that way you can find out sigma h okay.

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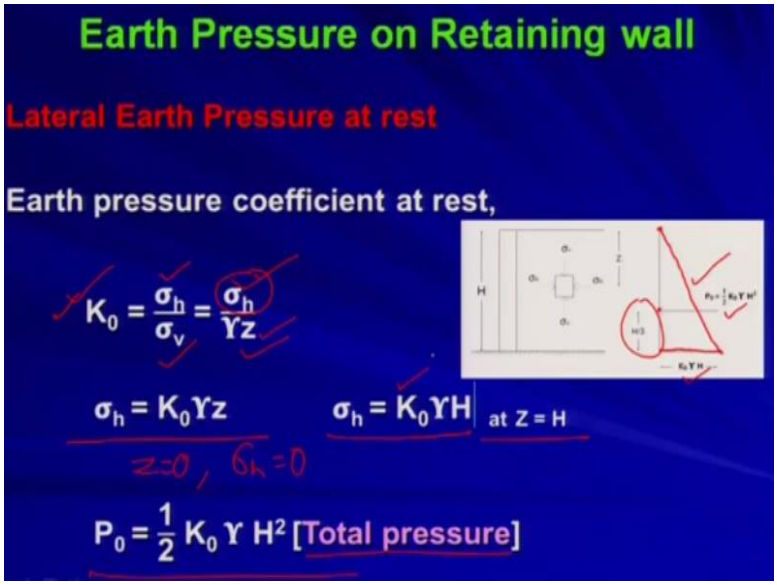


So, now first we are going to find out earth pressure at earth pressure coefficient at rest condition. So, earth pressure coefficient at rest condition is defined by K 0 which is nothing but sigma h by sigma v that is the definition of the earth pressure coefficient. Therefore, that is nothing but sigma h. Now what is sigma v that is nothing but gamma z am I right. So, if you know K 0 and if you know at what depth you are considering so therefore sigma v is known to you can find out sigma h.

So, from this I can write sigma h equal to K 0 into gamma z right. K 0 is nothing but the earth pressure coefficient at rest condition into gamma into z. Now by seeing this variation we can get the variation of the lateral pressure from top to bottom of the wall as linear. So, this is shown here. So, at z equal to 0 what is sigma h it is 0 okay. So, that is here. At z equal to h what is your sigma h? Sigma h is equal to K 0 into gamma h so that is here. So, this is the pressure.

So, it starts from 0 and ending at K 0 gamma h. So, this is the earth pressure distribution okay variation of earth pressure at rest condition along the wall height. Is that clear? So, at any depth you can find out earth pressure at rest condition okay. So, now the if you want to find out the total pressure that is the total thrust at rest condition that is nothing but the area of this triangle so area of this triangle is P 0 equal to half into K 0 into gamma H square right.

Because we are considering this all these problems as plane strain problem that is the direction normal to these board or normal to these plane okay is unit okay. So, this P 0 is equal to half into K 0 into gamma H square that is nothing but the total pressure at rest condition. That is the total thrust acting on the wall and which will be obviously acting at the CG of this triangle that is at H by 3 from the base of the triangle okay.

So, this is the point of application of this thrust and this is the total thrust. So, based on that you can design the wall. So, this much I mean the wall should be designed based on this pressure and at that label that pressure is acting or the total pressure rather total pressure is acting and based on that you can design the wall okay. That is earth pressure rest condition. Now in all this equations or the formulation we know how to find out sigma h we know how to find out the total pressure and all those things but provided if we know K 0.

K 0 is nothing but earth pressure at rest condition. If I know this K 0 value then only I can find out the magnitude of the total pressure total thrust pressure at different locations all those things right. Sigma h is only known when you know K 0 value. So, let us find out how to find out K 0.

