Geology and Soil Mechanics
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Lecture - 22
Seepage - B

Welcome back. So, in the last lecture basically we have talked about the seepage in soil and there we have seen that how we can construct the flow net and what are the different types of lines which will be conforming the flow net, the equipotential lines and the flow lines and then we have talked about the flow channels through which the flow actually the flow will be occurring and there we have seen or we have discussed the if you consider the square element, flow element then what will be the discharge, if you consider the rectangular flow element then what will be the discharge.

So, these things we have covered in the last lecture. I hope that you can still recall those things. So, from that point onward we will move forward. So, basically today we will be talking about the if you want to construct the flow nets in anisotropic soil.

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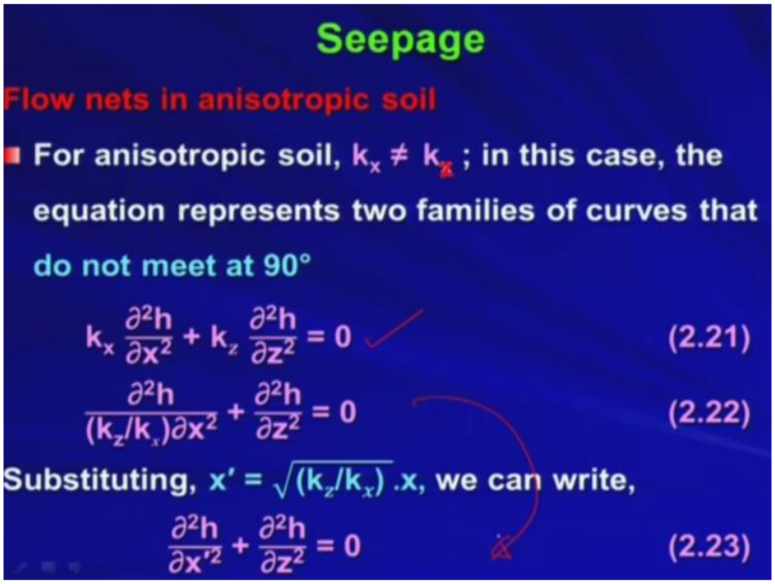

Seepage

Flow nets in anisotropic soil

- For anisotropic soil, $k_x \neq k_z$; in this case, the equation represents two families of curves that do not meet at 90°

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2.21)$$

$$\frac{\partial^2 h}{(k_z/k_x)\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2.22)$$

Substituting, $x' = \sqrt{(k_z/k_x)}.x$, we can write,

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (2.23)$$

So, for anisotropic soil, your k x is not equal to k z right. So, I mean what we discussed in the previous lectures that all the times we considered k x equal to k z and we have got the Laplace continuity equation which was nothing but del 2 h del x 2 plus del 2 h del z 2 equal to 0. So, that was the continuity equation for the isotropic soil, isotropic with respect to the flow okay or the permeability. So, if the k x is not equal to k z then the soil will be considered as anisotropic soil and in that situation how you will construct the flow net.

Because in this case the equation represents 2 families of curves that do not meet at 90 degree right. That is the major and important thing that basically you have the families of characteristics like families of say 2 curves like flow lines and the equipotential lines. Now in the earlier case we have seen when we have discussed about the flow nets construction and other things, at that time we have seen that the flow lines I mean all the flow lines intersect the equipotential lines at 90 degree right.

But in this because the equation of the Laplace equation that continuity equation will be telling that thing that you will be getting 2 families of curves which will be intersecting each other at 90 degree. But if you consider anisotropic soil where the continuity equation is given by k x del 2 h del x 2 plus k z del 2 h del z 2 equal to 0 and if your k x is not equal to k z then this equation okay will not tell the 2 families will tell about 2 families of curves which will be intersecting each other at 90 degree okay.

That quite obvious from the mathematics point of view. So, what we can do in that situation. So, basically we can modify this equation say equation 2.21 in this way say del 2 h by uh k z by k x del x square plus del 2 h del z 2 equal to 0. Now substituting x prime some different coordinate system say x prime is equal to say root over k z by k x into x. We can write that equation 2.22 will be taking the form of del 2 h del x prime 2 plus del 2 h del z 2 equal to 0.

So, now basically we are transforming the x axis in such a way that it will take x prime which will be equal to root over k z by k x into x okay and z will be remaining as it is. So, there is no issue with the z axis. So, now if you look at equation 2.23 so this equation will be telling there are 2 families of curves which will be intersecting each other at 90 degree okay. So, I mean in x prime z coordinate system okay. So, we are transforming the coordinate system, we are transforming the axis and then we are getting our desired equation okay.

