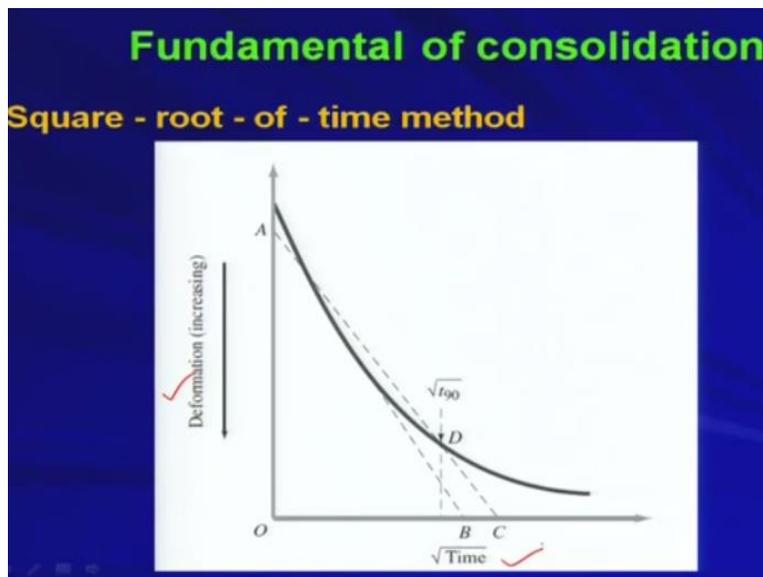


**Geology and Soil Mechanics**  
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**Lecture - 35**  
**Consolidation - E**

Welcome back. So, in the last lecture we just saw how to calculate the  $C_v$  that is the coefficient of consolidation by using logarithmic time method. So, now today in this lecture we will be seeing another method by which you can also calculate  $C_v$ . So, that is nothing but known as square root of time method.

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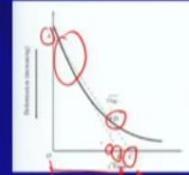


So, now what we do here? So, in square root of time method basically we have the same thing that is deformation versus root over time okay. So, we that is why the name is square root of time method. So, we plot the I mean consolidation plot in deformation versus root over time space okay. So, root over time along the x axis.

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## Fundamental of consolidation

### Square - root - of - time method



1. Draw a line **AB** through the early portion of the curve
2. Draw a line **AC** such that  $\overline{OC} = 1.15 \overline{OB}$ . The abscissa of point **D**, which is the intersection of **AC** and the consolidation curve, gives the square root of time for 90% consolidation  $\sqrt{t_{90}}$

Now basically what is what we are doing here? So, first I mean step is that draw a line AB through the early portion of the curve. So, draw a line AB so this is the line AB. So, this is A this is B so this line we are drawing that is a straight line we are drawing. So, draw a line AB through the early portion of the curve. So, early portion means this is the say early portion. So, along that early portion if we it is tangent or the asymptotic so this AB is nothing but the line which is protruding out from the early portion of the curve consolidation curve okay.

Now draw a line AC such that OC is always equal to 1.15 of OB. That means OC is 15% more than OB right. So, this, what is OB? This is your OB and this is your OC right. C is here. So, OC is always we are drawing another line AC starting from point A such that OC is always equal to 1.15 of OB fine there is no issue I mean very simple. The abscissa of point D right, the abscissa of point D, what is point D? Which is the intersection of AC and the consolidation curve okay.

So, you have drawn line AC and it is going to intersect the consolidation curve at point D right. So, we are picking up that point D so the abscissa of point D is basically gives the square root of time for 90% consolidation okay. That means I will be getting root over t 90 from the abscissa of point D on the times time I mean axis right. So, once we establish point D immediately you drop a line, vertical line point D, so that you get the square root of t 90. So, once you get square root of t 90 then basically the procedure is same.