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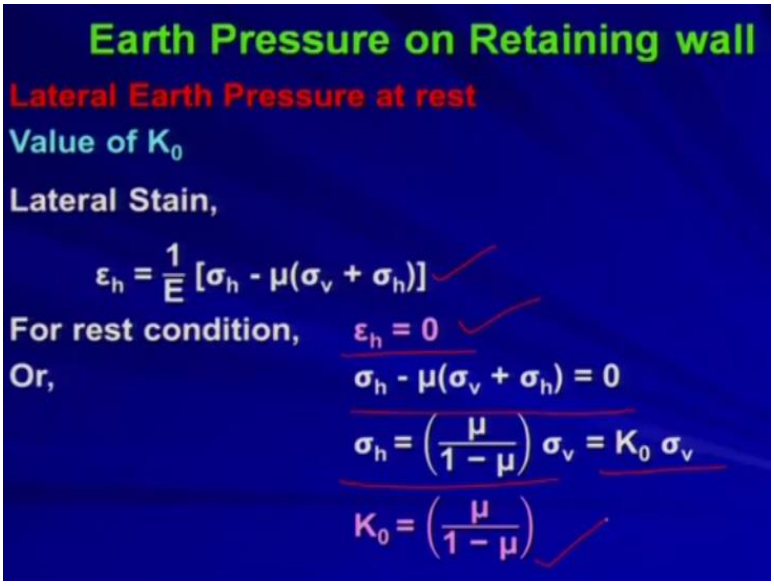


So, value of K 0. So, lateral strain generally I mean you know from the Hooke's law lateral strain is given by this expression. So, this is very simple that is coming from your mechanics. That is epsilon h that is your lateral strain is equal to 1 by E into sigma h - mu into sigma v + sigma h because this is the 2D problem 2D plane strain problem so that your epsilon h is given by this expression and this is coming from your Hooke's law right.

Now for rest condition what is epsilon h? epsilon h what is epsilon h that is the lateral strain that is the movement of the wall that is the strain happening in the soil. So, earth pressure at rest condition if you consider the rest condition at that time what is the lateral strain, lateral strain is zero, there is no movement in the soil backfill. So, epsilon h is zero. If epsilon h is zero so I can write sigma h - mu into sigma v + sigma h is equal to 0.

From this I can write sigma h is equal to mu by 1 - mu into sigma v which is nothing but K 0 sigma v. Therefore K 0 is mu by 1 - mu okay. So, what is mu? mu is the Poisson ratio. So, if you know the Poisson ratio you can find out the magnitude of K 0. So, once you know K 0 you can find out sigma h at different locations and as well as the total pressure or the total thrust on the wall at rest condition.

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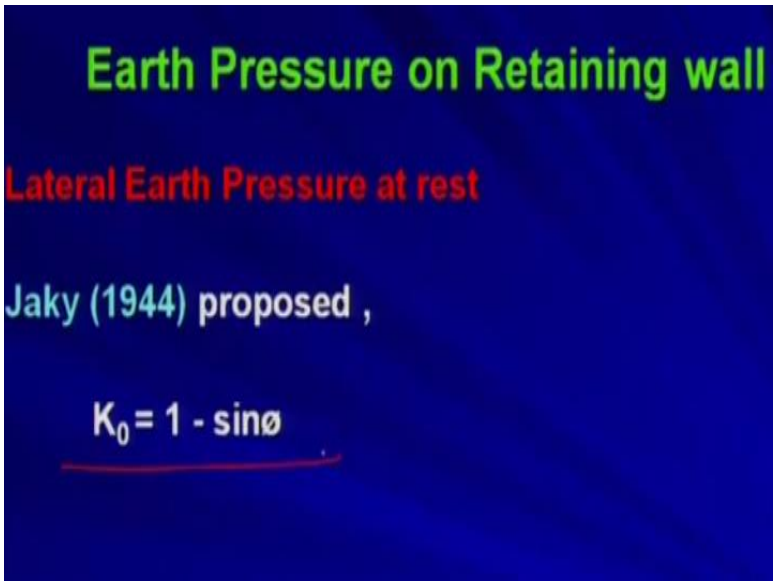


Now Jaky in 1944 he proposed that K 0 can be taken as 1 - sin phi where phi is the angle of internal friction of the soil and it has been found that this value of K 0 gives very good or reasonable result in the soil mechanics.