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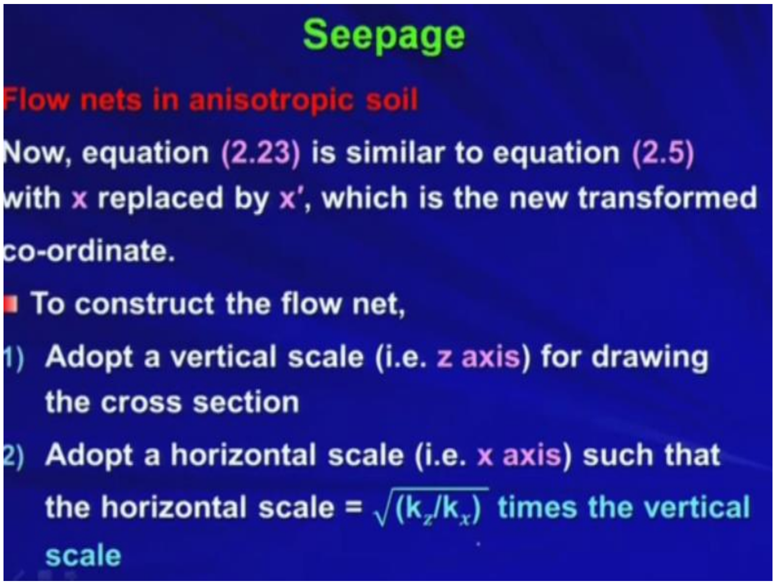

Seepage

Flow nets in anisotropic soil

Now, equation (2.23) is similar to equation (2.5) with x replaced by x′, which is the new transformed co-ordinate.

- To construct the flow net,

1) Adopt a vertical scale (i.e. z axis) for drawing the cross section
2) Adopt a horizontal scale (i.e. x axis) such that the horizontal scale = $\sqrt{(k_z/k_x)}$ times the vertical scale

So, now equation 2.23 is similar to equation 2.5 if you see if you recall or if you go back to the equation 2.5 which was giving me del 2 h del x 2 plus del 2 h del z 2 equal to 0. So, equation 2.23 is nothing but same equation as given in equation 2.5 with x replaced by x prime. That is the only difference we are making which is the new transformed coordinate okay so as I told you. To construct the flow net now adopt a vertical scale that is z axis for drawing the cross section okay so that you do.

So, now you can construct the flow net in anisotropic soil in by following these steps. Then second step is adopt a horizontal scale that is x axis such that the horizontal scale is equal to root over k z by k x times the vertical scale okay. So, now basically you are going to transform. Otherwise you will not be getting the families of curves which are intersecting each other at 90 degree okay. So, if you do this transformation then you will be getting that 90 degree intersection okay.

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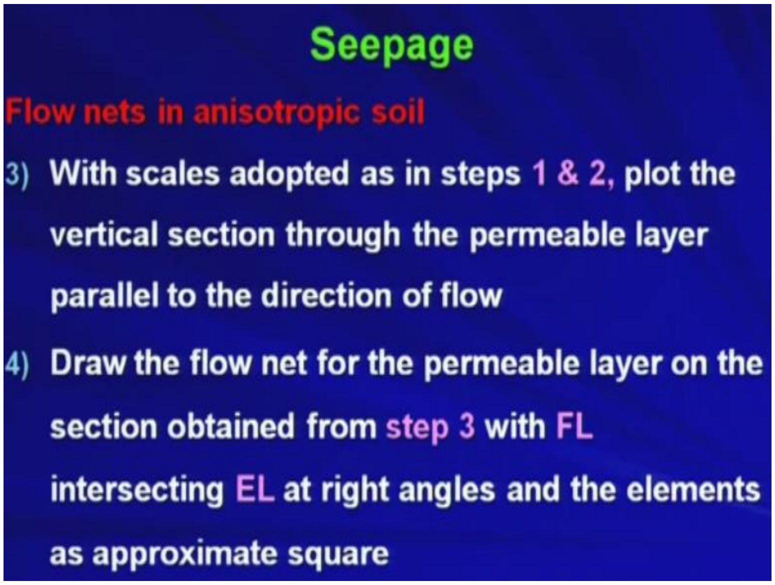

Seepage

Flow nets in anisotropic soil

3) With scales adopted as in steps 1 & 2, plot the vertical section through the permeable layer parallel to the direction of flow
4) Draw the flow net for the permeable layer on the section obtained from step 3 with FL intersecting EL at right angles and the elements as approximate square

Now with scales adopted as in steps 1 and 2 plot the vertical section through the permeable layer parallel to the direction of flow. So, the now the procedure will be remaining same whatever we followed for our pervious case like when the soil was isotropic and then draw the flow net for the permeable layer on the section obtained from step 3 with flow line intersecting equipotential lines at right angles and the elements as approximate square.