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## Fundamental of consolidation

### Square - root - of - time method

3. For 90% consolidation,  $T_{90} = 0.848$  [Eq. (4.36)]

$$T_{90} = 0.848 = \frac{C_v t_{90}}{H_{dr}^2}$$

$$\text{Or, } C_v = \frac{0.848 H_{dr}^2}{t_{90}} \quad (4.38)$$

Now for 90% consolidation your  $T_{90}$  is equal to 0.848 from equation 4.36 because your degree of consolidation, average degree of consolidation is 90% which is greater than 60%. So, you have got equation 4.36 to calculate the time factor  $T_v$  right. So,  $T_{90}$  is equal I mean you can calculate that and you can see  $T_{90}$  will be equal to 0.848. So, once you know the time factor at 90 degree 90% consolidation and that is nothing but 0.848.

So,  $C_v$  is not known to you  $T_{90}$  just now you have got from the procedure whatever steps you have just I mean used to obtain  $T_{90}$  okay. So, you have got root over  $T_{90}$  so you can get  $T_{90}$  and of course this is your drainage path. So, I mean it will be remaining same whatever we have considered just in the next I mean the previous class that I mean if you have the double drainage system I mean the drainage path will be the half of the thickness of the specimen.

If you have the single drainage system then you will be having the drainage path will be the total thickness of the specimen. So, from this expression I can calculate  $C_v$  equal to  $0.848 H_{dr}^2$  square divided by  $T_{90}$  right. So, equation 4.38 as well as in the previous class we have seen previous lecture we have seen how to calculate  $C_v$  from logarithmic time method. So, any method you can use to obtain or to find out  $C_v$  okay. So, now we will take couple of numerical problems okay on this consolidation whatever we have learnt so far. So, let us see the problem first okay.

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### Problem-16

- Two soil samples A and B, each 30 cm thick, were tested in a consolidometer. The initial void ratios of A and B under a pressure of 1 kg/cm<sup>2</sup> are 0.6 and 0.65 respectively. They decrease to 0.505 and 0.605 when the pressure is increased to 2 kg/cm<sup>2</sup>. To reach a 50% degree of consolidation the time taken by sample A is  $\frac{1}{4}$  of that required by sample B. Compute
  - $m_v$  for both the samples
  - The ratio of coefficient of permeability of the two samples

The problem says 2 soil samples A and B each 30 cm thick were tested in a consolidometer. The initial void ratios of A and B under a pressure of 1 kg/cm square are 0.6 and 0.65 okay that is the initial void ratio and respectively. So, they decrease to 0.505 and 0.605 when the pressure is increased to 2 kg/cm square okay. Now to reach a 50% degree, 50% degree of consolidation the time taken by sample A is one fourth of that of that required by sample B okay.

So, to reach 50% consolidation that means the time taken by A will be one fourth the time taken by sample B to obtain 50% degree of consolidation. Now you need to compute  $m_v$  coefficient of compressibility  $m_v$  for both the samples and the ratio of coefficient of permeability of the 2 samples okay. So, you need to find out the ratio of coefficient of permeability. Now let us let us start this problem okay. Let us see how to find out these things from this problem.

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$$\begin{aligned}
 (m_v)_A &= \frac{a_v}{1+e_0} = \left( \frac{\Delta e}{\Delta \sigma} \cdot \frac{1}{1+e_0} \right)_{\text{of } A} \\
 &= \frac{(0.6-0.505)}{(2-1)} \cdot \frac{1}{1+0.6} \\
 &= 0.059 \text{ cm}^2/\text{kg}
 \end{aligned}$$

So, problem 16, so basically you are calculating  $m_v$  of sample A. So, what was the expression of  $m_v$  could you remember, could you recall? The expression of  $m_v$  was  $a_v$  by  $1 + e_{\text{not}}$  where  $e_{\text{not}}$  is the, what is  $e_{\text{not}}$ ?  $e_{\text{not}}$  is the initial void ratio. Now again you can write what is  $a_v$ ?  $a_v$  was  $\Delta e$  by  $\Delta \sigma$  that is the ratio of change in void ratio to the change in I mean pressure right change in pressure. So, into  $1 + e_{\text{not}}$ . So, of A okay.