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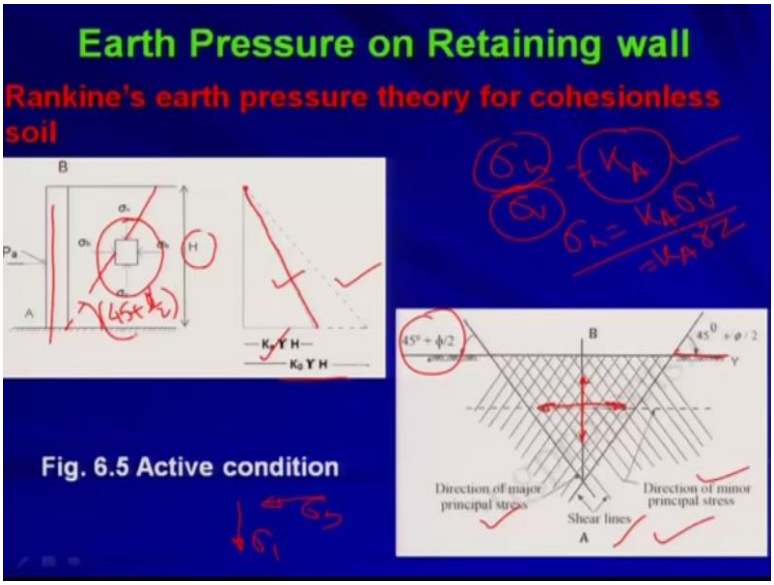


Now in case of active condition we are trying to obtain this kind of say or we are trying to develop this earth pressure coefficient or the relation between the vertical pressure and the horizontal pressure in case of active state. Now and that was proposed by Rankine's okay so therefore that is why it is known as Rankine's earth pressure theory for cohesionless soil. So, first we are considering the cohesionless soil that means c is 0 okay.

So, at that time we are considering again similar thing this is the wall so we are considering one say small element at some depth z okay from the ground surface H is the total depth of the wall so again the earth pressure distribution at active state will be so this is the earth pressure distribution at rest condition K 0 into gamma H already we have established that and already you you know that your earth pressure at active condition will be lesser than your earth pressure at rest condition right.

Now we need to find out the variation. So, we will see that this variation will be also linear and because your sigma h by sigma v is K a where K a is nothing but the earth pressure coefficient at active condition okay. We need to find out K a. Now this once you know K a which will define how to find out if you know sigma v you can find out sigma h which is nothing but the lateral earth pressure at active condition.

So, sigma h is nothing but K a into sigma v okay. So, this relation or this K a further can be written as K a into gamma z right. So, at z equal to 0 your sigma h at active condition will be 0 same okay as rest condition. Now when z equal to H at that time sigma h will be K a into gamma into H. So, that is written here K a into gamma into H okay. So, and the variation is linear so therefore this is your variation of the active earth pressure which will be lesser than your earth pressure at rest condition but it will be always linear okay.

So, now in this figure if you look at so in case of active state what is the direction of the major principle stress. The direction of the major principle stress is vertical. So, this is the direction of the major principle stress right. So, therefore what will be the direction of minor principle stress. That will be just 90 degree from the major principle stress direction so that will be simply horizontal.

This is your minor principle stress direction okay. So, now if you recall your Mohr circle representation in the Mohr circle in case of this kind of orientation when sigma 1 is acting in the vertical direction and sigma 3 is acting horizontal direction in this situation okay horizontal direction means sigma 3 is acting on the vertical plane and sigma 1 is acting on the horizontal plane in that situation in the Mohr circle if you recall from your mechanics point of view then basically the slip lines that means we will come to that point later on the slip lines will be making an angle 45 + phi by 2 with the horizontal, horizontal means the plane on which sigma 1 is acting.

