

Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture - 47
Problem on Shear Strength of Soil

Welcome back. So,, in the last lecture we have seen vane shear test and 2 important characteristics that is sensitivity and the thixotropy of soil. So, today we will be talking about the Skempton pore pressure parameters determination.

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Shear strength of soil

Skempton Pore Pressure Parameters

- Pore pressure parameters are empirical coefficients which are used to express the response of pore pressure to change in total stress under undrained condition
- Consider an element of soil which is elastic & isotropic, acted upon by compressive, principal stress increments $\Delta\sigma'_1, \Delta\sigma'_2, \Delta\sigma'_3$

Already you know what are different pore pressure parameters that is A and B, capital A and capital B we introduced that thing when we were talking about the different laboratory triaxial test okay; so anyway. Now we will be seeing in more detail how this pore pressure parameters can be developed or can be found out okay. So, pore pressure parameters are empirical coefficients right which are used to express the response of pore pressure to change it change in total stress under undrained condition okay.

So, this pore pressure parameter will basically help you to find out the pore change in I mean to get the total pore water pressure because of some change or the variation in the stress under undrained condition because if it is not undrained condition you will not be getting the pore water pressure. If it is a drained condition then complete water will be draining out and there will be no pore pressure development right.

So, this thing is only valid when you are talking about the undrained situation okay. So, now consider an element, now we are trying to develop from the basics from the basics means from the mechanics point of view. So, consider an element of soil which is elastic okay and isotropic okay acted upon by compressive principle stress increments sigma 1 prime, sigma 2 prime, and sigma 3 prime. So, delta sigma 1 prime. So, this is the stress increment.

So, delta sigma 1 prime, delta sigma 2 prime, delta sigma 3 prime on the applied stress say. This is the stress increment we are applying. Now what are these components? So, sigma 1 prime, delta sigma 1 prime is nothing but the stress increment that is the major principle stress increment okay effective major principle stress increment. Then delta sigma 2 prime is the major intermediate principle stress increment and delta sigma 3 prime is the minor principle stress increment okay. So, we are considering all the principle stresses major, intermediate, and minor.

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Shear strength of soil
Skempton Pore Pressure Parameters

- The soil element decreases in volume

Volumetric strain (ϵ_V) = $\frac{\Delta V}{V}$

- If ϵ_1 , ϵ_2 & ϵ_3 are strains in the directions of the three principal stresses. Then

$$\epsilon_1 = 1/E [\Delta\sigma'_1 - \nu(\Delta\sigma'_2 + \Delta\sigma'_3)]$$

$$\epsilon_2 = 1/E [\Delta\sigma'_2 - \nu(\Delta\sigma'_1 + \Delta\sigma'_3)]$$

$$\epsilon_3 = 1/E [\Delta\sigma'_3 - \nu(\Delta\sigma'_1 + \Delta\sigma'_2)]$$

Now under this application of the stress increment you will be getting some volume reduction. That is nothing but the volume volumetric strain. So, the soil element decreases in volume because of the application of the stress increment effective stress increment and that volumetric strain is given by delta V by V where delta V is the volume change and capital V is the original volume okay. So, that is nothing but epsilon V.

Now if epsilon 1, epsilon 2, and epsilon 3 are strains in the directions of the 3 principle stresses okay. Now epsilon 1 is along major principle stress direction. Epsilon 2 is along intermediate principle stress direction and epsilon 3 is along minor principle stress direction. So, we can

express all the strain components like epsilon 1, epsilon 2, and epsilon 3 by using the Hooke's law like this epsilon 1 is equal to 1 by E into delta sigma 1 prime - Nu into where Nu is the Poisson ratio into delta sigma 2 prime + delta sigma 3 prime. Similarly, I can express epsilon 2 and epsilon 3 right okay.

