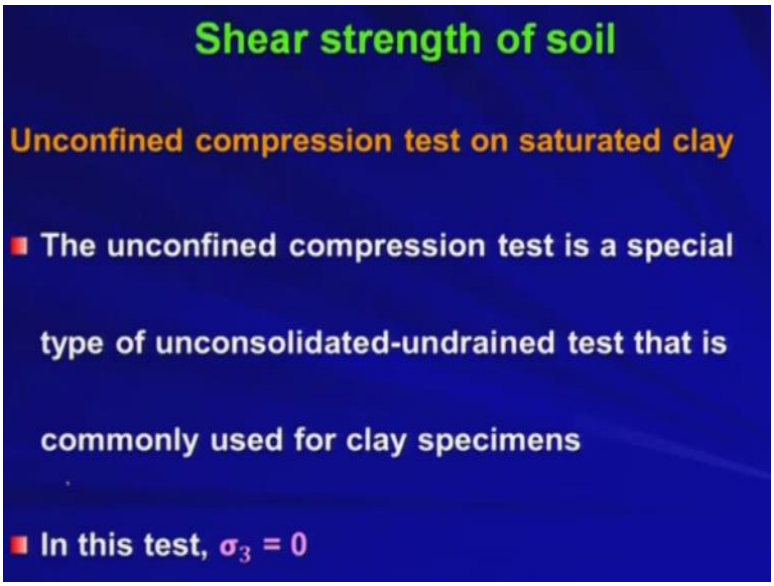
Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture - 44
Shear Strength of Soil

Welcome back. So, today we will be seeing another type of say test by which you can find out the shear strength of soil that is nothing but unconfined compression test on saturated clay right.

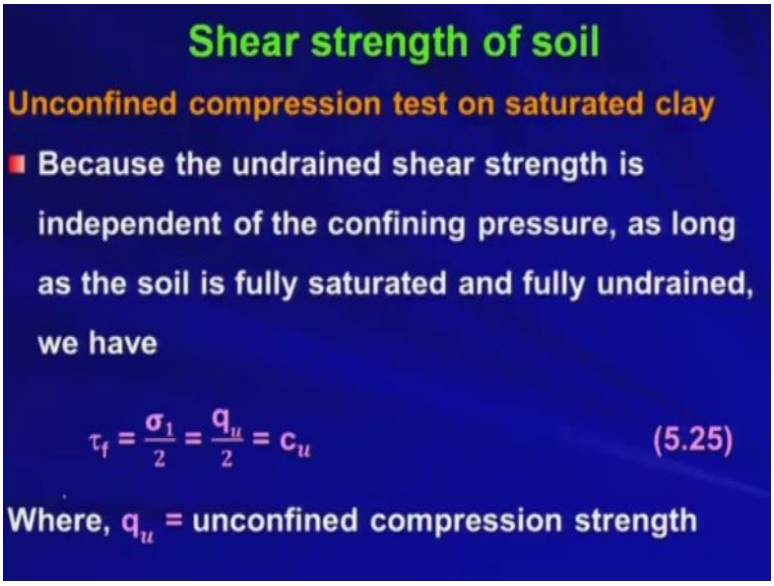
(Refer Slide Time: 00:25)



So, the unconfined compression test is a special type of unconsolidated undrained test that is commonly used for clay specimens. So, this test that is this unconfined compression test is basically used for your clay specimen and this is a special type of UU test whatever you have seen earlier right. So, unconsolidated undrained means you are not considering drainage at any point of time during the test right. So, it is pretty similar to that.

Only difference is that in this test sigma 3 is 0. Now if sigma 3 is 0 then what you can think of? So, if sigma 3 is 0 that means your confining pressure that is the cell pressure is not there. So, that means you are considering a cylindrical soil sample and you are shearing it with the application of only axial stress that is your sigma 1 right. So, there is no cell pressure or all-round cell pressure acting on the soil sample.

(Refer Slide Time: 01:26)



So, because the undrained shear strength is independent of the confining pressure as long as the soil is fully saturated and fully undrained we have so basically before that I mean already we have discussed this thing in detail that in case of undrained shear strength if you are considering I mean if you have if you are talking about UU test and if you are trying to find out the undrained shear strength basically it is independent of the confining pressure right.

So, I mean no matter confining pressure you use I mean you will be getting the failure line which will be touching the Mohr circle and this confining pressure I mean whatever confining pressure you will be applying based on that basically you will be getting the deviator stress. So, the confining pressure really does not matter in case of undrained shear strength determination right.