Sigma 1 is acting on the horizontal plane because sigma 1 is acting in the vertical direction so sigma 1 is acting on the horizontal plane. So, the line which is making an angle 45 + phi by 2 with the horizontal plane will be the slip lines okay the shear lines right. So, there will be 2 shear lines because from the top you will be having one failure envelope and the bottom you will be having another failure envelope so you will be getting 2 shear lines and those shear lines and that is why this failure as I told you this will be making 45 + phi by 2 because that is your shear line. Along that line basically the failure will be happening okay.

So, I told you that we will be discussing this thing later on anyway. So, we will see this thing in more detail when we will be talking about the Mohr circle representation.

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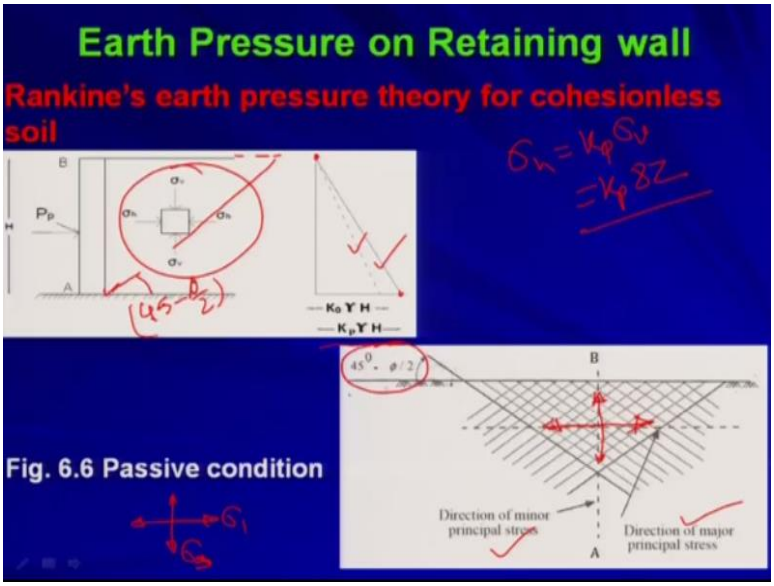


Now talking about the passive condition at that time also we are considering this state of stress okay. So, one soil element we are considering. Here also your sigma h is equal to K p into sigma v which is nothing but K p into gamma into z okay. So, now you see this dotted line is your earth pressure at rest condition that is the variation at earth pressure at rest condition okay. Now this solid line will be giving you the earth pressure at passive state.

So, at z equal to 0 it is 0 at z equal to H it is K p into gamma H which will be greater than your earth pressure at rest condition agreed. So, earlier in case of active state you got the lesser value than the rest condition. Now you are getting the higher value than the rest condition which is quite obvious from our earlier discussion. Again, in this situation in case of passive state if you see what is the direction of the major principle stress?

Direction of the major principle stress is horizontal right. In case of active state direction of major principle stress was the vertical, now it is horizontal. That means you are pushing, this is your main action right, you are pushing the wall towards the backfill so that should give you the major principle stress. So, what is your minor principle stress? What is the direction of the minor principle stress? That is nothing but vertical.

So, this is the so this is your major principle stress direction and this is your minor principle stress direction which is just reverse than the active state okay and if you plot so that means now basically your sigma 1 is acting horizontally and sigma 3 sigma 3 is acting vertically. So, in this situation in this condition okay so this condition if you try to plot in the Mohr circle you will be getting the shear lines which will be making an angle 45 - phi by 2 with the plane on which sigma 3 is acting right.

Sigma 3 is acting on horizontal plane. Now sigma 3 is acting on horizontal plane. So, 45 - phi by 2 is the angle between the shear lines and the horizontal plane on which sigma 3 is acting. Now this thing will be clear when you will be talking about the Mohr circle representation. So, therefore you will be getting the failure surface which will be inclined 45. So, already we have seen so this shear lines will be making an angle 45 - phi by 2 with the horizontal plane okay.