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Shear strength of soil
Skempton Pore Pressure Parameters

$$\frac{\Delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1-2\nu}{E} [\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3]$$

Or, $\frac{\Delta V}{V} = \frac{3(1-2\nu)}{E} \left[\frac{\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3}{3} \right]$

Or, $\left(\frac{\Delta V}{V} \right) \times \frac{3}{\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3} = \frac{3(1-2\nu)}{E} = C_v \quad (5.48)$

■ Equation (5.48) means that the ratio of ε_v to the average effective stress change $\frac{\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3}{3}$ is a constant, equal to $\frac{3(1-2\nu)}{E}$ for a perfectly elastic material, called 'compressibility of the soil skeleton' C_v

Now what is your volumetric strength? So, from the mechanics if you recall your mechanics background so delta V by V is the volumetric strength which is nothing but equal to epsilon 1 + epsilon 2 + epsilon 3 exactly right so which is equal to from the previous expression so we previous expressions we can express this delta V by V that is the volumetric strain is equal to 1 - 2 Nu divided by E into delta sigma 1 prime + delta sigma 2 prime + delta sigma 3 prime okay which can be further written that I mean some modification or some I mean change we are making that is delta V by V again we are multiplying 3 here and we are dividing this whole thing by 3 okay.

So, which is nothing but delta V by V into 3 by the summation of all stress components is equal to 3 into 1 - 2 Nu divided by E which is known as and which we are expressing as C v which is nothing but the compressibility of the soil skeleton okay. Now what is this? This left-hand side if you look at the left hand side what is this? So, in equation 5.48 means that the ratio of epsilon V to the average effective stress change so this is nothing but the average effective stress change right so whatever sigma 1 prime whatever changes are made so this is the average of that so all 3 stress components divided by 3 okay.

So, this the ratio of these 2 that is epsilon V the ratio of epsilon V to the average effective stress is a constant and is equal to $\frac{1 - 2\nu}{E}$ right. This is a constant. All the times it will be constant. So, whatever volumetric strain you are getting and whatever average effective stress is there the ratio will be always constant and for a perfect elastic material okay C_v is called compressibility of the soil skeleton okay. Is that clear? So, this C_v is basically a I mean if you look at this C_v is a constant kind of thing.

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Shear strength of soil
Skempton Pore Pressure Parameters

- Let Δu be the increase in pore pressure when no drainage is allowed

$$\Delta\sigma'_1 = \Delta\sigma_1 - \Delta u, \Delta\sigma'_2 = \Delta\sigma_2 - \Delta u, \Delta\sigma'_3 = \Delta\sigma_3 - \Delta u,$$

$$\Delta V = VC_v(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u) \quad (5.49)$$

- Now the change in volume of pore fluid (water + air) ΔV_w due to increase in pore pressure Δu must be equal to the volume change given in equation (5.49)

So, now in the next slide basically you will be, now let Δu be the increase in pore water pressure when no drainage is allowed okay. So, $\Delta\sigma'_1$ is equal to $\Delta\sigma_1 - \Delta u$ total stress - pore water pressure. Similarly, I can express $\Delta\sigma'_2$ and $\Delta\sigma'_3$. The total stress minus the pore water pressure which is getting developed is giving you the effective stress. So, ΔV from the previous expression I can write V into C_v into the $\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3$ I mean basically this is nothing but $\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3$ right.

So, now in terms of total stress I mean parameters $\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u$. That is the pore water pressure which will be giving me the volume change that is the Δv . So, now the change in volume of pore fluid try to understand so we are coming from the basics. Now the change in volume of pore fluid that is water plus air. So, pore fluid means it is made of water and air the mix of that right.

So, now the change in volume of pore fluid ΔV_w due to increase in pore pressure Δu must be equal to the volume change given in equation because only the pore fluid can experience the volume change I mean the soil I mean grains the solid materials or the solid grains of the soil they will not be experiencing any volume change. So, whatever volume change is happening due to the increase in pore water pressure that is solely due to the volume change in the pore fluid right. So, ΔV_w if it is the volume change in the pore fluid that must be equal to ΔV right.

