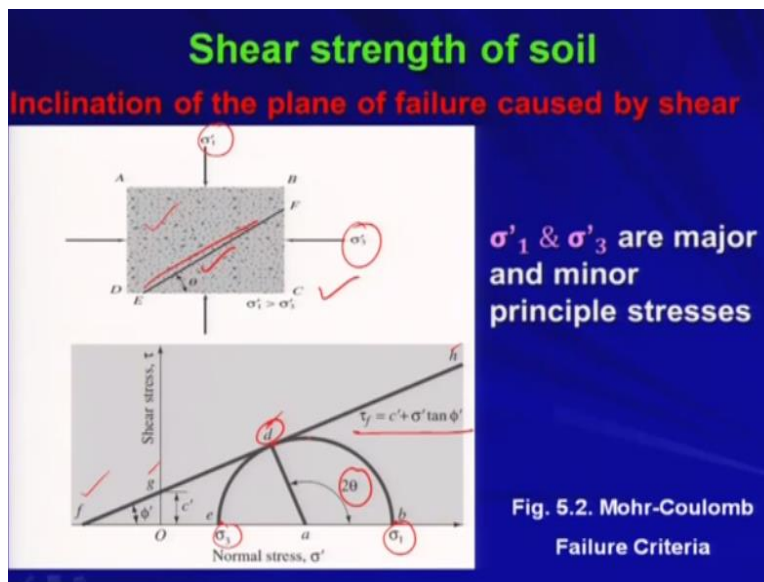


Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture – 39
Shear Strength of Soil - B

Welcome back. So, in the last lecture we stopped here. So, basically, we were discussing about the Mohr-Coulomb Failure Criteria. So, there we had seen that the failure is not happening at the maximum normal stress or at the maximum shear stress rather failure is happening some critical combination of shear stress and normal stress which is nothing but this point d.

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So, which is nothing but this point d. So, point d is telling about the failure right. So, therefore the angle I mean if you recall your Mohr circle representation so basically the angle between the major principle stress and the failure plane so therefore the failure plane I mean if you consider the pole method the failure plane will be making an angle say theta because 2 theta is the angle in the stress field that is nothing but in the Mohr circle representation.

So, if you talk about the physical plane, so this is your physical plane. In that physical plane, basically that will be I mean coming as theta only right half of that angle. So, 2 theta is the angle between the major principle plane and the failure plane right because this is your failure point this is your major principle stress point. So, this is the angle, 2 theta is the angle between major principle plane and the failure plane and that is shown in the physical plane like this theta.

So, theta is the angle between the so this EF if your failure plane right which is making an angle theta with the major principle plane okay. So, I hope you have understood this concept. So, now what whatever you have understood now so that thing we are going to represent in the mathematical expression.

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Shear strength of soil

Inclination of the plane of failure caused by shear

- The radial line ab defines the major principle plane, CD & the radial line ad defines the failure plane, EF
- It can be shown that,

$$\angle bad = 2\theta = 90^\circ + \phi'$$

Or, $\theta = 45^\circ + (\phi'/2)$ (5.4)

Again $\frac{ad}{\epsilon} = \sin \phi'$ (5.5)

Now the radial line ab so this radial line ab defines the major principle plane agreed? So, I mean if you recall your Mohr-Coulomb Mohr circle representation whatever you have learnt in your say mechanics right. So, the radial line ab defines the major principle plane CD okay CD means whatever we have seen in the physical plane and the radial line ad so this is the radial line and this radial line ad defines the failure plane EF okay agreed?

Okay, now it can be shown that angle bad , angle bad , so bad angle bad is equal to 2θ which is nothing but equal to 90 degree plus ϕ prime. Now why it is 90 degree plus ϕ prime because this angle is 90 degree. This is the radial line and this is the tangential line. So, this is this failure envelope is the tangential line.

So, therefore this angle is 90 degree. So, this angle is 90 degree, this angle is say ϕ prime so therefore this angle will be 90 degree plus ϕ prime which is nothing but equal to 2θ so this we have got. Now from this I can write θ is equal to 45 degree plus ϕ prime by 2 right ϕ prime by 2 right. Now what we are getting here? Now ad by fa right so again ad by fa is equal to $\sin \phi$ prime right. So, ad by fa so this by this because this is a I mean this angle is 90 degree right so ad by fa is equal to $\sin \phi$ so this angle is ϕ .