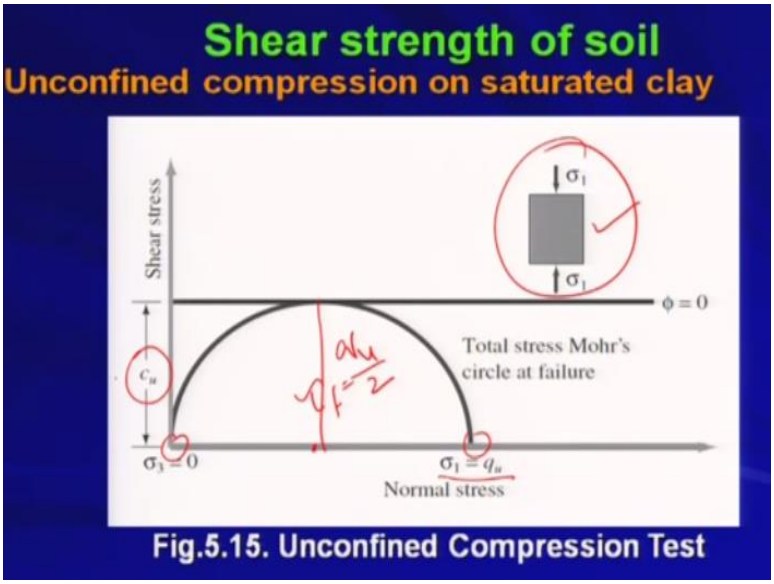
So, therefore as long as soil is fully saturated that means you are not considering the partially saturated soil that means there is no air (()) (02:26) or the there is no air particle or I mean the void space is completely filled with water. So, there is no air inside the soil specimen. In that situation and if you consider fully undrained condition that there is no drainage happening in the soil sample during shearing.

Therefore, the tau f that is the failure shear stress that means that is nothing but your shear strength is nothing but sigma 1 by 2 right. So, basically what is happening? You are not considering sigma 3 that is all-round confining pressure is not there. You are just applying sigma 1 in the axial direction and you are shearing it till the failure.

So, you will be getting the Mohr circle okay with sigma 3 is 0 and there is some value of sigma 1 and that will cause the failure and therefore the tau f that is the failure shear stress that is the

shear strength is nothing but sigma 1 by 2 that is nothing but the radius of the Mohr circle am I right. That is the radius of the Mohr circle which is nothing but q u by 2 so and is nothing but Cu that is undrained cohesion. So, where q u is nothing but the unconfined compression strength. So, the q u is defined as the uncompression unconfined compression strength in the soil mechanics.

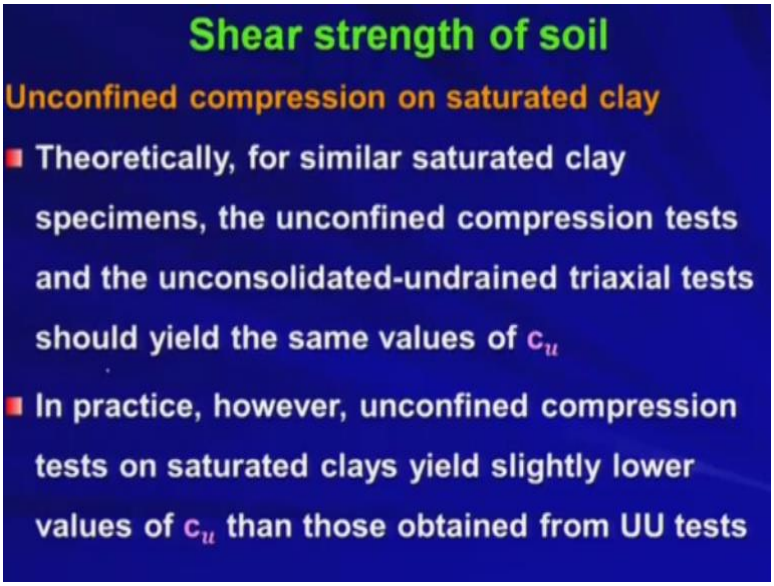
(Refer Slide Time: 03:49)



So, basically if you look at this figure so it will be clear to you. Basically in case of unconfined compression test you have the situation like that you do not have any sigma 3 only sigma 1 is acting axial direction and therefore so this state of stress if you want to plot in shear stress versus normal stress space with the help of Mohr circle then this will be your sigma 3 that is 0 and this will be your sigma 1 that is nothing but q u that is your unconfined compression strength okay and this point is the center and this is basically your q u by 2 and that is nothing but tau f right.

So, already we have seen that thing okay in the previous equation. So, this is your shear strength right so in this situation you will not be getting any phi angle so everything I mean the failure envelope will be horizontal and will be parallel to the normal stress axis therefore all the times you will be getting the cohesion. So, in case of soil so clay clayey soil you will be getting this kind of behaviour from the unconfined compression test setup okay.

(Refer Slide Time: 05:05)



So, theoretically for similar saturated clay specimens the unconfined compression tests and the unconsolidated undrained triaxial test should yield the same values of Cu. Do you agree with this or not? Because for similar saturated clay specimen already we know that if we talk about UU test UU test is not dependent on the confining pressure. Now if it is not dependent on the confining pressure then it is there is I mean it is no matter whatever confining pressure you are considering you will be getting the same strength right.

So, that means I mean in that situation you can consider the confining pressure is 0 also because it is independent of the confining pressure so you can also consider confining pressure is 0. So, that is why it is it is told that the similar saturated clay specimens the unconfined compression test and the unconsolidated that is UU triaxial test should yield the same values of Cu theoretically okay but practically in practice however unconfined compression test on saturated clays yield slightly lower values of Cu than those obtained from UU test okay but theoretically they should be same because the undrained strength is not dependent on the confining pressure.