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Shear strength of soil
Skempton Pore Pressure Parameters

- If n is the porosity & C_w is the compressibility of the pore fluid

$$\Delta V_w = nVC_w\Delta u \quad (5.50)$$

- Again, $\Delta V = \Delta V_w$

Or, $C_v(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u)/3 = nC_w\Delta u$

- In the conventional triaxial test,

$$\Delta\sigma_2 = \Delta\sigma_3$$

So, if n is the porosity and C_w is the compressibility of the pore fluid then I can write ΔV_w that is the volume change in the pore fluid is equal to n into V into C_w and that is happening due to the pore water pressure Δu I can write that so from the basic principle or the basic mechanics knowledge or the concept right so again just now I have told you that ΔV that is the change in volume change in total volume must be equal to the change in pore fluid I mean volume of pore fluid right because soil grains will not be experiencing any volume change.

So, therefore C_v into whatever you have seen in the previous expression C_v into this is equal to $n C_w$ into Δu . So, from this expression I am getting this and from the previous expression I have got this. So, in the conventional triaxial test now this is the relation we have established agreed. We have established the relation C_v into $\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3$ into Δu divided by 3 is equal to n into C_w into Δu .

This is the general relation. Now in the conventional triaxial test already you have seen. Now if you recall we have applied the all-round cell pressure where your $\Delta\sigma_2$ was equal to $\Delta\sigma_3$ that means there was no concept of intermediate principle stress rather intermediate principle stress and minor principle stress they become same right when we talked about the triaxial test.

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Shear strength of soil
Skempton Pore Pressure Parameters

$$\Delta u = \frac{1}{1 + n \frac{C_w}{C_v}} [(\Delta\sigma_1 + 2\Delta\sigma_3)/3]$$

$$= \frac{1}{1 + n \frac{C_w}{C_v}} [(\Delta\sigma_1 - \Delta\sigma_3 + 3\Delta\sigma_3)/3]$$

$$\Delta u = \frac{1}{1 + n \frac{C_w}{C_v}} [\Delta\sigma_3 + (\Delta\sigma_1 - \Delta\sigma_3)/3]$$

Can be written as,

$$\Delta u = B [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \quad (5.51)$$

So, if that is so then Δu can be expressed like this from the previous expression which can be further written as this okay $\Delta\sigma_1 - \Delta\sigma_3 + 3$ into some modification we are doing which can be further written as this what Δu is equal to that is the pore water pressure increment is equal to this.

Now if you look at this expression here this is the part and this is the whole part okay. Now it can be written this expression can be written in this fashion that is Δu is equal to B into $\Delta\sigma_3 + A$ into $\Delta\sigma_1 - \Delta\sigma_3$. Now we are expressing these 2 I mean this expression or this equation by this expression given in equation 5.51.

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Shear strength of soil
Skempton Pore Pressure Parameter

Where, **B & A** are called “Skempton’s Pore pressure parameters”

It can be written as,

$$\Delta u_1 = B \Delta \sigma_3$$

$$\Delta u_2 = BA(\Delta \sigma_1 - \Delta \sigma_3)$$

$$\Delta u_1 + \Delta u_2 = \Delta u$$

■ Δu_1 is the change in pore pressure due to an increase in cell pressure $\Delta \sigma_3$

Now from this where B and A are called as Skempton pore pressure parameter. So, now I hope that you have understood that how these 2 parameters are coming into the picture from the basic principle of mechanics okay. So, earlier we introduced that Skempton pore pressure parameters. At that time, we never talked about how these parameters are coming into the picture but now you are quite clear how these parameters are coming into the picture right this B and A.

So, it can be written as that if you see the previous expression right if you see the previous expression equation 5.51 so basically, I can write delta u 1 is equal to B into delta sigma 3 and delta u 2 is equal to BA into delta sigma 1 - delta sigma 3 right. So, total pore water pressure increment is equal to delta u 1 + delta u 2.

Now if you recall when we talked about the triaxial test at that time we I mean introduced A parameter as well as B parameter in different I mean A parameter and AB I mean multiplication of B and A okay or this B parameter and this multiplication of B and A in different conditions and different situations right. So, this delta what is delta u 1? Delta u 1 is the change in pore water pressure due to an increase in cell pressure delta sigma 3 okay. So, as you increase delta sigma 3 you are getting delta u 1. That is the change in pore water pressure.

