

Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture - 43
Shear Strength of Soils

Welcome back. So, in the last lecture we were seeing or we were discussing about the CU test.

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Shear strength of soil

Triaxial Shear Test

Consolidated Undrained test (CU)

- CD tests on clay soils take considerable time
- For this reason, CU tests can be conducted on such soils with pore pressure measurements to obtain the drained shear strength parameters
- Because drainage is not allowed in these tests during the application of deviator stress, the tests can be performed rather quickly

And we concluded that CU test is more versatile in nature because CU test is faster as compared to the CD test as well as from CU test whatever you will be getting whatever results you will be getting you can also get that result from CD test. So, therefore if you perform CU test you will be getting both the representation, representation with respect to the total stress, representation with respect to the effective stress right.

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Shear strength of soil

Triaxial Shear Test

Consolidated Undrained test (CU)

- At failure

$$\bar{A} = \bar{A}_f = \frac{(\Delta u_d)_f}{(\Delta \sigma_d)_f} \quad (5.15)$$

- Normally Consolidated clays : $\bar{A}_f = 0.5 - 1$
- Over Consolidated clays : $\bar{A}_f = -0.5 - 0$

Now at failure okay, in case of consolidated undrained test at failure, \bar{A} is nothing but \bar{A} is equal to $\Delta u_d / \Delta \sigma_d$ where Δu_d is the pore water pressure developed at the failure and $\Delta \sigma_d$ is the deviatoric stress at failure right. So, for normally consolidated clay generally \bar{A}_f varies from 0.5 to 1 that means if it is 1 that means the pore water pressure is basically equal to the total I mean the deviatoric stress right.

So, it generally varies from 0.5 to 1 whereas in case of over consolidated clay \bar{A} varies from -0.5 to 0. Why -0.5 because you are getting the negative pore water pressure if you recall right in case of dense sand or in case of over consolidated clay right so in case of normally consolidated clay and the loose sand will be coming in the one set and over consolidated clay and dense sand will be coming in the one set. Now in case of consolidated clay basically \bar{A} will be coming as negative. Why because your pore water pressure that means your numerator is becoming negative right. Negative pore water pressure you are getting due to the dilation of the material.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

- In UU tests, drainage from the soil specimen is not permitted during the application of σ_3 as well as $\Delta\sigma_d$
- The total pore pressure u in the specimen at any stage of deviator stress application

$$u = u_c + \Delta u_d \quad (5.16)$$

Where, u_c = Pore pressure increase due to σ_3
 Δu_d = Pore pressure increase due to $\Delta\sigma_d$

Now coming to the third type of triaxial test that is unconsolidated undrained test that is known as UU test. So, in UU test drainage from the soil specimen is not permitted during the application of σ_3 as well as $\Delta\sigma_d$. Now what does it mean? So, in case of CD test you allowed drainage okay you allowed consolidation when you applied σ_3 that means cell pressure and when you applied $\Delta\sigma_d$ that is deviatoric stress.

So, there are 2 parts in which basically I mean in case of in any triaxial test basically you have 2 steps. In one step you first apply the cell pressure and another step you shear it with the help of $\Delta\sigma_d$ that is the deviatoric stress. Now when you are applying the cell pressure in case of CD test when you are applying the cell pressure when you are applying the shearing I mean deviatoric stress in both the situations you allowed drainage right.

You allowed consolidated right in case of first step and you allowed drainage when you sheared it. Now in case of CU test that is consolidated undrained test in that situation you allowed drainage okay when you applied σ_3 but you did not allow drainage when you sheared it with the help of $\Delta\sigma_d$ that is deviatoric stress right. But in case of UU test in no case in no situation in no steps right you are allowing drainage.

So, that means drainage from soil specimen is not permitted during the application of σ_3 that is in the first step as well as $\Delta\sigma_d$ that is happening in the second step okay. So, the total pressure the total pore water pressure u in the specimen at any stage of deviatoric stress application is nothing but $u_c + \Delta u_d$ right. Now 2 distinct basically you will be getting the pore pressure generation in 2 steps.