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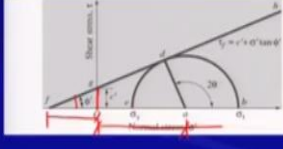
Shear strength of soil

Inclination of the plane of failure caused by shear

$$\overline{fa} = \overline{fO} + \overline{Oa} = c' \cot \phi' + \frac{\sigma'_1 + \sigma'_3}{2} \quad (5.6)$$

$$\text{And, } \overline{ad} = \frac{\sigma'_1 - \sigma'_3}{2} \quad (5.7)$$

$$\sin \phi' = \frac{\frac{\sigma'_1 - \sigma'_3}{2}}{c' \cot \phi' + \frac{\sigma'_1 + \sigma'_3}{2}}$$

$$\text{Or, } \sigma'_1 = \sigma'_3 \left(\frac{1 + \sin \phi'}{1 - \sin \phi'} \right) + 2c' \left(\frac{\cos \phi'}{1 - \sin \phi'} \right) \quad (5.8)$$


Now f_a is equal to now you see this f_a is equal to fO plus fO plus Oa okay. So, fO plus Oa is equal to what is fO ? fO is nothing but $c' \cot \phi'$. So, this angle is ϕ' this is c' prime okay so $c' \cot \phi'$ plus $\sigma'_1 + \sigma'_3$ by 2. What is σ'_3 ? This is your σ'_3 , from Oe is your σ'_3 . Similarly, Ob is your σ'_1 . So, $\sigma'_1 + \sigma'_3$ by 2 which will be nothing but Oa agreed? Okay.

Now again we can write ad what is ad ? ad is nothing but the radius of the Mohr circle. That is nothing but $\sigma'_1 - \sigma'_3$ by 2 okay. So, $\sin \phi'$ the radius of the Mohr circle will be σ'_1 that is the major principle stress minus the minor principle stress divided by 2 that will be your radius. Now once you know all those things like f_a , ad and all those things so you put in the expression of $\sin \phi'$ so $\sin \phi'$ whatever what $\sin \phi'$ is $\sin \phi'$ is equal to ad divided by f_a . So, in place of f_a I can write this.

In place of ad I can write this so that comes like this okay. Now from this if you try to simplify this thing this equation then you will be getting σ'_1 is equal to $\sigma'_3 \frac{1 + \sin \phi'}{1 - \sin \phi'} + 2c' \frac{\cos \phi'}{1 - \sin \phi'}$. I am not talking about the prime all the times because prime is written so it is quite understood that this parameters are effective parameter. So, all the times I am not talking about the prime, σ' , c' , ϕ' . So, I am skipping those things. However, that is already accepted that this parameters are effective parameters anyway. So, equation 5.8 is giving me the relation between the major

principle stress and the minor principle stress along with the c that is the cohesion and phi. That is the angle of internal friction.

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Shear strength of soil

Inclination of the plane of failure caused by shear

Or, $\frac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2[45^\circ + (\phi'/2)] = \tan^2\alpha'$

& $\frac{\cos\phi'}{1 - \sin\phi'} = \tan[45^\circ + (\phi'/2)] = \tan\alpha'$

Therefore, $\sigma'_1 = \sigma'_3 \tan^2\alpha' + 2c' \tan\alpha'$ (5.9)

So, now $1 + \sin\phi'$ by $1 - \sin\phi'$ mathematically I can express $\tan^2 45^\circ + \phi'$ by 2 which is nothing but equal $\tan^2 \alpha'$. So, α' stands for $45^\circ + \phi'/2$ okay. In place of $45^\circ + \phi'/2$ I am writing α' okay. So, that is nothing but $\tan^2 \alpha'$. Similarly, $\cos\phi'$ by $1 - \sin\phi'$ I can write mathematically that is nothing but $\tan 45^\circ + \phi'/2$ which is nothing but $\tan \alpha'$.