So, now let us identify this parameter. I mean do you know these parameters by the way? So, let us see that. In the problem, what is  $\Delta e$ . if you go back to the problem  $\Delta e$  is said as so initial void ratio of sample A was 0.6 when your pressure was 1 kg/cm square. Now if you increase the pressure to 2 kg/cm square then the void ratio decreases to 0.505. That means the  $\Delta e$  is nothing but  $0.6 - 0.505$  right.

Now what is your  $\Delta \sigma$ ? That is the change in pressure right. So, what is the change in pressure? Earlier it was 1 kg/cm square and this change in void ratio is happening by increasing the pressure by an amount I mean 2 kg/cm square. So, I mean so basically you are getting that is your 2 that is the final stress increment I mean total stress minus the initial pressure. So, that is your  $\Delta \sigma$  right into  $1 + e_{\text{not}}$ . What was  $e_{\text{not}}$ ?  $e_{\text{not}}$  was 0.6 that is the initial void ratio for sample A. So, if you calculate this thing you will be getting 0.059 centimeter square per kg. So, this is your  $m_v$  for sample A okay. Now similarly I can find out the  $m_v$  for sample B. How can I find out that?

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$$\begin{aligned}
 (m_v)_{4B} &= \frac{\Delta e}{\Delta \sigma} \cdot \frac{1}{1+e_0} \\
 &= \frac{(0.65-0.605)}{(2-1)} \cdot \frac{1}{1+0.65} = 0.027 \text{ cm}^2/\text{kg}
 \end{aligned}$$

So,  $m_v$  of B is equal to the same thing  $\Delta e$  by  $\Delta \sigma$  into  $1$  plus  $e$  not. What is the value of  $\Delta \sigma$  for sample B? If you go back to the problem it is said that  $e$  not was  $0.65$  for B and it comes down to  $0.605$  okay with the increase in pressure. So, your  $\Delta e$  is equal to  $0.65 - 0.605$  divided by the same increment of pressure is happening for sample B also into  $1$  plus  $e$  not,  $e$  not is  $0.65$ .

So, if you put the if you calculate these values you will be getting  $0.027$  centimeter square per kg okay. So, the first part has been obtained. That is  $m_v$  for both the sample. So, you have got  $m_v$  for sample A is nothing but  $0.059$  centimeter square per kg whereas  $m_v$  for sample B is  $0.027$  centimeter square per kg okay. So, now coming to the second part of this problem.

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P-16  
 Since both samples reach the same degree of consolidation, they have the same value of  $T_v$

$$\frac{(C_v)_A}{(C_v)_B} = \frac{t_B}{t_A} \cdot \frac{(H_{dv})_A^2}{(H_{dv})_B^2}$$

$$T_v = \frac{C_v t}{H_{dv}^2}$$

$$(T_v)_A = (T_v)_B$$

Now since both samples reach the same degree of consolidation they have the same value of time factor  $T_v$  agreed? Because you can calculate if you once you know the degree of consolidation average degree of consolidation then you can find out  $T_v$  by equation by the equations whatever we have obtained so if your  $U$  that is the average degree of consolidation is 0 to 60% you have got you have got one equation or expression for calculating  $T_v$  and if it if  $U$  is greater than 60% then you have got another expression for calculating  $T_v$  anyway.

So, if I talk about same degree of consolidation for both the sample that means it is in the problem it is given the 50% degree of consolidation okay that is the matter of concern. So, if both the samples are experiencing the same degree of consolidation or I mean if we are concerned for same degree of consolidation for both the samples then of course our  $T_v$  will be same right from the equation whatever we have seen in the last lecture anyway.