So, then the matter will be remaining same right. So, basically now you are constructing flow nets with the help of flow lines and equipotential lines. Now they will be meeting each other at 90 degree and the elements you just consider whatever I mean thing you have considered for the isotropic soil, the same will be applicable here. So, elements will be approximate square.

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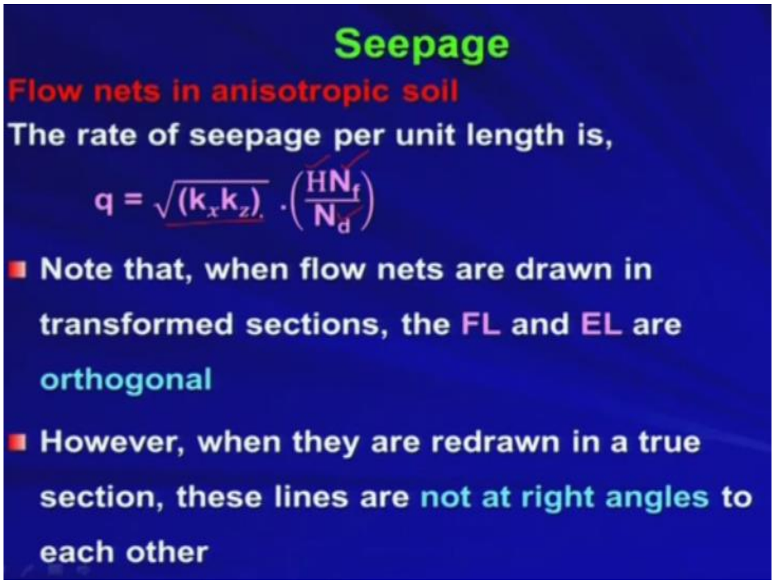

Seepage

Flow nets in anisotropic soil

The rate of seepage per unit length is,

$$q = \sqrt{(k_x k_z)} \cdot \left(\frac{HN_f}{N_d}\right)$$

- Note that, when flow nets are drawn in transformed sections, the FL and EL are orthogonal
- However, when they are redrawn in a true section, these lines are not at right angles to each other

Now flow, the rate of seepage per unit length, so now we are going to find out the rate of seepage. So, rate of seepage per unit length will be q small q equal to root over k x into k z into HN f N d right HN f by N d. So, that I mean you if you know this N f that is number of flow channels and number of equipotential drop N d and the head difference between upstream and downstream side say H.

So, then and if you know the permeability quotient or permeability in x and z direction, you can find out the rate of seepage per unit length of the I mean barrier of the I mean actual problem okay. So, note that when flow nets are drawn in transformed sections, the flow line and equipotential line are orthogonal. So, please make a note that when because this transformation is made to take the advantage of this 90 degree intersection or the orthogonal intersection of flow line and equipotential line.

So, when you are transforming in the transformed section so when the flow nets are drawn in the transformed section, basically the flow lines and equipotential lines they must be orthogonal because by taking the advantage you are getting that thing from the mathematical expression. However, when they are redrawn in a true section that means first you have got the true section or the true problem then you are transforming that thing in some transformed section where the flow lines and equipotential lines they are intersecting each other at 90 degree.

Now again you are redrawing the same thing from the transformed section to true section. Now in the true section when they are redrawn these lines are not a right angle not at right angles to each other. So, please remember this thing. So, in the actual or the true section they are not really meeting at 90 degree whereas in the transformed section they are meeting at 90 degree because we have taken the advantage by transforming the coordinate system.

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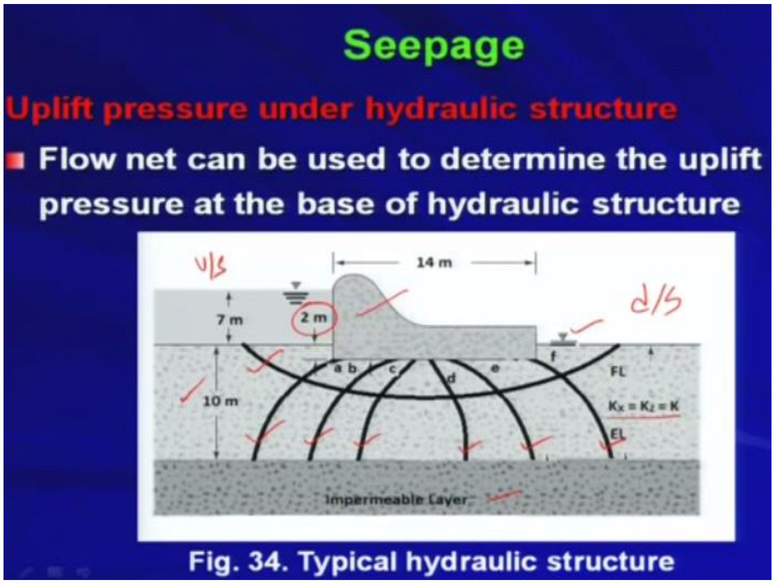