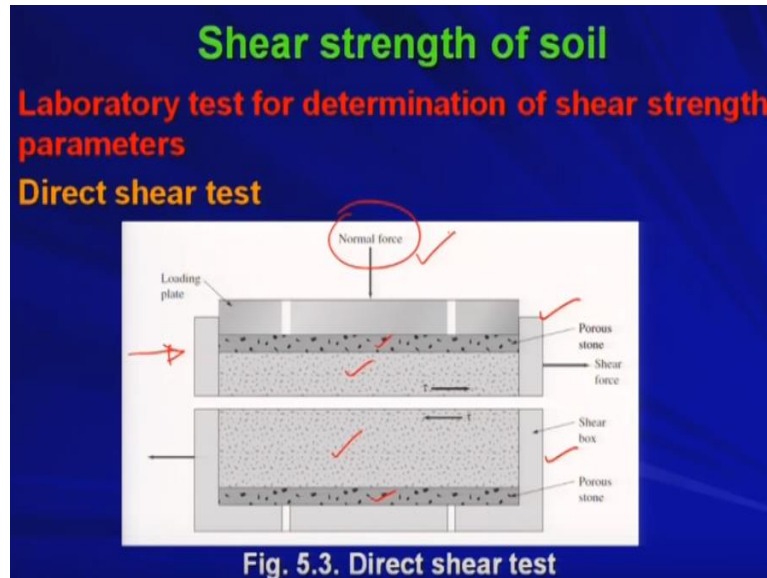
Therefore, the relation whatever we have got in equation 5.8 becomes σ_1 is equal to $\sigma_3 \tan^2 \alpha' + 2c \tan \alpha'$. So, this is the relation. So, this is your Mohr-Coulomb Failure Criteria. So, Mohr-Coulomb Failure Criteria says that is nothing but $\sigma_1 + \sigma_3 \tan^2 \alpha'$ is equal to $\sigma_3 \tan^2 \alpha' + 2c \tan \alpha'$.

Now we will be talking about the laboratory experiments available to determine these parameters okay like c and ϕ' if you so therefore what you get from this equation. So, if you know c that is the cohesion and the angle of internal friction ϕ' so if you know these 2 parameters basically you can calculate and of course if you know the σ_1 or σ_3 you can calculate the other parameters.

At the same time if you know c and ϕ' you can establish the Mohr-Coulomb Failure Criteria and then whatever stress you are choosing you can say whether this stress is coming below the failure criteria or coming just I mean on the failure I mean line or it is not feasible. So, these kind

of say I mean concept or this kind of say idea you will be getting from this once you establish the failure criteria.

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Now the laboratory test, the laboratory test for determination of shear strength parameters, so first we will discuss about direct shear test. So, what we do here. So, basically here you will be having 2 halves so this is the top half, this is the bottom half of the shear box. So, this is known as shear box okay. Now you will fill the sand so this is filled with sand. You will fill the sand in that particular mould. So, this is a kind of mould which will be having 2 halves.

So, this is your top half, this is your bottom half, and this is the mould. Inside the mould you will be filling the sand for which I mean not sand for any kind of material for which you want to find out the shear strength parameters like c and ϕ . So, if I talk shear strength parameters that means c and ϕ will be counted as shear strength parameters okay. So, I want to find out the parameters. Now on top of the sand porous stone to get the pore water pressure dissipated from the soil body.

Similarly, you will put another say porous stone at the bottom the function is same. It will try to dissipate the water from the soil body okay. So, there is no I mean the things will be completely effective right. So, because you are not allowing any kind of pore water pressure building up in the soil matrix okay. Now what you are applying what you are doing so these are 2 porous stones.

Now you are applying normal force okay by some means by some say mechanism you are applying some normal force on the which will be which will be developing the normal stress right because this normal force you are once you are applying so along that plane basically or any plane if you consider along that plane if you divide the plane area then you will be getting the normal stress acting on that particular plane okay. Now you are shearing.

Now you are applying some shear movement along this direction and you the one top half will be moving okay with respect to the bottom half. So, basically this is something like this is the mould. Now so one half is allowed to move over the other half okay so basically you will be getting this kind of movement that means you are shearing you are shearing the soil. So, the shearing is happening at that plane and slowly you will be getting some shear force okay or the shear stress is getting developed along that plane and at certain point you will be getting the failure.

So, that means the combination of normal stress due to this I mean so this is fixed for the whole experiment. So, you are applying one normal force and you are executing the complete shearing that means complete failure so that means you are getting the combination of sigma and tau for a failure. Then similarly you will be increasing the normal force okay and then again you will shear it you will go to the failure then you will be getting another combination of sigma and tau for which you are getting the failure and so on you will be going on.