The first step you are applying the cell pressure. At that time, you will be getting some generation of the pore pressure and in the second step when you are shearing it when you are applying the axial deviatoric stress at that time also you will be generating the pore water pressure. So, that will be coming in the second step. So, in both the steps whatever pore water pressure you will be getting or you will be generating that will be the total pore water pressure at any point of time right. So, total pore water pressure u is nothing but u_c plus Δu_d where u_c is pore pressure increase due to σ_3 that is happening in the first step and Δu_d is pore pressure increase due to $\Delta \sigma_d$ that is happening in the second step okay.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

- Earlier we know that,

$$u_c = B \cdot \sigma_3 \quad \& \quad \Delta u_d = \bar{A} \cdot \Delta \sigma_d \quad (CU)$$

$$u = B \sigma_3 + \bar{A} \Delta \sigma_d = B \sigma_3 + \bar{A} (\sigma_1 - \sigma_3) \quad (5.17)$$

- This test is usually conducted on saturated clay and silt specimens
- The added axial stress at failure $(\Delta \sigma_d)_f$ is practically same regardless of the chamber confining pressure

Now earlier we know that u_c if you recall whatever we have discussed in case of CU test at that time u_c was equal to B into σ_3 right okay and Δu_d is equal to \bar{A} into $\Delta \sigma_d$ that is coming from CU test. So, this is coming from CU test so B is the Skempton pore pressure parameter and this is coming from your CU test that is again \bar{A} is your Skempton pore pressure parameter.

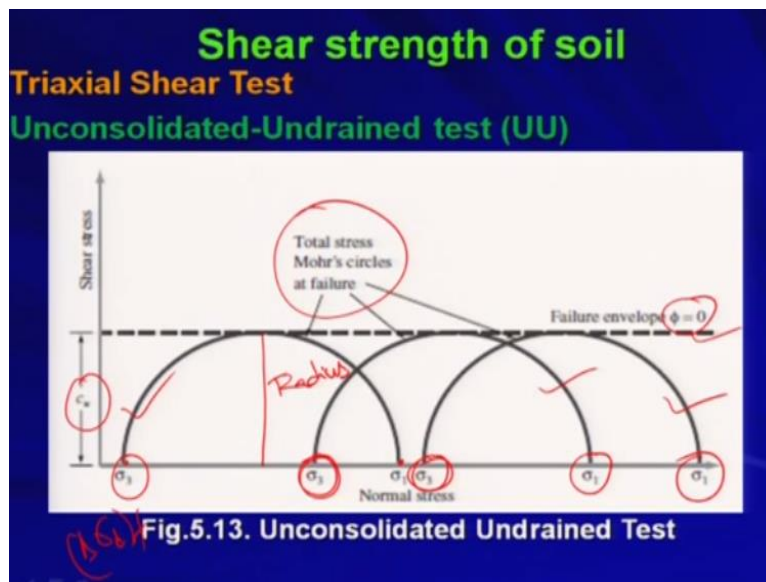
Therefore, u is nothing but $B \sigma_3 + \bar{A} \Delta \sigma_d$ which is equal to $B \sigma_3 + \bar{A} (\sigma_1 - \sigma_3)$. What is $\Delta \sigma_d$? $\Delta \sigma_d$ is the deviatoric stress that means the difference between the major principle stress and minor principle stress. So, $\sigma_1 - \sigma_3$ is coming into the picture. So, this equal 5.17 is very important. So, that I mean if you know

the pore pressure Skempton pore pressure parameters basically you can calculate the total pore pressure developed during the test okay.