So, we can write  $C_v A$  by  $C_v$  what is  $C_v$  coefficient of consolidation is equal to  $t B$  that is the time by  $t A$  into  $H dr$  square for A,  $H dr$  square for B right. I hope that you remember  $T_v$  is equal to  $C_v t$  by  $H dr$  square. So, that was the expression we have derived earlier in the lecture right. So, now what we can get? So,  $C_v$  by A that is the ratio we are calculating. So,  $C_v A$  is the  $C_v$  value of for A sample and  $C_v$  value for B sample is  $C_v B$  is nothing but equal to from this expression basically because your  $T_v A$  and  $T_v B$  both are same right because you are thinking about the same degree of consolidation okay.

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The image shows a green chalkboard with handwritten mathematical derivations. At the top, the ratio of consolidation coefficients is calculated as follows:

$$\frac{(C_v)_A}{(C_v)_B} = \frac{4}{1} \times \frac{(30/2)^2}{(30/2)^2} = 4$$

Below this, the word "Therefore," is written. To the right, a boxed formula defines the coefficient of consolidation:

$$C_v = \frac{k}{m_v \gamma_w}$$

Using this definition, the ratio of consolidation coefficients is further derived as:

$$\frac{k_A}{k_B} = \frac{(C_v)_A (m_v)_A \gamma_w}{(C_v)_B (m_v)_B \gamma_w}$$

$$= \frac{4 \times 0.059}{0.027} = 8.71$$

So, from this we can write so  $C_v A$  by  $C_v B$  is equal to now what is  $t_B$  by  $t_A$ . So, that is given in the problem. So, if you see the problem to reach a 50% degree of consolidation the time taken by sample A is one fourth of that required by sample B. So,  $t_B$  by  $t_A$  that is nothing but 4 by 1 okay multiplied by  $H_{dr}^2 A$  and  $H_{dr}^2 B$ . So, that is nothing but I mean if I consider that I mean the in the consolidometer basically what you do?

You put the porous stone on top as well as the at the bottom right. If you put the porous stone at top as well as the bottom that means this is a double drainage system. Am I right? So, water will be expelling out from the top as well as from the bottom. So, your  $H_{dr}^2$  will be why 30, 30 cm was the thickness of the sample so  $30 \times 2$  whole square divided by  $30 \times 2$  whole square; the thickness of the sample, both the samples is same.

So, from this I can get  $C_v A$  the ratio of  $C_v$  for sample A and for sample B is 4 okay. Now therefore I know that do you remember this I mean if you recall that thing that  $C_v$  is equal to  $K_v$  by  $m_v$  into  $\gamma_w$  right. So, long back I mean in the previous lectures we have derived that or we have got it okay. So,  $C_v$  is equal to  $K_v$  by  $m_v$  into  $\gamma_w$ . So, if that is true so I can write  $K_A$  that is the coefficient of permeability of sample A divided by coefficient of permeability of sample B is equal to  $C_v A$   $m_v A$  into  $\gamma_w$  divided by  $C_v B$  into  $m_v B$  into  $\gamma_w$ .

So, if we put the values, so 4 into 0.059 divided by 0.027 is equal to 8.71 is the ratio. So, that means the ratio of coefficient of permeability of 2 samples that is  $K_A$  to  $K_B$  is nothing but 8.71. So, we have done the or we are done with the problem. So, now basically you have seen we have used this logic because your  $T_v$  must be same for both the samples because you are thinking about same degree of consolidation for both the samples and so the  $T_v$  is obtained from this expression I mean I mean this expression will give you the value  $C_v$  and again  $C_v$  can be obtained from this expression. So, you can calculate the coefficient I mean ratio of coefficient of permeability. Now we will take the next problem.