So, therefore this failure envelope or the failure surface in case of passive state will be making an angle 45 - phi by 2. So, that is the reason that is why you are getting this failure surface is inclined at 45 - phi by 2 with the horizontal in case of passive state whereas in case of active state you got the active I mean the failure envelope or the failure plane is inclined at an angle 45 + phi by 2 with the horizontal right. So, now if you see this Mohr circle representation it will be more clear.

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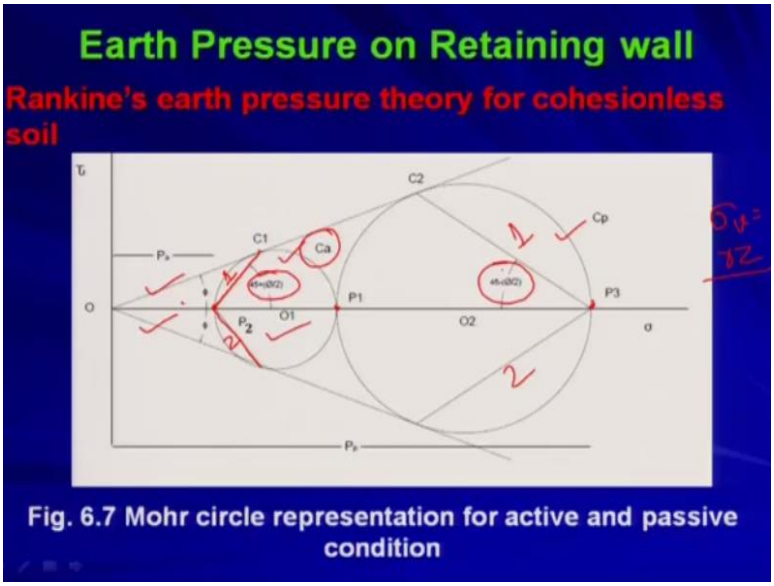


So, this is the Mohr circle C a that is Mohr circle for active state and C p is the Mohr circle for passive state. So, and as I told you your sigma v is same for passive state or active state whatever state you consider sigma v is gamma z because you are not changing the location at which you are considering that soil element whatever soil element whatever soil element we considered. So, sigma v will be remaining same for active and passive state whereas your sigma 3 for active state and sigma 1 for passive state will be different.

This P 1 point will be talking about this sigma v. So, you are starting from this. So, active state circle this circle is giving you the active state and therefore this point P 2 is nothing but your active earth pressure that is your minor principle stress and minor principle stress is nothing but the lateral earth pressure in case of active state and that is eventually your active earth pressure. Whereas in case of passive state this P 1 becomes the minor principle stress whereas this P 3 point is becoming the major principle stress which is giving you the lateral pressure and which is nothing but the passive earth pressure okay.

So, now as I told you this so this is the plane this is the first shear line and this is the second shear line right. So, because you are you will be getting the Mohr-Coulomb failure envelope on both the sides. So, this is your first shear line. This is your second shear line which is making an angle 45 + phi by 2. So, I am not going too much detail about how to draw this how to obtain this shear line and how to get this first shear line second shear line because this is the part of your mechanics and which is eventually your prerequisite of this course.

So, you must know that how to get these planes on which the failure is happening. So, this is your first shear line which is making an angle 45 + phi by 2 in case of active state. So, that is why you are getting the failure line or the failure envelope which is making an angle 45 + phi by 2 with the horizontal but on the other side in case of passive state you are getting this angle as 45 - phi by 2 right so if you recall I mean just for sake of brevity okay so this point will be becoming as the pole that is the origin of planes and this point will be becoming as pole which is nothing but the origin of planes in case of passive state.