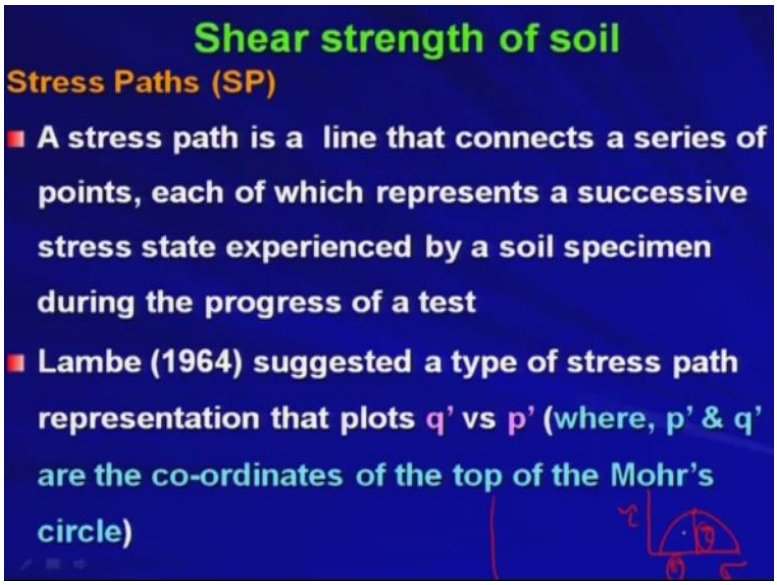
So, therefore you will not be you will be getting same amount of shear strength from both the test UU test as well as unconfined compression test. However, in practice your unconfined compression test result that is the value that Cu value obtained from unconfined compression test that UC test is slightly lower than the Cu obtained from the UU test. This is generally seen in the practical situation.

However, I mean in theory they should be same. Now coming to the concept of stress paths this is another very fundamental concept and you must understand this concept and once you

understand this concept though this is beyond the scope of our study in this particular course however the stress path will be very useful as well as helpful when you will be talking about the critical state soil mechanics.

So, that means cam clay model and other things there at that time shear I mean stress path will be playing a major role or the important role to understand the concept okay. However, so that is beyond the scope so I will not talk about that type of say critical state soil mechanics or the failure envelope and all those things. However, we should know what is stress path I mean how we can establish the stress path and what significant information I will be getting from the stress path.

(Refer Slide Time: 07:46)



So, a stress path is a line that connects a series of points each of which represents a successive stress state experienced by a soil specimen during the progress of a test okay. So, it says that stress path is a line it could be linear it could be nonlinear that means curved line okay that connects a series of points.

So, you will be getting a series of points and through which you will be getting I mean if you join those points basically you will be getting the line and that line will be indicated as the stress path. Now what are those points so each of which represents so I mean each of these points will represent a successive stress state that means suppose you are doing some consolidated drained test or if you are doing some consolidated undrained test whatever test you are performing so

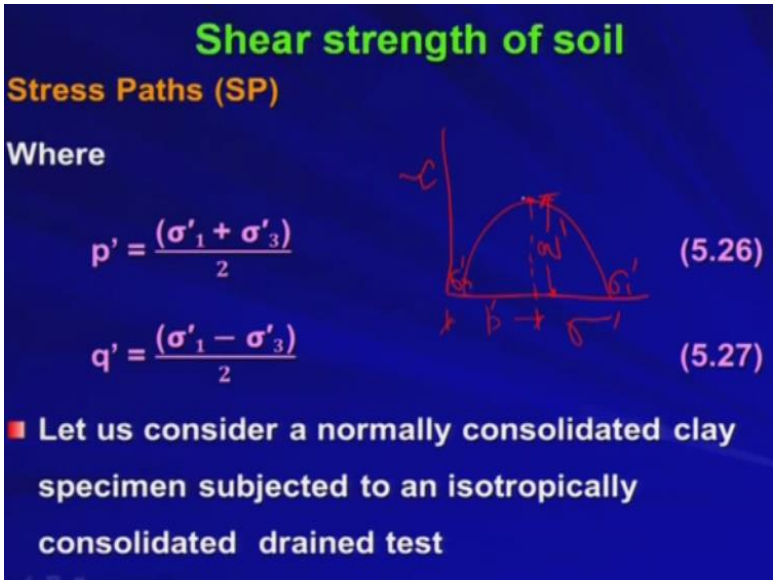
basically what you are doing first, initially you are applying the confining pressure and then you are shearing it through the deviatoric stress.

Now at each and every point you are getting different state of stress right different combinations of your sigma I mean sigma 3 is constant so different combinations of your sigma 3 and sigma 1. That combination is getting different. That point is I mean not a fixed point it is a variable point right during the progress of the test. So, during that time so each point if you see each point will tell you about how the stress has progressed I mean how the test has progressed right.

So, the if you see the stress path then you will be seeing okay the test that whatever test you are talking about that test has progressed along this path. That means each point on the stress path line or the stress path will be indicating the state of stress at different situation or different state of say test okay. Lambe in 1964 suggested a type of stress path representation that plots q prime versus p prime that is I mean popularly known as qp plot okay where p prime and q prime are the coordinates of the top of the Mohr circle.