Seepage

Uplift pressure under hydraulic structure

- Flow net can be used to determine the uplift pressure at the base of hydraulic structure

Fig. 34. Typical hydraulic structure

Now flow net can be used to determine the uplift pressure at the base of hydraulic structure. So, basically as you have seen so flow net the main purpose of the flow net is to calculate how much seepage or how much flow is happening I mean through the permeable layer. So, that information you are getting if you construct the flow net if you know the number of flow channels, if you know the number of equipotential drops if you know the head and other things you can find out the rate of seepage right.

Already you have seen that whether you consider square element whether you consider rectangular element, that is not the matter of fact but the main thing is that you are getting the rate of seepage but you can get other information from the flow net. So, what is that other information? That is you can find out or you can determine the uplift pressure under the hydraulic structure okay by constructing the flow net.

Now we will see that how we can do that. Now suppose this is the problem actually. See you have say some kind of say small barrier okay it is a concrete kind of barrier which is holding the water on the upstream side, this is your upstream side okay. The depth of the water at the upstream side is say 7 meter and this wall or this concrete block is embedded at 2 meter in the ground okay and in the downstream side this is the water level.

So, the difference between the water at upstream and this is your downstream side. So, upstream and downstream side is say 7 meter right. So, this is the head that is the capital H. Now the depth of permeable layer is say 10 meter okay and then followed by the impermeable layer here. Now we are considering the soil is isotropic with respect to the flow that means k x equal to k z equal to k.

Now we are constructing the flow net. We can construct the flow net right by following the or by following the philosophy for the constructing the flow net. Now this is nothing but your one of the flow line right whereas this these are all your equipotential lines okay. These are all equipotential lines. Now basically if you can find out the pressure or the vertical pressure at point small a, small b, small c, small d, small e, and small a, then basically you can find out and you can if you can plot the pressure distribution then basically you will be getting the pressure at the base of the hydraulic structure.

So, this is your concrete hydraulic structure. So, at the base of that structure you can find out and then based on that uplift pressure basically you can determine the stability by considering the weight and all those things. So, that comes under the stability analysis of the geotechnical structure. So, now I mean how do we find out the uplift pressure at the base of this hydraulic structure? Let us see

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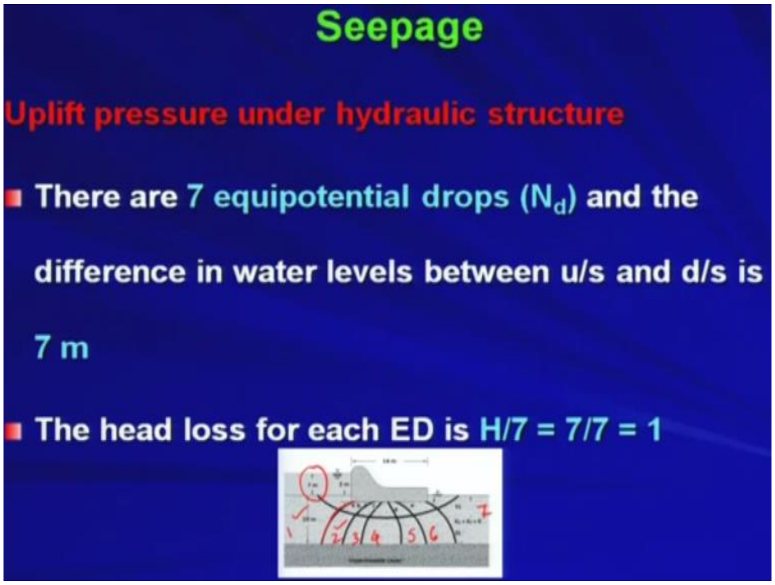

Seepage

Uplift pressure under hydraulic structure

- There are 7 equipotential drops (N_d) and the difference in water levels between u/s and d/s is 7 m
- The head loss for each ED is H/7 = 7/7 = 1

So, basically if you see so there are how many number of equipotential drops in the flow net I mean this flow net whatever we have perceived though we have shown only one flow line but how I mean we are showing all the equipotential lines. Now how many equipotential drops are there in this flow net. So, if you can count this is 1, 2, 3, 4, 5, 6, and 7. So, you have got 7 equipotential drops.