So, you will be increasing sigma and you will be finding out the failure I mean shear stress required for failure right. So, in that way you will be getting different combinations of sigma and tau and then you plot that thing and then you I mean this points basically this combination if you plot in sigma and tau space basically these points will be coming in sigma and tau space and then if you join that thing with a straight line that will be giving you the I mean failure envelope Mohr-Coulomb failure envelope right. So, the idea is like that. So, once you know the Mohr-Coulomb failure criteria or the failure envelope from there you can find out c and phi okay. So, now let us see how we can extract the or how we can represent interpret the results obtained from this direct shear test.

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Shear strength of soil

Direct shear test

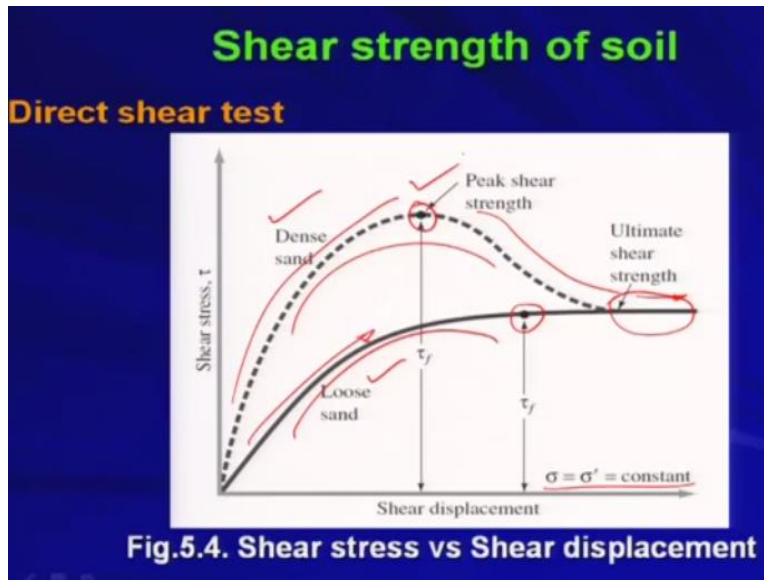
- The soil specimen may be square or circular in plan
- The size of specimen
 - 51mm x 51mm x 25mm
 - 102mm x 102mm x 25mm
- Shear force is applied by moving one-half of the box relative to the other
- The shear test can be either stress controlled or strain controlled

The soil specimen may be square or circular in plan. So, the box whatever mould I told you that mould could be in could be circular or square in cross section. The size of the specimen is 51 mm by 51 mm by 25 mm. I mean 2 kinds I mean 2 types or 2 sizes are available. The first one is 51 by 51 by 25 mm. Another one is 102 by 102 by 25 mm okay. So, sometimes you need the smaller specimen.

Sometimes I mean depending on the soil gradation you need say bigger size or bigger mould. Now shear force is applied by moving one-half of the box related to the other as I told you right. The bottom part will be moving and the top I mean bottom part bottom part will be there and the top part will be there so basically you will be getting the relative movement. So, one part will be fixed. Another part will be shearing.

So, I mean another part will be sliding over the other part. So, in that way basically you will be getting the shear force which will be developed at the interface or at the plane of the separation. The shear test can be either stress controlled or strain controlled. Now what do you understand by stress controlled and strain controlled. Now we will come to that point.

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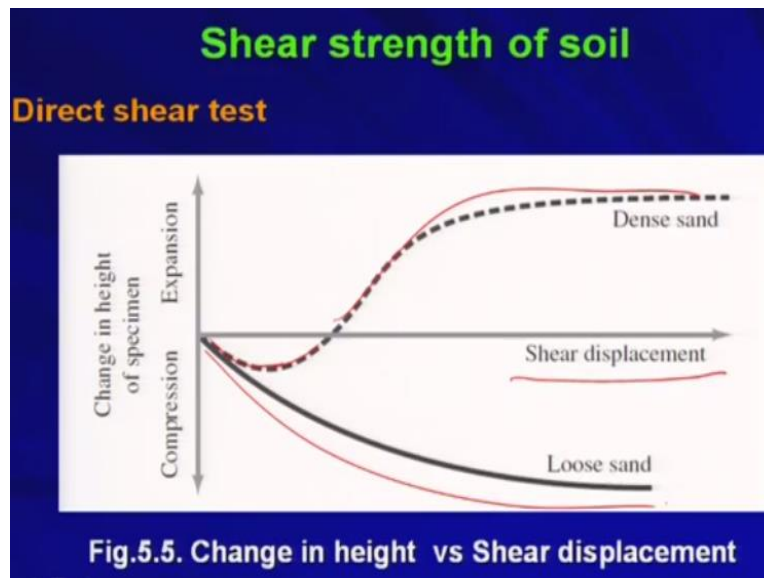
Now before that if you use dense sand and if you use loose sand. Now at this point you are quite familiar to this these terms dense sand loose sand. It depends on the relative density right. So, already you we covered this thing. So, based on that we can say okay this is my dense sand this is my loose sand. Now if you try to find out the shear stress versus shear displacement relation in dense sand and loose sand basically you will be getting the relation like that for dense sand it is first it will increase it will reach the peak and then it will fall and it will be becoming constant almost constant right.