Now what do you mean by top of the Mohr circle? So, if you try to draw the Mohr circle this is your say tau versus sigma plot and if you plot a Mohr circle okay so the it says that you are plotting the stress path in q prime p prime space where p prime and q prime are the coordinates of the top of the Mohr circle. Top of the Mohr circle is here right. So, top of the Mohr circle is lying here. So, this is the coordinate. So, this is your sigma and this is your say tau for this point. So, these 2 things will be talking about p and q. So, they are expressed like that.

(Refer Slide Time: 10:47)



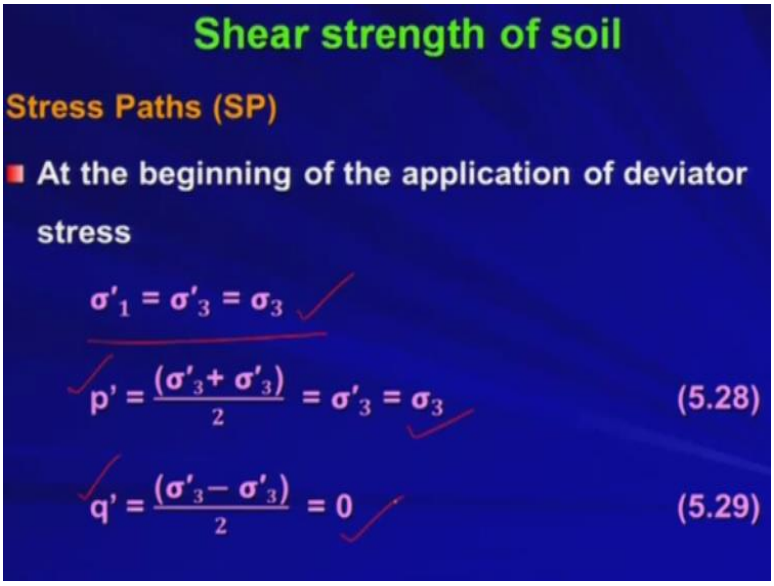
So, p prime is expressed as sigma 1 prime + sigma 3 prime everything is expressed in terms of effective stress so sigma 1 prime + sigma 3 prime by 2 okay so that means if you recall so this is your Mohr circle this is your tau this is your sigma. So, this is the center of the Mohr circle and this is the peak point okay of the Mohr circle. So, p prime this is nothing but your p prime right from here to here right.

Sigma 1 this is your sigma 1 prime say if I plot in terms of effective stress this is your sigma 3 prime. So, sigma 1 prime + sigma 3 prime by 2 is nothing but the distance of the center from the origin agreed okay. Similarly, q prime, what is q prime? q prime is sigma 1 prime - sigma 3 prime by 2 what is that? That is nothing but the radius. So, this is your q prime okay. So, this is your q prime.

So, sigma 1 prime - sigma 3 prime by 2 that is nothing but the radius of the Mohr circle. So, that is why this p prime and q prime this will indicate the highest point of the Mohr circle okay. So, highest point is lying here only. Now let us consider a normally consolidated clay specimen subjected to an isotropically consolidated drained test. What we are considering, we are considering some kind of test and let us see how we can develop the stress path for that particular test. So, we are considering a normally consolidated clay specimen subjected to an isotropically consolidated drained test okay.

So, you are considering normally consolidated clay so they are there will be only phi so c will not be there as already we have seen okay for the consolidated drained test that there is no c interset because c interset you will be getting for the over consolidated clay. So, you are I mean talking about the normally consolidated clay. So, you will be getting only 1 line which is inclined at an angle phi that is angle of internal friction with the sigma axis right and this is isotropically consolidated drained test we are considering.

(Refer Slide Time: 13:10)

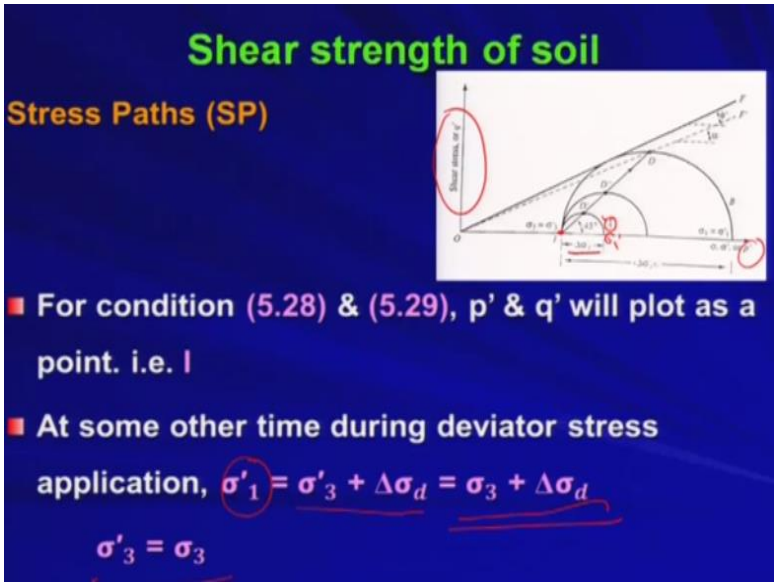


So, at the beginning of the application of deviatoric stress right at the beginning of the application of that means there are 2 stage right I hope that you still recall there are 2 stages of the triaxial test. One is that you are applying the confining pressure you are consolidating that thing. In case of CD test, we are doing that right. In case of CD test first you are consolidating some sample with the help of your confining pressure or the cell pressure and then after that you are shearing it with the help of deviatoric stress.