So, P 2 is your origin of planes that is the pole in case of active state and so from this point to the failure point if you join so on that plane you will be getting the failure. Similarly, this is the point P 3 will be the pole in case of passive state. So, if you join P 3 and C 2 so you will be getting the failure. So, this is your first shear line this is your second shear line which will be making an 45 - phi by 2 angle with the horizontal and that is why you are getting the failure line or the failure envelope is inclined at an angle 45 - phi by 2 in case of passive state. I hope that you have understood this okay.

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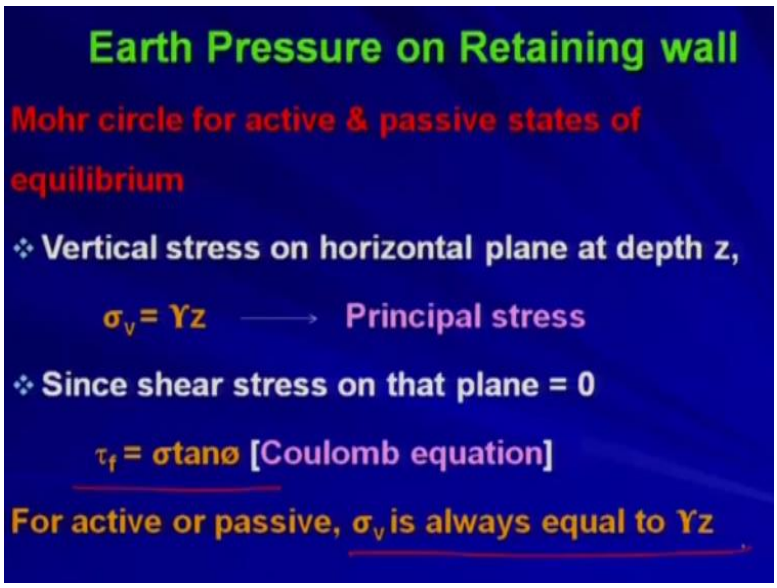


Now coming to the Mohr circle for active and passive states of equilibrium so vertical stress on horizontal plane at any depth z is nothing but sigma v which is nothing but equal to gamma z this is your principle stress right one of the principle stress. If you are considering active state this will be your major principle stress. If you consider the passive state it will be your minor principle stress right but it will be your one of the principle stress okay.

Since shear stress on the plane is 0 because the principle I mean the on that plane basically your shear stress is 0 so that is why it is becoming the principle stress right and from the Coulomb's equation I know tau f equal to sigma tan phi because c is not there we are not considering the cohesive soil for the time being as we told that we are considering only cohesionless soil cohesive soil will be coming later on.

So, the for active or passive state sigma v is always equal to gamma z please try to remember. Sigma v could be minor could be major depending on the situation whether you are dealing with active and passive state but sigma v will be unchanged and sigma v will be equal to gamma z whether you are considering active state or whether you are considering passive state no matter whatever state you are considering.

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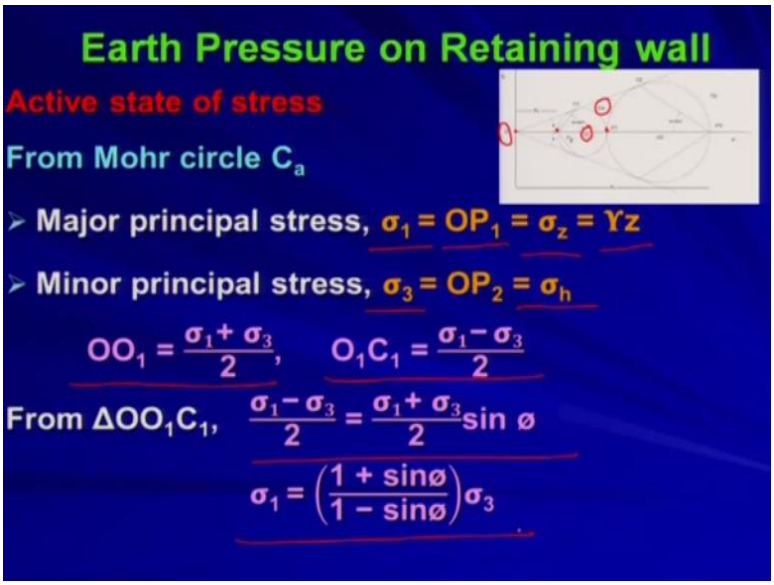