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## Problem-17

- An area is underlain by a stratum of clay layer of thickness 6 m. The layer is doubly drained and has the coefficient of consolidation of  $0.5 \text{ m}^2/\text{month}$ . How long would it take for a surcharge load to cause a settlement of 50 cm, if the same load causes a final settlement of 60 cm.

So, next problem says that an area is underlain by a stratum of clay layer of thickness 6 m. The layer is doubly drained and has the coefficient of consolidation of  $0.5 \text{ m}^2/\text{month}$ . How long would it take for a surcharge load to cause a settlement of 50 cm if the same load causes a final settlement of 60 cm right. So, basically in the problem you have one clay stratum okay so which is experiencing some double drainage system that means it I mean water can go out from the top as well as from the bottom.

The thickness of the stratum is 6 m and the coefficient of consolidation is given. Now basically the thing is that, that stratum will be experiencing 100% degree of consolidation right at 60 cm I mean settlement right. The problem I mean it is given in the problem. If the same load that means you are increasing the load, you are putting some surcharge if the same load causes a final settlement of 60 cm.

So, when you are talking about the final settlement that means it is the settlement corresponding to the 100% degree of consolidation. That means when your  $U$ , capital  $U$  is equal to 100% that means your soil or the stratum will be experiencing 60% settlement right. Now if the strata is experiencing 50 cm settlement then what will be the degree of consolidation? This is very simple right.

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P-17

$$U = \frac{50}{60} = 0.8333 = 83.33\%$$

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$

$$= 1.781 - 0.933 \log(100 - 83.33)$$

$$= 0.641$$

So, I can calculate the so problem 17 so your degree of consolidation for 50 cm settlement is nothing but 50 by 60 right. So, which is nothing but 0.8333 which is nothing but 83.33%, very simple right. Because your 60 cm settlement is your final settlement. Final settlement means when you will be getting 100% degree of consolidation then only you can say this is my final settlement.

So, 60 cm is the settlement at the this is the final settlement at U equal to 100%. So, what will be the magnitude of U at 50 cm settlement. That is nothing but 83.33%. So, that means if the soil is if the degree of, average degree of consolidation is 83.33% then the soil will be experiencing the 50 cm settlement okay. So, this is the only tricks in the problem. Other things are very straightforward.

Okay so if you have degree of consolidation U equal to 83.33% then you can calculate T v right. How you calculate T v? Because your U is greater than 60%. So, which equation you will be using so that you have in the lecture 1.781 if you look at 0.933 log 100 minus U in percentage okay. So, if you put this 1.781 - 0.933 log 100 - 83.33. So, this will give me 0.641. So, your T v is equal to 0.641 at 83.33% degree of consolidation okay.

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$$C_v = \frac{T_v H_{dr}^2}{t}, t = \frac{(0.641) (6/2)^2}{C_v}$$

$$= \frac{(0.641) (6/2)^2}{0.5}$$

$$= 11.538 \text{ months}$$

So, once you know  $T_v$  you can find out  $C_v$  which is nothing but  $T_v H_{dr}^2$  by  $t$  okay. So,  $T_v$  just now you have calculated 0.641. Now what is  $H_{dr}^2$ . You are getting double drainage system or the doubly drained system. So, your total thickness of the stratum is 6 m that will be divided by 2 whole square right because the water can go out from the top as well as from the bottom divided by the time. Time is given. I am sorry, so time is not given.

So,  $C_v$  is given rather okay. So, this is the expression. So, from that I can write  $t$  equal to  $C_v$  which is nothing but 0.641 by 6 by 2 whole square divided by  $C_v$  is how much 0.5 meter square per month. So, 0.5 meter square per month is your  $C_v$ . So, once you get all those things you will be getting 11.538 months. That means the stratum that clay stratum of 6 m thickness will experience 50 cm settlement okay in 11.538 months okay if other things are remaining as it is okay.

I will stop here today. So, in the next lecture we will try to take some more examples, more numerical examples, to understand the consolidation in a better way and then we will move to the next topic. Thank you very much.