So, before application of the deviatoric stress at the beginning so sigma 1 prime is equal to sigma 3 prime is equal to sigma 3 agreed because you are applying all-round cell pressure. So, sigma I mean so radial direction whatever stress is there the top direction also the stress will be same the axial direction. So, the sigma 1 will be nothing but equal to sigma 3 will be nothing but equal to total stress that is sigma 3 say okay.

So, in this situation I can calculate p prime by the equation given just now okay. So, sigma 3 prime + sigma 3 prime by 2 which is nothing but sigma 3 prime and it is eventually equal to sigma 3 right. The effective stress and total stress must be same under CD test okay. Now what will be the value of q prime? That is simply 0 right because you are getting sigma 1 is equal to sigma 3, so q prime is 0.

(Refer Slide Time: 14:42)



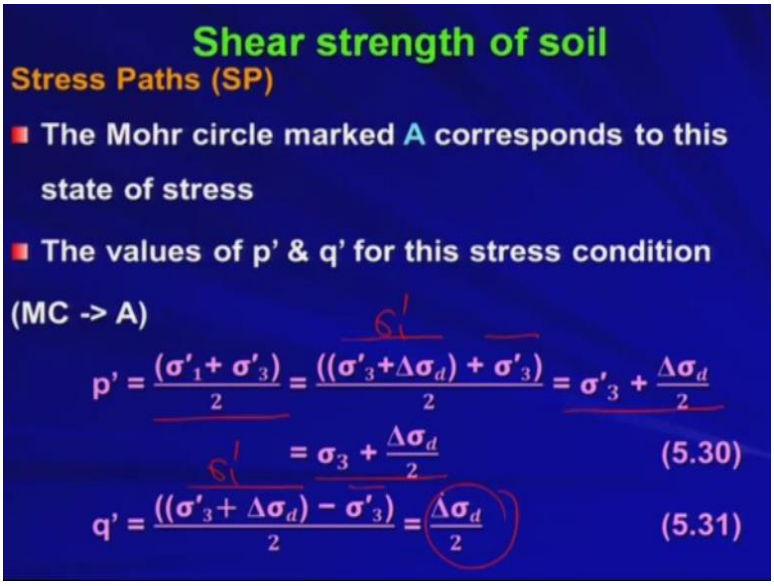
So, now if you see this so for condition 5.28 so what you have got? See p prime is sigma 3 and q prime is 0. Now in this space q prime versus p prime space we are plotting okay. So, the for condition 5.28 and 5.29 p prime and q prime will plot as a point that is I so that is the point. So, this is the point which will be indicated by 5.28 and 5.29 already we have seen where we are getting sigma p prime is equal to sigma 3 and q prime is equal to 0 right. So, this is the point.

So, I point will indicate that. That is nothing but the initiation of the test. So, that means you are just starting the test before applying the deviatoric stress this is the condition this is the situation. So, this point whenever I am saying this point so basically, I will be understanding this is isotropically consolidated sample okay. So, shear is not happened till now okay.

Now at some other time during the deviatoric stress application so now you are applying the deviatoric stress at any other time so sigma 1 prime will be equal to sigma 3 prime + delta sigma d agreed? So, sigma 3 prime will be your cell pressure that is already there and that is that will be always there during the test plus the deviatoric stress sigma delta sigma d.

So, sigma 3 prime + delta sigma d is equal to sigma 1 prime which is nothing but because you are considering drained test so that is nothing but equal to the total stress that is sigma 3 + delta sigma d okay. Now what about sigma 3 prime? Sigma 3 prime is already equal to sigma 3 and that will be remaining constant throughout the test so sigma 1 prime I have got sigma 3 + delta sigma d and sigma 3 prime is equal to sigma 3.

(Refer Slide Time: 16:42)



Now the Mohr circle marked A corresponds to this state of stress. So, now if you see this figure basically the Mohr circle A this is the Mohr circle A so that will indicate so this Mohr circle will indicate that this is your sigma 1 prime and this is your sigma 3 prime. So, this Mohr circle will indicate that state of stress am I right? So, you started the test only through a point okay and then you are increasing the deviatoric stress therefore your sigma 1 is getting I mean increased and that is why you are getting the failure.