So, this test is usually conducted on saturated clay and silt specimens okay. Generally, for saturated clay and silt specimens this UU test is conducted. The added axial stress at failure that is $\Delta\sigma_d$ is practically same regardless of the chamber confining pressure. So, we will see that. So, basically the thing is that $\sigma_1 - \sigma_3$ is nothing but $\Delta\sigma_d$ right agreed. $\sigma_1 - \sigma_3$ that is the difference between the major and minor principle stress is nothing but $\Delta\sigma_d$.

So, at failure basically the whatever σ_1 you are getting that is the major principle stress and whatever σ_3 , σ_3 is constant during the test right so the difference between the major and minor principle stress at failure is nothing but $\Delta\sigma_d$ and that will be remaining same okay regardless of the chamber confining pressure whatever confining pressure you consider. As you go on increasing the chamber confining pressure and automatically your σ_1 will be also increasing right at the failure. So, whatever you do your difference between these 2 will be remaining same okay. So, we will see that.

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Now this is a typical say plot for shear stress versus normal stress in case of unconsolidated undrained test. Here you can see so this is these are all total stress Mohr's circles at failure okay. So, this is I mean you are doing the test at I mean in the same specimen but at different confining pressure right. So, this is one circle this is another Mohr circle this is another Mohr circle. In

every situation, what you are doing you are just doing the test, performing the test at different confining pressure at different cell pressure right. So, if you consider this is your one cell pressure basically you are getting the failure when it is sigma 1 is the major principle stress.

Similarly, this is your modified say cell pressure. You will be getting the failure at I mean you will be getting the failure for this major principle stress whereas when your cell pressure is here you will be getting the failure for this major principle stress sigma 1 and therefore I mean in all the situations in all the case if you look at the sigma 1 - sigma 3 is nothing but your delta sigma d f right. The difference is remaining same. Therefore, you will be getting one horizontal failure envelope where phi equal to 0 and you will be getting some intercept at the y axis that is the shear stress axis and that is nothing but your cohesion okay.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

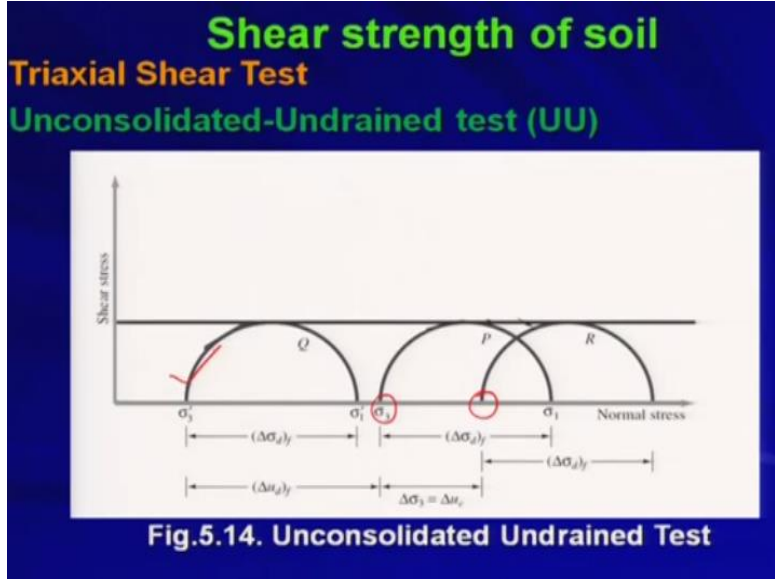
■ We get,

$$\tau_f = c = c_u \quad (5.18)$$

Where, c_u = undrained shear strength and is equal to the radius of the Mohr's circle

So, therefore we get tau f equal to c only cohesion is equal to Cu where Cu is the undrained shear strength and is equal to the radius of the Mohr circle as you have seen from this figure Cu is nothing but the radius of the Mohr circle. This is your radius okay.