So, in case of active state of stress so from the Mohr circle C a as I told you in the active state of stress the major principle stress sigma 1 is equal to O into OP 1 okay so O is the origin okay and P 1 is this point. So, OP 1 is your sigma 1 which is nothing but your sigma z that is that means sigma v right sigma z and sigma v both are same so in the z direction that is why I have written sigma z to understand this thing more clearly.

So, sigma z is equal to gamma z. So, sigma 1 that is the major principle stress in case of active state is gamma z. Now the what is the minor principle stress then? Minor principle stress is nothing but sigma 3 which is nothing but OP 2 okay. So, OP 2 is nothing but sigma h which needs to be determined. Sigma h is your active pressure. So, OO 1 OO what is OO 1 that is the

distance between the center of the Mohr circle here this is O 1 center of the Mohr circle and origin okay.

So, OO 1 is sigma 1 + sigma 3 by 2. I do not think that I should explain this thing in more detail because already we have explained enough in in the chapter when we discussed about shear strength. So, OO 1 equal to sigma 1 + sigma 3 by 2. Similarly, O 1 C 1 is the radius that is nothing but sigma 1 - sigma 3 by 2. So, from triangle OO 1 C 1 we get this relation right. So, from this I can write sigma 1 is equal to 1 + sin phi by 1 - sin phi into sigma 3.

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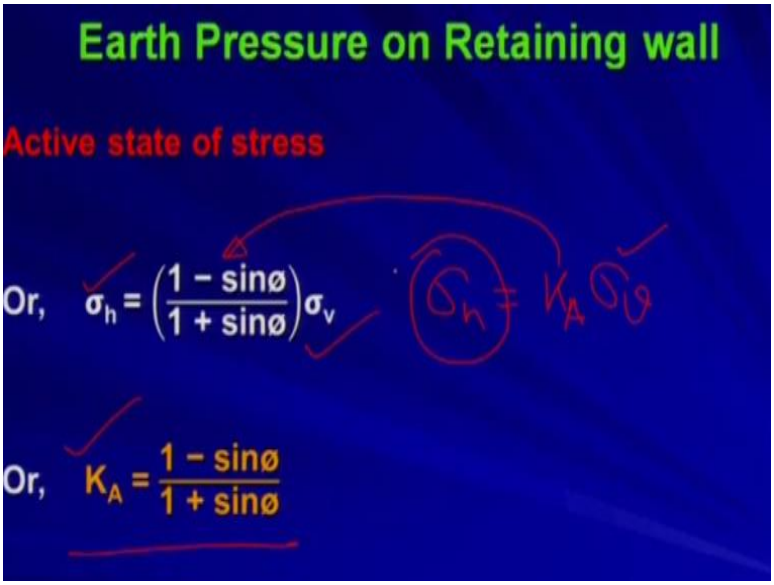


So, what does it mean sigma 1 so if you see the previous slide sigma 1 is equal to 1 + sin phi by 1 - sin phi into sigma 3. Now what is sigma 1? Sigma 1 is nothing but your sigma v right that is nothing but your gamma z. What is sigma 3? Sigma 3 is your lateral earth pressure that is sigma h right. So, I can write sigma h is equal to 1 - sin phi by 1 + sin phi into sigma v. Now from the relation of your earth pressure coefficient this therefore I can write this is equal to sigma v. So, now K A is nothing but 1 - sin phi by 1 + sin phi.

So, if you know the angle of internal friction phi you can find out what is your active earth pressure coefficient right. So, once you know K A and already you know sigma v you can find out sigma h. That is how much active earth pressure is exerted on the wall by the backfill. So, I will stop here today. So, in the next lecture we will talk about or we will try to find out the magnitude of K p and we will move further. Thank you very much.