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Shear strength of soil

Skempton Pore Pressure Parameter

- Δu_2 is the change in pore pressure due to an increase in the deviatoric pressure $(\Delta\sigma_1 - \Delta\sigma_3)$
- For completely saturated soil, the only pore fluid is water, compressibility of water $C_w \ll C_v$,

$$\frac{C_w}{C_v} \approx 0 \Rightarrow B = 1$$
- In a dry soil, the compressibility of pore fluid (air only) $C_w \gg C_v$, $\frac{C_w}{C_v} \approx \infty \Rightarrow B = 0$

So, Δu_2 is the change in pore water pressure due to an increase in the deviatoric stress or the deviatoric pressure that is $\Delta\sigma_1 - \Delta\sigma_3$ okay and combine effect will be giving you the total pore pressure increment that is Δu . So, Δu_1 is happening in the first stage Δu_2 is happening in the second stage of the triaxial test and the combination of these 2 will give you the total pore water pressure increment.

Now for completely saturated soil only pore fluid is water agreed and compressibility of water C_w is very less than C_v that is the complete volume I mean soil skeleton, volume compressibility of soil skeleton right. Therefore, I can write C_w by C_v is approximately equal to 0 because C_w is very less than because water is I mean water is incompressible right. Only the pore fluid is water in case of saturated soil right.

So, the if the soil if the water is completely incompressible then I can write C_w by C_v is approximately equal to 0 and which will give me B equal to 1 if you just see the previous expressions the B equal to 1. In a dry soil, the compressibility of the pore fluid so in the dry soil what is your pore fluid only air there is no water so air is highly compressible. So, C_w is very greater than C_v . If that is so then C_w by C_v I can say that is infinite okay very high value infinite value so B becomes 0. So, B basically varies from 0 to 1.

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Shear strength of soil
Skempton Pore Pressure Parameter

- B varies from 0 to 1 depending on the degree of saturation S,

If B = 1 (for S = 100%)

$$\Delta u = \Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3)$$

For a partially saturated soil

$$\Delta u = B \Delta \sigma_3 + \bar{A} (\Delta \sigma_1 - \Delta \sigma_3)$$

Where, $\bar{A} = AB$

So, B varies from 0 to 1 depending on the degree of saturation S. So, as it is completely saturated B will become 1. As the it is becoming completely dry that means degree of saturation is 0 at that time B becomes 0 right. So, B becomes B varies from 0 to 1 depending on the degree of saturation. Now if B is 1 that means the degree of saturation is 100%. That means the pore fluid is completely made of water or filled with water. So, there is no air. So, in that situation delta u so B is 1 so delta sigma 3 + A into delta sigma 1 - delta sigma 3.

So, basically what I mean to say if you consider a 100% saturated soil okay in the triaxial test if you know the value of A and if you know the increment of sigma 1, increment of sigma 3 you can find out the pore pressure how much pore pressure is getting generated during the test at different stage. Now for a partially saturated soil delta u is given by this expression where A bar is nothing but the multiplication of A into B.

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Shear strength of soil

Skempton Pore Pressure Parameter

- B can be determined in an Unconsolidated Undrained test
- The cell pressure is increased by $\Delta\sigma_3$ & the corresponding increase in pore pressure Δu_1 is measured in the 1st stage; then $B = \frac{\Delta u_1}{\Delta\sigma_3}$

Now B can be determined in an unconsolidated undrained test. The cell pressure is increased by $\Delta\sigma_3$ and the corresponding increase in pore pressure Δu_1 is measured in the first stage then B is given by this. That means if you measure Δu_1 right. So, you are doing some unconsolidated undrained test. That means in the first stage also you are closing the valve in the second stage also you are closing the valve. So, in both the stages you are measuring the pore water pressure. So, in the first stage so B is given by this expression right. B is nothing but Δu_1 by $\Delta\sigma_3$. Now in the first stage when you are applying only the cell pressure and you are not allowing the consolidation of the soil sample so basically some pore water pressure will be getting built up. So, that pore water pressure is nothing but Δu_1 . So, Δu_1 by $\Delta\sigma_3$ that is the increment of cell pressure will give you the value of B. So, in that way B can be calculated or B can be determined.