That means, so from this zone to this zone, so this is given by this is separated by this equipotential lines right. So, similarly, I can find out how many number of equipotential drops are there in the flow net. Now so the difference in water level between upstream and downstream that is given that is already I told you that will be 7 minute.

So, these 2 informations we are getting from this flow net construction and the head loss for each therefore I mean as you follow whatever we have discussed in the previous lecture, so the head loss for each equipotential drop will be how much H by 7 that is nothing but 1 right. So, H is 7 that is the total head loss. Now for each equipotential drop okay, what will be the head loss? That will be 7 by 7 that is 1 right simply.

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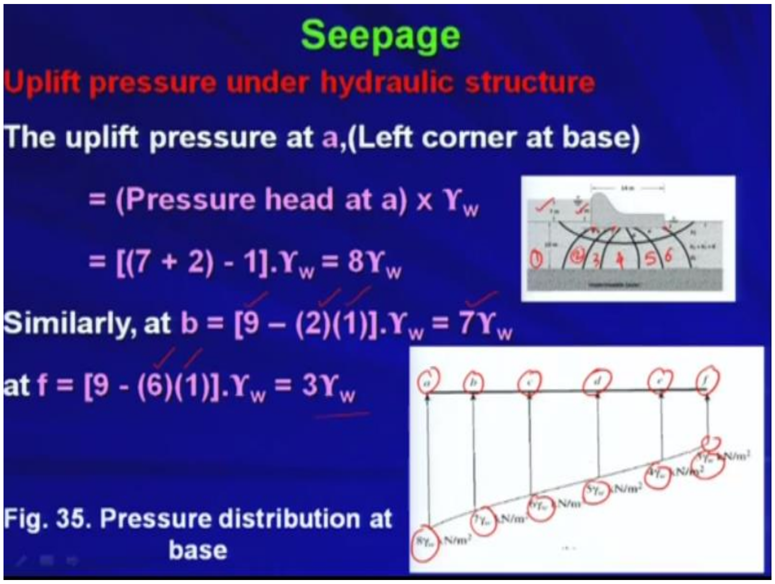

Seepage

Uplift pressure under hydraulic structure

The uplift pressure at a,(Left corner at base)

$= (\text{Pressure head at a}) \times \Upsilon_w$

$= [(7 + 2) - 1].\Upsilon_w = 8\Upsilon_w$

Similarly, at b = $[9 - (2)(1)].\Upsilon_w = 7\Upsilon_w$

at f = $[9 - (6)(1)].\Upsilon_w = 3\Upsilon_w$

Fig. 35. Pressure distribution at base

Now the uplift pressure at a that is the left corner like this, so left corner at the base okay. So, that is nothing but pressure head at a multiplied by the unit weight of water that is gamma w. Now what is the pressure head at a. So, 7 + 2 is the total depth right, 7 + 2 is the total depth of water. Now when you are reaching at a so basically you are exhausting one equipotential drop. Do you agree with that or not. So, please try to understand this thing fundamentally.

So, 7 + 2, 7 this 7 + 2 that is the embedment. So, this is the point a that elevation of point a or the depth of point a from the top water surface at the upstream side is nothing but 7 + 2 that is 9. Now when you are coming to point a basically you are exhausting the first equipotential drop. This one you are exhausting. So, 7 + 2 - 1 this much of loss you are accumulating that is 1 is the loss. So, 7 + 2 that is 9 that is the total head minus the loss that is 1 is equal to 8 into gamma w. So, that is the uplift pressure at a.

Similarly, you can calculate for other points. Say at b so if you if you see this is my point b. Now at b what will be the uplift pressure. So, what is the head at b. Head is I mean the depth of b point is 7 + 2, 9 the same because a, b, c, d all those things are lying along the base of the hydraulic structure so their elevation or their depth is same right. So, 9 is the total depth minus how much loss you are accumulating by going from the upstream side to point b so there are 2 equipotential drops basically first one and the second one.

So, 2 equipotential drops you are exhausting so the loss will be 2 into 1 so 9 that is coming from the depth and 2 that is the number of equipotential drops okay to reach point b and 1 is the loss for each equipotential drop. So, you are getting 7 w so that is your uplift pressure at the point b along the base of the hydraulic structure. Similarly, at f that is the last point that is the right most corner point at the base so what will be the uplift pressure at that point? What is the depth?