Whereas in case of loose sand it will be monotonously or continuously increasing and it will reach the I mean constant path or the ultimate path. So, basically at the ultimate state okay both dense sand and loose sand is giving you the same shear strength okay. However, the peak shear strength for dense sand is much more higher than the peak shear strength of the loose sand okay. So, another thing you can conclude from this the peak shear strength and the ultimate shear strength both are almost same for the loose sand whereas the peak shear strength is much more higher than the ultimate shear strength for dense sand right.

So, that you are I mean this is for a constant normal I mean test so that means what you are doing basically. You are applying some normal force as I told you and you keep that thing constant and then you are gradually shearing. So, in the process of shearing you can develop this kind of this curves you are getting developed when you are shearing. So, gradually the shear resistance or the shear stress is increasing with the shear displacement and it will be reaching I mean reaching at

the peak I mean that is for dense sand and then it will fall down right. Similarly, for loose sand it will be continuously increasing and it will be becoming constant right okay.

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Now what is happening in case of change in height of specimen that means so you have the height of the specimen as 25 mm right as you have seen that is the mould depth right. So, change in height of the specimen so that is happening because of I mean this 2 types of sand you are using dense sand and loose sand. So, as you progress the shear displacement as you are applying the shear force that means gradually the shear displacement is happening okay in the mould.

So, basically the dense sand will exhibit some amount of reduction or the compression in the volume initially and I mean that means the material will be compressed and then it will dilate it will expand and this expansion is shown like that. Whereas in case of loose sand it is always compression. That means the volume whatever volume you consider initially as you are shearing it the volume will be getting reduced. Whereas in case of dense sand initially it will be getting reduced and then it will be increased right. So, that is happening in dense sand. So, this with shear displacement this is the change in height of the specimen happening in the experiment.

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Shear strength of soil

Direct shear test

- In stress controlled tests, the shear force is applied in equal increments until the specimen fails
- In strain controlled tests, a constant rate of shear displacement is applied to one-half of the shear box
- The advantage of strain controlled test is that in the case of dense sand, peak shear resistance i.e. at failure as well as lesser shear resistance i.e. at a point after failure called ultimate strength can be observed and plotted

Now as we told about stress control test or the strength control now what are these tests? Now in stress control test the shear force is applied in equal increments until the specimen fails okay. So, what you are what you are doing here? So, you are in case of stress controlled the I mean your normal stress is constant right. Now you are applying say Δ amount of shear stress and you are waiting for some time.

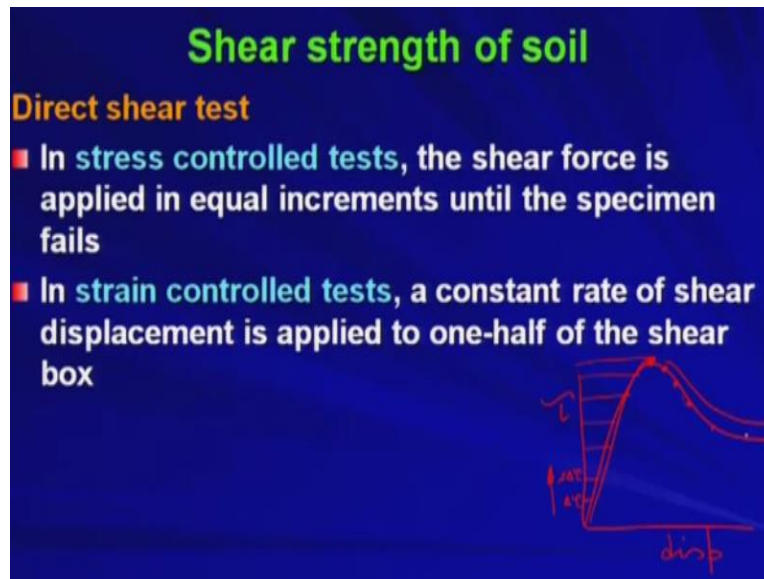
Then you are increasing to Δ amount of shear stress then you are waiting then 3Δ then 4Δ and so on you are going on increasing. That means you are controlling the stress right. So, first you have applied Δ amount $\Delta\tau$. Then say $2\Delta\tau$ then $3\Delta\tau$ $4\Delta\tau$ and gradually you are increasing the shear stress and ultimately you will be getting the failure right. So, that is known as stress controlled.