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If you look at this figure basically in this figure what we are doing we are doing the UU test in a similar type of soil in 2 different specimen say one specimen is experiencing this much of cell pressure another specimen is experiencing this much of cell pressure and basically you are going to find out the effective stress Mohr circle the effective stress Mohr circle is this and we will see that your effective stress Mohr circle will be the unit whereas your total stress Mohr circle could be anything. So, if you perform say n number of test you will be getting n number of total stress Mohr circles okay at failure but you will be getting the unique effective stress Mohr circle for UU test. We will see that.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

Reason for obtaining same added axial stress

$(\Delta \sigma_d)_f$

- If a clay specimen (No. 1) is consolidated at a chamber pressure σ_3 and then sheared to failure without drainage, the total stress conditions at failure can be represented by the Mohr circle 'P'
- The pore pressure developed in the specimen at failure equal to $(\Delta u_d)_f$

Now what is the reason for obtaining same added axial stress $\Delta\sigma_d$. What is the reason? I mean already we have told that $\sigma_1 - \sigma_3$ is always becoming $\Delta\sigma_d$ and that will be regardless of whatever say confining pressure you are considering that difference between the major and minor principle stress will be always $\Delta\sigma_d$ why? What is the reason behind that?

So, if a clay specimen say No. 1, No. 1 clay specimen is consolidated at a chamber pressure σ_3 and then sheared to failure without drainage okay the total stress consolidation total stress conditions at failure can be represented by Mohr circle P right. So, what we are doing here. If a clay specimen so we are considering one soil specimen clay soil specimen that is No. 1 soil specimen is consolidated at a chamber pressure σ_3 and then sheared to failure without drainage okay that means we are doing some CU test kind of thing okay drainage the total stress conditions at failure can be represented by the Mohr circle P.

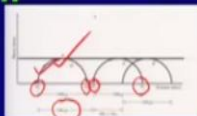
So, this is the Mohr circle which will be representing that condition okay the total stress condition. The pore pressure developed in the specimen at failure equal to Δu_d as we have discussed as we have seen in case of your consolidated undrained test right. So, there basically we consolidated the material at chamber pressure σ_3 but we did not allow the drainage during the shearing right but so at that time the pore pressure developed in the specimen at failure was equal to Δu_d agreed if u recall your CU test okay.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)



- Thus the major and minor principal effective stress at failure are

$$\sigma'_1 = [\sigma_3 + (\Delta\sigma_d)_f] - (\Delta u_d)_f = \sigma_1 - (\Delta u_d)_f \quad (5.19)$$

$$\sigma'_3 = \sigma_3 - (\Delta u_d)_f \quad (5.20)$$

- Circle Q is the effective stress Mohr circle
- Now, let us consider another similar specimen (No. 2) that has been consolidated under σ_3 with initial pore pressure equal to 0

Now thus the major and minor principle effective stress at failure are σ_1' is equal to $\sigma_3 + \Delta\sigma_d - \Delta u_d$. So, $\sigma_3 + \Delta\sigma_d$ is nothing but the total σ_1 right that is the major principle stress total major principle stress minus the pore water pressure which is getting developed at the failure. So, $\sigma_1 - \Delta u_d$ okay is your σ_1' .

Whereas σ_3' is nothing but $\sigma_3 - \Delta u_d$ agreed. Already we have discussed this. So, circle Q is the effective stress Mohr circle. So, this is the circle which will be representing the effective stress Mohr circle if you see that. So, basically you had σ_3 here now this σ_3 minus this will give you σ_3' right. $\sigma_3 - \Delta u_d$ will give you σ_3' .

Whereas this was your σ_1 . So, $\sigma_1 - \Delta u_d$ will give you σ_1' . So, this circle is basically your effective stress Mohr circle right. Now let us consider another similar specimen, similar specimen means similar type of soil similar specimen we are considering and we are denoting we are representing that specimen is No. 2 okay and for that I mean let us consider this another similar specimen that has been consolidated under σ_3 with initial pore pressure equal to 0 right. So, we can do that. So, we are doing this okay.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