Depth is same that is 9 minus how many equipotential drops you are exhausting. So, if you count 1, 2, 3, 4, 5, and 6. So, 6 equipotential drops you are exhausting. Each equipotential drop will be having say loss of 1. So, you are getting 3 gamma w. I hope that you have understood the concept right. So, therefore you can draw the pressure distribution at the base that is uplift pressure distribution at the base so that means at point a you are getting 8 gamma w kilo newton per meter square.

At point b you are getting 7 gamma w kilo newton per meter square. At point c, you are getting 6 gamma w. At point d, you are getting 5 gamma w. At point e, you are getting 4 gamma w and at the last point that is f you are getting 3 gamma w okay. So, this is the uplift pressure distribution. Now if you I mean what is the use of this. Now if you want to find out the stability that means uplift movement or whatever those things you can calculate.

So, basically from this pressure distribution curve what is the total force acting at the base of the foundation towards the vertically upward direction that you can find out and based on that you can find out the stability with respect to the uplift right. So, this is very useful to calculate or to find out the stability of the structure. So, and that you have got just by constructing the flow net. So, you have got or you have got the information about the advantage of the flow net construction.

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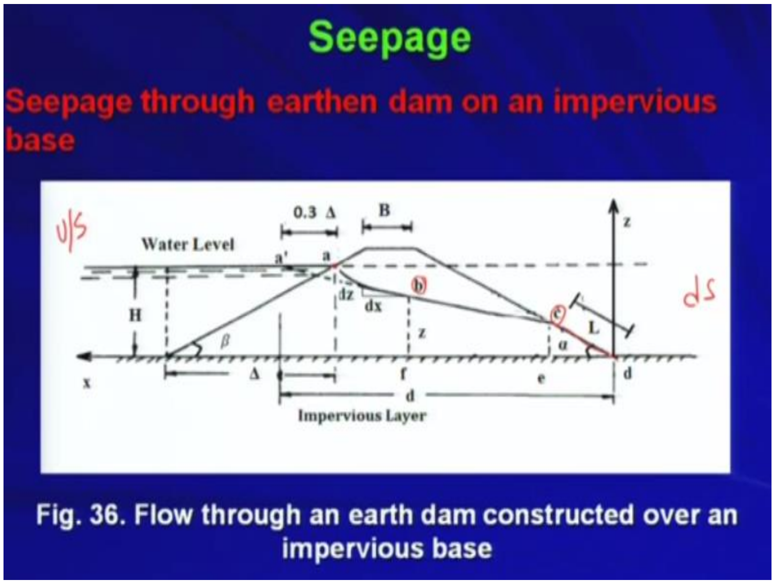

Seepage

Seepage through earthen dam on an impervious base

Fig. 36. Flow through an earth dam constructed over an impervious base

Now coming to the seepage through earthen dam on an impervious base. So, now this thing also we can find out say now if you have one earthen dam so that you can see from this figure okay. So, the earthen dam is there. Now this earthen dam is resting on the impervious base that means the flow if the flow is happening then the flow is happening through the body of the earthen dam right. So, earthen dam is constructed to store the water.

So, this is your upstream side okay where you are storing the water and the water level is say h okay and this is your downstream side okay. Now basically if you see the flow through the earthen dam what will happen? So, this is the entry point okay point a small a is the entry point of the water right and it will go all the way to through b and c and then finally it will flow along the surface of the downstream side okay downstream slope. I hope you have understood.

So, basically the topmost flow line is nothing but a, b, c as well as d right. So, this is the topmost flow line along which the flow is happening. So, below that line okay up to the impervious layer the soil I mean flow will be occurring and above a, b, c, d you do not have any flow. So, basically you can construct the flow rate for that there is no issue because the flow is happening. So, once the flow is happening you can construct the flow net. But now we are interested to find out the seepage through the earthen dam. Let us find out how we can find out the seepage.