So, here actually you are controlling the stress. Whereas in case of strain controlled test a constant rate of shear displacement is applied to one-half to the shear box. That means here actually what you are doing. You are applying constant that means your shear displacement that is the rate because you are applying the shear movement right, shear displacement. That rate of shear displacement is constant.

So, at the constant rate you are shearing the material. That means you are controlling the strain. That means the shear displacement you are controlling. You are not controlling the shear stress okay and you are going to the failure. So, this 2 things are happening. I mean you can have 2 options. Either you go for stress control test or strain control test depending on the requirement okay.

Now the advantage of strain controlled test is that in the case of dense sand peak shear resistance that is at failure as well as lesser shear resistance that is at a point after failure called ultimate strength can be observed and plotted. Now what does it mean right.

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Now if you look at the, this is your shear stress versus your displacement, shear displacement you plotted right and in case of dense sand you got like that right do you remember just now we have seen. Now in case of stress controlled test what you are controlling you are controlling stress. Now you are controlling stress means first you are applying $\Delta\sigma$ $\Delta\tau$ say okay. Then in the next increment you are going for $2\Delta\tau$ and gradually you are increasing okay and you are reaching say this peak okay.

So, what you are going what you are doing you are gradually increasing the shear stress that is the stress controlled test you are performing okay. In the stress, controlled test once you reach the peak then further this part from this point to this point this part you cannot capture because this part cannot happen with the increase of stress. Have you understood this thing? So, this rising part you can capture by increasing the stress with similar increment or the same increment that means you are controlling the stress and you are getting that.

Once you reach the peak this falling part you cannot control with the increase in stress right. So, now this point from this point to this point you cannot increase the stress rather stress is decreasing right. Strain is increasing right. The displacement is going on increasing but your

stress is decreasing. So, this falling part you cannot you will not be able to capture by stress controlled test.

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Shear strength of soil

Direct shear test

- In stress controlled tests, the shear force is applied in equal increments until the specimen fails
- In strain controlled tests, a constant rate of shear displacement is applied to one-half of the shear box
- The advantage of strain controlled test is that in the case of dense sand, peak shear resistance i.e. at failure as well as lesser shear resistance i.e. at a point after failure called ultimate strength can be observed and plotted

The slide also features a red hand-drawn diagram of a stress-strain curve. The curve starts at the origin, rises to a peak (labeled 'peak shear resistance'), and then descends to a lower level (labeled 'ultimate strength'). The peak is marked with a red arrow pointing to it, and the ultimate strength is also marked with a red arrow. The curve is drawn on a grid.

Whereas the same thing if you do that the same thing if you do that with strain controlled test now what is happening you see. In strain controlled test you are controlling the displacement. The rate of displacement is constant. So, this is your constant displacement you are considering. You are going on right so continuously you are increasing the displacement right there is no issue. So, if you continuously increasing increase the displacement you will be getting you will be able to capture the whole curve right whatever was not possible in case of stress controlled test.

In case of strain controlled test now you are able to capture the whole plot or the whole I mean say portion right of the stress strength curve right. That is why this is written. The advantage of strain controlled test is that in the case of dense sand, peak shear resistance that is at failure, that is there, as well as the lesser shear resistance at a point after failure that is called as ultimate that is here right, can be observed and plotted in case of strain controlled but in case of stress controlled you cannot go, this part is not possible for your stress controlled test okay.

So, I will stop here today. So, in the next lecture we will be saying or we will be discussing the direct shear test in more detail and we will be finding out the equations okay applicable to the direct shear test and then we will move further with different other kind of say laboratory tests. Thank you very much.