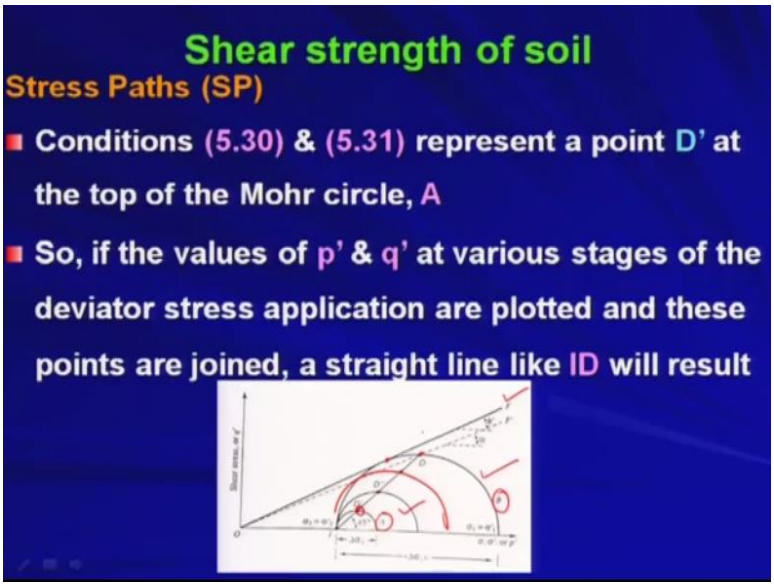
Now you are going up to the failure say or at any stage not up to the failure say at any stage your sigma 1 prime will be obtained by this expression so that is nothing but sigma 3 + delta sigma d which will be giving you the sigma 1 prime and sigma 3 of course that will be there. So, this circle Mohr circle A will represent that state of stress agreed okay. The values of p prime and q prime for this stress condition for say Mohr circle A now we are going to find out p prime and q prime when the state of stress is represented by the Mohr circle A.

So, at that time p prime is equal to that will be remaining same that expression is constant that is the distance of the Mohr circle center to the from the origin right. So, sigma 1 prime + sigma 3 prime by 2 which is nothing but so sigma 1 prime is this plus sigma 3 prime so ultimately you will be getting sigma 3 prime + delta sigma d by 2 and because you are considering drained test it is equal to the total stress.

So, sigma 3 + delta sigma d by 2 okay. Now in case of this condition what is the magnitude of q prime? q prime is sigma 3 prime + delta sigma d that is nothing but sigma 1 prime. This is your

sigma 1 prime right so this is also your sigma 1 prime. So, this your sigma 1 prime - sigma 3 prime by 2 which will be nothing but delta sigma d by 2 right.

(Refer Slide Time: 18:52)



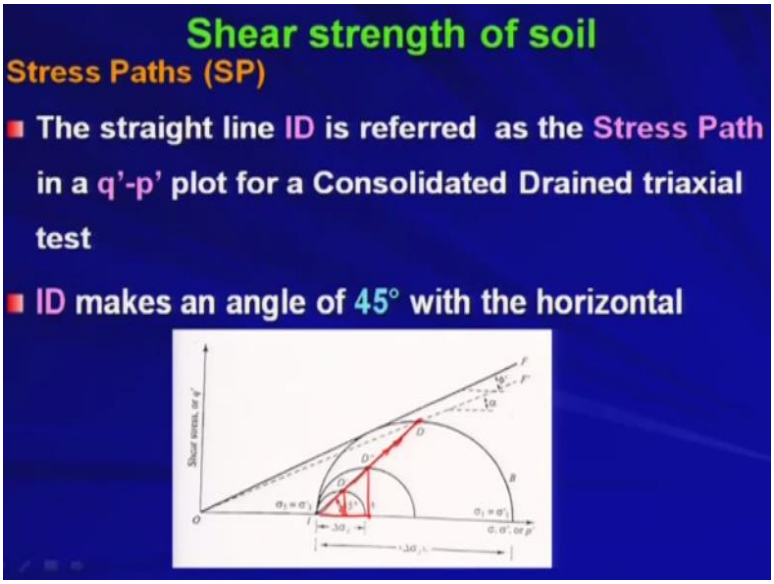
So, these conditions 5.30 and 5.31 these conditions represent a point d prime at the top of the Mohr circle right, so this is the point agreed. So, this is the point that is nothing but the topmost point of the Mohr circle A which will be representing the p prime and q prime at that state of stress agreed okay. Now so if the values of p prime and q prime at various stages of the deviatoric stress application are plotted and these points are joined then a straight line like ID will result okay.

So, now I mean we are continuously monitoring the test and we are continuously monitoring the sigma 1 prime so deviatoric stress basically so sigma 1 prime will be always equal to sigma 3 prime + delta sigma d so at some point at some stage you have got Mohr circle A then at the next stage when you are increasing the deviatoric stress you have got this circle and another circle say something like that and so on and ultimately you have got this Mohr circle when this Mohr circle will be talking about the failure.

That Mohr circle B will be talking about the failure because it is touching with the failure envelope OF agreed? So, this is touching at this point. So, till for circle A there is no failure. This is the progress of the test as the test is progressing you are getting different Mohr circles right. So, when you reach the Mohr circle B at that time you are getting the failure and at that time you are denoting that point okay.

So, because these are the points you are getting I told you that you will be getting several points and then you will be joining these points to get the stress path. So, this ID is your is a kind of straight line agreed? So, this is this will be your straight-line okay and this straight line will be known as the stress path which will be defining the state of stress for the isotropically consolidated drained test okay understood how the stress path is getting developed okay.