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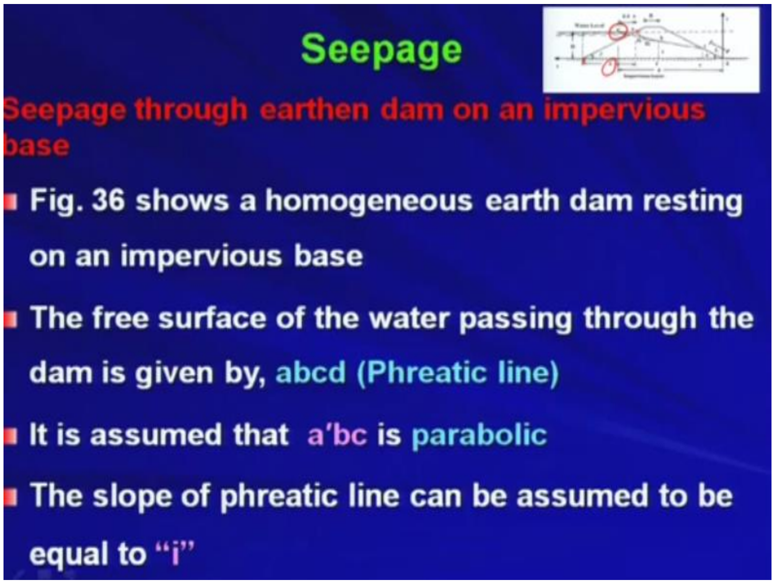

Seepage

Seepage through earthen dam on an impervious base

- Fig. 36 shows a homogeneous earth dam resting on an impervious base
- The free surface of the water passing through the dam is given by, abcd (Phreatic line)
- It is assumed that a'bc is parabolic
- The slope of phreatic line can be assumed to be equal to “i”

So, basically what we are doing, this figure whatever figure I have shown you just now that shows a homogeneous earth dam resting on an impervious layer as I told you. The free surface of the water passing through the dam is given by a, b, c, d and that is known as very important term that is known as Phreatic line. So, top most flow line is nothing but the Phreatic line. So, that is the free surface of the water passing through the water is given.

So, that means just I mean that that flow line on top of that you will be having the atmospheric pressure. So, a, b, c, d is the topmost flow line and the flow is happening below that line right. The flow is happening through the earth dam body below that line itself. It is assumed that a prime bc is parabolic. So, basically what we are doing here so those things those steps I will be talking about later on.

Say suppose this point is nothing but a that is the actual entry point of the flow okay through the dam. Now this point a is located from the from this point by an amount delta okay. Now I am choosing or I am assuming the actual flow is happening from point a prime which is point 3 delta distance away from actual point a okay and we are assuming that a prime bc you can see from the figure a prime bc this is nothing but a parabolic curve.

That is the assumption and that holds good, it has been observed from the experience as well as from the calculation that it holds good so that is the assumption that a prime which is 0.3 delta distance apart or away from a towards the upstream side so that point a prime and b and c if you connect those points basically you will be getting one parabolic curve. So, your Phreatic line basically will take a shape of parabola. So, the slope of Phreatic line can be assumed to be equal to i that is the hydraulic gradient. Now what I mean to say the downstream slope is nothing but this right. So, this is your downstream slope. Now if the downstream slope is not very steep, so if the downstream slope is very flatter, then basically you can consider or you can assume the slope of the Phreatic line is your is equal to your hydraulic gradient that is i.

If it is so, if it is assumed I mean this assumption is logical and this assumption holds good also I mean there is no problem in this assumption and we will see that when the problem will be coming at that time what could be the different assumptions. So, that we will see later on. So, at this moment we are assuming 2 things. One thing is that a prime bc is the parabola okay or parabolic rather and the slope of the Phreatic line that is equal to i right hydraulic gradient.

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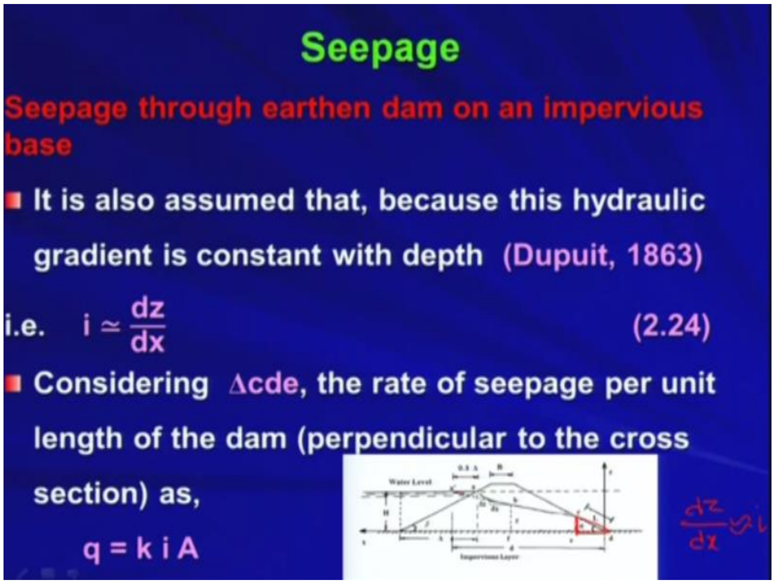