- If the chamber pressure is increased by $\Delta\sigma_3$ without drainage, the pore pressure increase is equal to

$$\Delta u_c = \Delta\sigma_3 \quad (5.21)$$
- At this time effective confining pressure is,

$$\sigma_3 + \Delta\sigma_3 - \Delta u_c = \sigma_3 + \Delta\sigma_3 - \Delta\sigma_3 = \sigma_3 \quad (5.22)$$

Same as specimen No. 1

If the chamber pressure is increased by $\Delta\sigma_3$. So, at σ_3 we are applying σ_3 and then we are allowing the consolidation and we are allowing the pore pressure is becoming zero so with that situation we are starting and then what we are doing we are

increasing $\Delta \sigma_3$ that is the cell pressure we are increasing okay without considering the drainage. So, if the chamber pressure is increased by $\Delta \sigma_3$ without drainage the pore pressure increase is equal to Δu_c and that is happening actually during the first step right when you are dealing with the only cell pressure or the confining pressure.

So, Δu_c is equal to $\Delta \sigma_3$ because whatever additional I mean cell pressure you are applying the same amount will be transferred to the pore water because you are not allowing the drainage. So, soil particles will not take care of any kind of load extra load right. So, everything will be taken care of by the water itself pore free. So, Δu_c is equal to $\Delta \sigma_3$. So, at this time effective confining pressure is $\sigma_3 + \Delta \sigma_3 - \Delta u_c$ right.

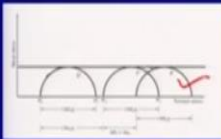
So, what is your total confining pressure $\sigma_3 + \Delta \sigma_3$ and what is your effective confining pressure $\Delta \sigma_3 + \sigma_3 - \Delta \sigma_3$ minus the pore water pressure so pore water pressure is how much, Δu_c . Again, Δu_c is nothing but equal to $\Delta \sigma_3$. So, you will be getting σ_3 right. So, at this time your effective confining pressure is nothing but your σ_3 right. So, same as specimen No. 1 okay.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

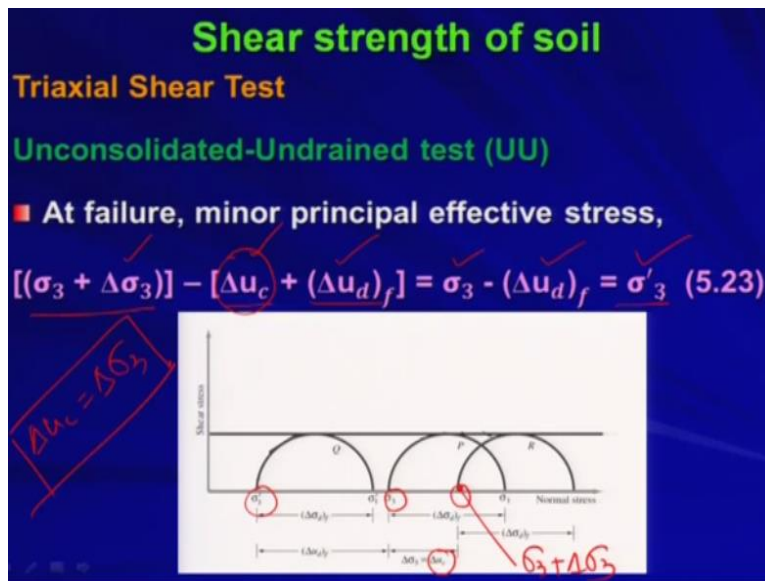


- Hence, if the specimen No. 2 is sheared without drainage, it should fail at the same deviator stress that was obtained for specimen No. 1
- The total stress Mohr circle will be R

Okay so in case of this thing whatever we have discussed so hence if the specimen No. 2 is sheared without drainage so basically now you are getting the same kind of condition that is the in case of previous slide if you look at the effective confining pressure was σ_3 right and which was same as for specimen No. 1.