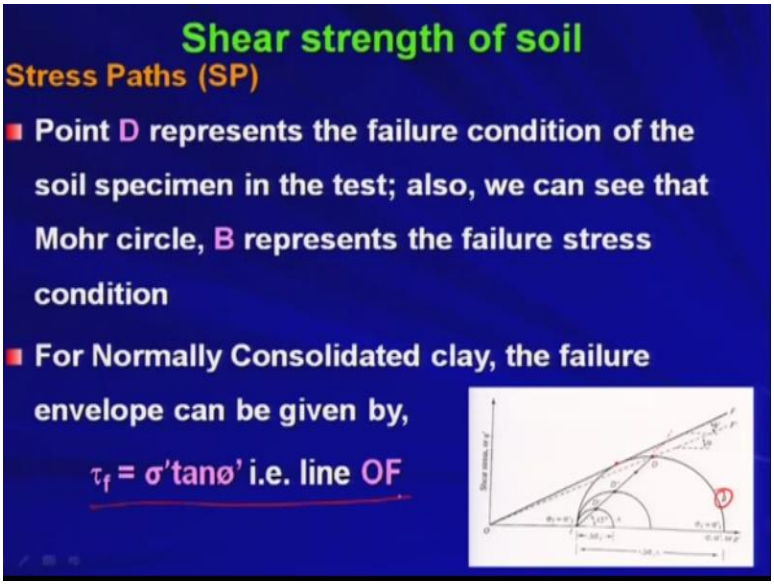
(Refer Slide Time: 21:05)



Now the straight line ID is referred as the stress path in a q prime p prime plot for a consolidated drained triaxial test as I told you. So, now you have got this is your stress path. So, if I get this kind of stress path I will be knowing how the test has progressed. That means all these points on the stress path will be denoting the state of stress okay at every stage of the test okay and ultimately when you will reach point D so that Mohr circle corresponding Mohr circle will be talking about the failure.

Now ID makes an angle of 45 degree with the horizontal because this angle is 45 degree agreed because this is also radius this is also radius this is also radius this is also radius so this angle is 45 degree okay. So, ID is a straight line and it will make an angle 45 degree with the horizontal.

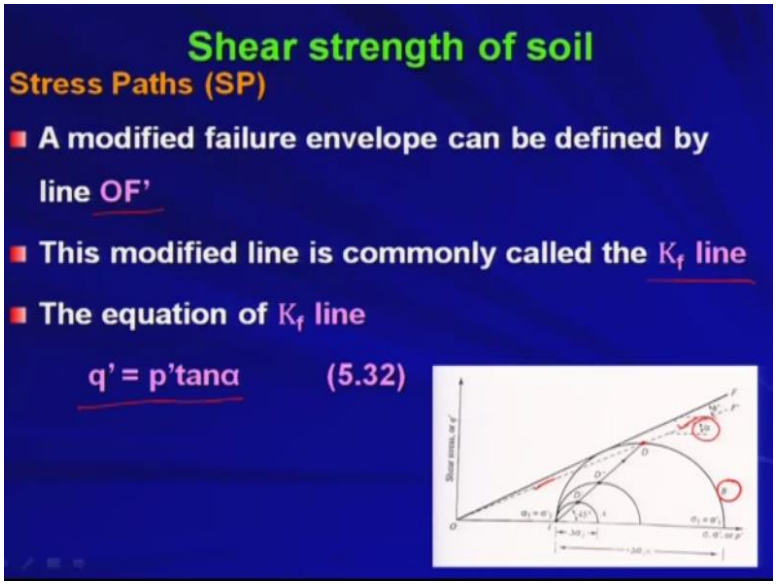
(Refer Slide Time: 22:05)



Now point D represents the failure condition of the soil specimen in the test agreed? Because when you reach at point D after that you cannot extend the stress path like that because you cannot have the access because soil has already failed. So, when you reach at point D at that time the specimen will tell I have failed in the test right. So, also, we can see that Mohr circle B okay represents the failure stress condition because it is touching the failure envelope at this point.

So, Mohr circle B is touching the failure envelope OF and therefore it will give you the failure and the corresponding point on the stress path is nothing but point D that is the topmost point of the Mohr circle. For normally consolidated clay the failure envelope can be given by tau f is equal to sigma prime into tan phi prime that is line OF agreed. So, that is not new, that already we have seen when we talked about the CD test okay. So, tau f that is the failure envelope is equal to sigma prime tan phi prime and that is denoted by the line OF.

(Refer Slide Time: 23:16)