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Shear strength of soil

Skempton Pore Pressure Parameter

- \bar{A} is measured during the 2nd stage of the triaxial test
- If Δu_2 is the pore pressure increases due to $(\Delta\sigma_1 - \Delta\sigma_3)$ with cell pressure being constant ($\Delta\sigma_3 = 0$),

$$\bar{A} = \frac{\Delta u_2 (\Delta\sigma_1 - \Delta\sigma_3)}{\Delta\sigma_1 - \Delta\sigma_3}$$
- For a fully saturated soil, A can be determined easily in Consolidated Undrained test

Now \bar{A} is measured during the second stage of the triaxial test. So, in the undrained unconsolidated undrained test you are performing so \bar{A} is measured in the second stage of triaxial test. Second stage means when you are applying the deviatoric stress. So, if Δu_2 is the pore water pressure increases due to $\Delta\sigma_1 - \Delta\sigma_3$ that is the deviatoric stress right with cell pressure being constant, cell pressure is constant that means $\Delta\sigma_3$ is 0 then \bar{A} is given by this agreed. So, \bar{A} is given by this.

So, for a fully saturated soil \bar{A} can be determined easily in consolidated undrained test okay. So, \bar{B} you have determined and \bar{A} you need to determine. So, \bar{A} for a fully saturated soil \bar{A} can be determined easily in consolidated undrained test. So, the first stage is not required. Consolidated undrained means in the first stage you are allowing drainage so therefore Δu_1 is 0. So, only you are measuring Δu_2 . So, that is only required. So, to find out \bar{A} . So, once you obtain \bar{B} and then you do consolidated undrained test to find out the magnitude of \bar{A} okay because you need Δu_2 only.

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Shear strength of soil

Skempton Pore Pressure Parameter

- In this test, $\Delta\sigma_3 = 0$ ✓

$$A = \frac{\Delta u}{(\Delta\sigma_1 - \Delta\sigma_3)} = \frac{\Delta u}{\Delta\sigma_1}$$

$\Delta u = \Delta u_1 + \Delta u_2$

$$A = \frac{\Delta u}{\sigma_1 - \sigma_3}$$

- 'A' depends upon the strain, anisotropy, sample disturbance and the OCR

So, in this test $\Delta\sigma_3$ is 0 what is that consolidated undrained test $\Delta\sigma_3$ is 0. So, A is nothing but this. So, Δu divided by $\Delta\sigma_1$. So, what is Δu in this consolidated undrained test because Δu_1 is 0 that is nothing but Δu_2 . So, Δu_2 is nothing but your Δu okay. So, from this you can find out A. So, you measured this you know σ_1 you know σ_3 you can find out A.

So, A depends upon the strain anisotropy, sample disturbance, and OCR. So, these are the parameters which will govern or which will I mean affect the magnitude of A. What are those things? Strain right, anisotropy that is the how much strain is happening in the soil sample anisotropy if any anisotropy is present in the soil or not sample disturbance so how much disturbance is happening in the sample, and of course the over consolidation ratio that is OCR.

So, these are the things will affect A. So, now it I hope that is this is clear to you that how we can find out these 2 parameters by performing the triaxial test. So, if you want to know the magnitude of A basically you perform consolidated undrained test and you get these things from the test and you get the parameter A and if you want to find out the parameter B then you obtain or you perform some unconsolidated undrained test and from there you can find out B.

So, these 2 parameters already we have seen that how we can establish these parameters how these parameters have come into the picture and how we can find out these parameters from the laboratory experiment laboratory triaxial test. So, I will stop here today and this will conclude the shear strength chapter. This chapter was quite big and I took little bit more time so that you understand this concept because this is very important chapter in the soil mechanics. Everything,

all the designed aspects, all the designed philosophy in the geotechnical engineering is lying or is based on this shear strength.

So, I will stop here today. Thank you very much. So, next class we will take some numerical examples on shear strength. Thank you.