So, therefore if the specimen No. 2 is sheared without drainage right without drainage we are shearing it, it should fail at the same deviatoric stress because I mean basically what you are doing you are taking different samples from the same soil. So, it will exhibit similar kind of nature. So, it should fail, the failure should happen at the same deviatoric stress that was obtained for specimen No. 1. So, the total stress Mohr circle will be R. So, this is your total stress Mohr circle for the specimen No. 2.

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So, at failure minor principle effective stress was $\sigma_3 + \Delta\sigma_3$ that was the enhanced confining pressure - $u_c - u_d f$ so why minus, minus Δu_c , so why minus Δu_c ? So, that means this part is getting developed when you increase the confining pressure from σ_3 to $\Delta\sigma_3$ right. You did not allow the drainage so it has been developed okay.

Now you did not allow the drainage during shearing also. So, therefore at failure $\Delta u_d f$ has been developed right. So, ultimately you will be getting so this Δu_c is nothing but $\Delta\sigma_3$ right already we have seen. So, you are getting $\sigma_3 - \Delta u_d f$ which is nothing but your σ'_3 right. So, if you look at this figure basically this is your σ_3 right.

So, you are allowing the new σ_3 that is nothing but so this point is basically $\sigma_3 + \Delta\sigma_3$ and due to that you are getting some pore water pressure generation Δu_c okay. Now I mean once you do this so minor principle effective stress is becoming that is your total what is this, this is your total minor principle stress. So, total minor principle stress minus your

delta u c minus delta u d f will be giving you the effective minor principle stress right. So, that is becoming sigma 3 prime.

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Shear strength of soil
Triaxial Shear Test
Unconsolidated-Undrained test (UU)

■ Major principal effective stress

$$[(\sigma_3 + \Delta\sigma_3) + (\Delta\sigma_d)_f] - [\Delta u_c + (\Delta u_d)_f]$$

$$= [\sigma_3 + (\Delta\sigma_d)_f] - (\Delta u_d)_f$$

$$= \sigma_1 - (\Delta u_d)_f$$

$$= \sigma'_1 \quad (5.24)$$

Similarly the major principle effective stress the major principle effective stress is sigma 3 + delta sigma 3 that is the cell pressure all-round cell pressure plus the deviatoric stress sigma d f at failure minus the pore water pressure that is delta u c which is happening during the increase of delta sigma 3 and delta u d f that is the pore water pressure which is happening during shearing. So, again this these 2 are same right. So, sigma 3 + delta sigma d f - u d f which is equal to this what is this this is your sigma 1.

So, sigma 1 - delta u d f which is nothing but sigma 1 prime. So, you are getting this. So, now ultimately you are here. Now your effective major principle stress is sigma 1 prime. So, whatever you do whatever you are doing I mean by increasing the cell pressure you are getting 2 different total stress Mohr circles right P and R but for both the cases your effective stress Mohr circle is same that is Q right. That we I mean I meant to I mean say I mean at the starting that means that is the reason that is why you are getting the same.

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Shear strength of soil

Triaxial Shear Test

Unconsolidated-Undrained test (UU)

- Thus the effective Mohr circle will still be Q, because strength is a function of effective stress
- Note that the dimension of P, Q & R are all the same

Thus, the effective Mohr circle will still be Q because strength is a function of effective stress okay. No matter whatever you do your total stress Mohr circle will be I mean n number of I mean confining pressure you change n number of total stress Mohr circle you will be getting but your effective stress Mohr circle will be unique. So, your effective stress Mohr circle will be more I mean logical to consider during design.

So, note that the dimension of P, Q, R are all the same. So, dimensions are not different right. So, the radius of the Mohr circle P, radius of the Mohr circle Q, radius of the Mohr circle R, all are same okay. So, therefore they are remaining same. Only thing is that you are getting different total stress Mohr circles and for that corresponding unique I mean effective stress Mohr circles will be unique okay. Thank you very much.

So, I will stop here today. So, in the next lecture we will be considering different kinds of tests and different kinds of issues related to the shear strength chapter. Thank you very much.