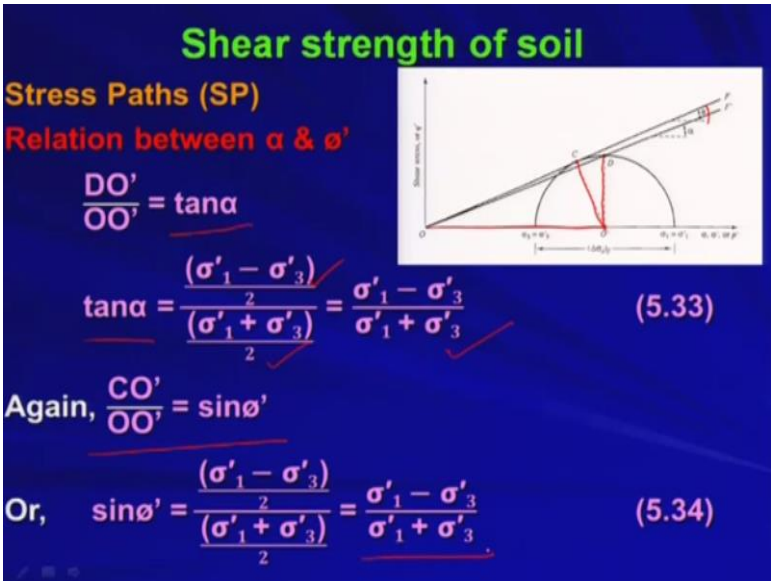


So, a modified failure envelope can be obtained by line OF prime okay. So, now you see this dotted line so this dotted line okay this OF prime is giving you the modified failure envelope. How you are getting OF prime? So, you have got the failure I mean failure Mohr circle that is B. Once you got the failure Mohr circle B and you can establish the point D because that will be the topmost point of the Mohr circle at failure and that will be also lying on the stress path so once you know the point D you join that point D with the origin and you will be getting one line OF prime and that is nothing but your modified failure envelope okay.

So, when you are joining the stress path okay. So, stress path the points on the stress path will be talking about the I mean the points whichever will be lying on the stress path that points all the points will be talking about the stages of stress application on the specimen and ultimately when you are getting the failure if you join that point with the origin you will be getting the modified failure envelope.

So, this modified line is commonly called as K f line in soil mechanics. So, maybe in some book in some literature in some journal papers or research papers you will be seeing if you see the K f line if they are mentioning about the K f line so K f line is denoted by this modified failure envelope that is OF prime. So, OF prime is nothing but your K f line. Now the equation of K f line is nothing but q prime is equal to p prime tan alpha where alpha is the angle of inclination of OF prime with the horizontal okay. So, q prime is equal to p prime tan alpha.

(Refer Slide Time: 25:11)

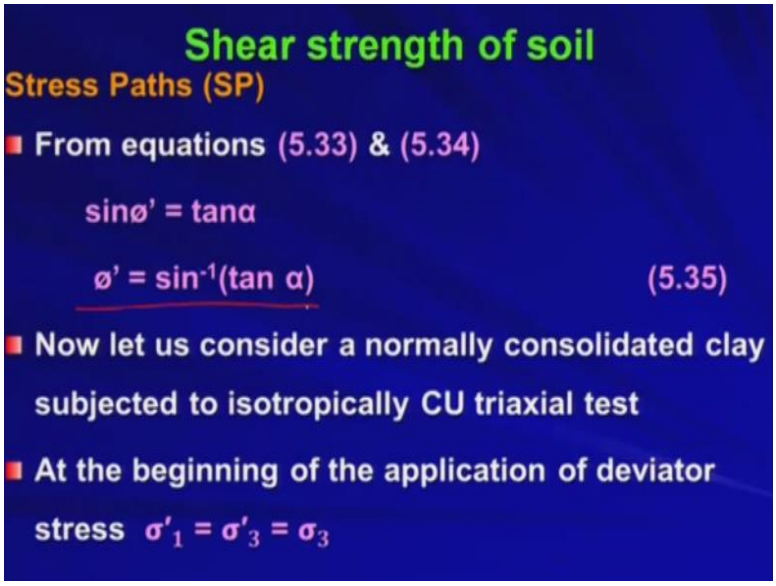


Now we are going to establish the relation between alpha and phi prime. So, is there any relation or not that we are looking for first and then we will be trying to establish the relation between alpha and phi prime. What is alpha? That is the angle of inclination of the modified failure envelope and what is phi prime that is the angle of inclination of the original failure envelope right. So, in this figure DO prime, DO prime by OO prime is equal to tan alpha okay.

So, therefore tan alpha is equal to, what is the coordinate of DO prime and OO prime? DO prime is nothing but the radius of the Mohr circle so that is given by this sigma 1 prime - sigma 3 prime by 2 and what is OO prime that is nothing but the distance of the Mohr circle center from the origin so that is nothing but sigma 1 prime + sigma 3 prime by 2 and we are getting sigma 1 prime – sigma 3 prime by sigma 1 prime + sigma 3 prime right.

Similarly, I can get CO prime by OO prime. So, CO prime is this okay. So, C is nothing but the point where the Mohr circle is touching the original failure envelope okay CO prime by OO prime that is nothing but sin phi prime okay because this angle is phi prime okay. So, sin phi prime is again is equal to sigma 1 prime - sigma 3 prime by sigma 1 prime + sigma 3 prime okay.

(Refer Slide Time: 26:54)



So, from equations 5.33 and 5.34 we can write because both right-hand side is same. So, sin phi prime is equal to tan alpha. So, therefore we can write phi prime is equal to sin inverse tan alpha. So, this is the relation between phi prime and alpha. So, phi prime is equal to sin inverse tan alpha right. So, that means if you know the inclination or the angle of internal friction of the original failure envelope or the original soil sample then basically if you want to find out the modified failure envelope then you can find out alpha from this expression right and once you know phi that means you can find out alpha and therefore you can find out the location or the orientation of K f line okay.

Now let us consider a normally consolidated clay subjected to isotropically CU triaxial test okay. What we are considering? We are considering now the normally consolidated clay, again normally consolidated, we are not talking about the over consolidated we are talking about the normally consolidated clay subjected to isotropically CU that is consolidated undrained triaxial test okay.

So, I will stop here today. In the next class, we will take this example and we will try to develop the stress path for this isotropically CU triaxial test. Thank you very much.