Seepage

Seepage through earthen dam on an impervious base

- It is also assumed that, because this hydraulic gradient is constant with depth (Dupuit, 1863)

i.e. $i \simeq \frac{dz}{dx}$ (2.24)

- Considering Δcde, the rate of seepage per unit length of the dam (perpendicular to the cross section) as,

$q = k\,i\,A$

So, it is also assumed that because this hydraulic gradient is constant with depth right so then i is nothing but dz dx so whatever we have seen so the z is here so I am considering this is my origin that is the 2 point okay so that is my origin. So, x is in this direction and z is in this direction. So, this dz by dx if you see here okay so dz by dx is nothing but your i approximately equal to i right the slope of the Phreatic line.

So, slope of the Phreatic line is nothing but dz dx okay and which is approximately equal to i whatever we have assumed. Now considering triangular cde now coming to this figure once again so what I mean to say so this is your this is the parabola okay. So, this is the shape of this curve is the parabola and the slope of this line is dz dx and which is approximately equal to i based on our assumption.

Now considering triangular cde, so this is the triangle okay. The rate of seepage per unit length of the dam that is perpendicular to the cross section as given by q is equal to k into i that is the velocity into a that is the cross-sectional area. So, I am considering the that triangle the flow is happening along ce. So, k i multiplied by A that is the cross-sectional area will be giving me the seepage or the I mean flow or the quantity of seepage or the rate of seepage right.

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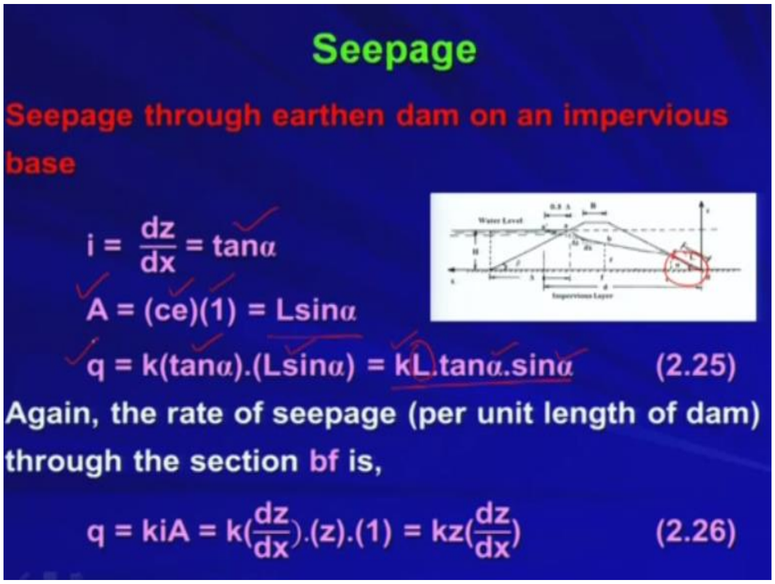

Seepage

Seepage through earthen dam on an impervious base

$$i = \frac{dz}{dx} = \tan\alpha$$

$$A = (ce)(1) = L\sin\alpha$$

$$q = k(\tan\alpha).(L\sin\alpha) = kL.\tan\alpha.\sin\alpha \qquad (2.25)$$

Again, the rate of seepage (per unit length of dam) through the section bf is,

$$q = kiA = k\left(\frac{dz}{dx}\right).(z).(1) = kz\left(\frac{dz}{dx}\right) \qquad (2.26)$$

So, basically, so i is given by dz dx equal to, which is nothing but tan alpha. So, if you look at the figure so i is the that dz dx is the slope i is approximately equal to dz dx already we have seen. Now in this case dz dx if you can find out what will be the dz dx at point c that is nothing but tan alpha okay. So, A that is the cross-sectional area along line ce is nothing but ce that is the length of that or the height of that line ce into unit length normal to the screen or normal to the board okay. So, we can find out A is equal to L sine alpha from the triangular relation okay.

So, therefore q is nothing but k into i where i is your tan alpha into L sine alpha which is nothing but your A. So, I am getting your rate of seepage is k L into tan alpha sine alpha. Now what is L L is the I mean distance from the from this point d and to point c so that is completely unknown to me at this point of time. So, I know k, I known alpha, I know other things, but I do not know the magnitude of L. So, once I know the magnitude of L I can find out the rate of seepage through this dam okay.

So, I will stop here today okay. In the next class, we will continue and we will see that how we can find out this L. Thank you.