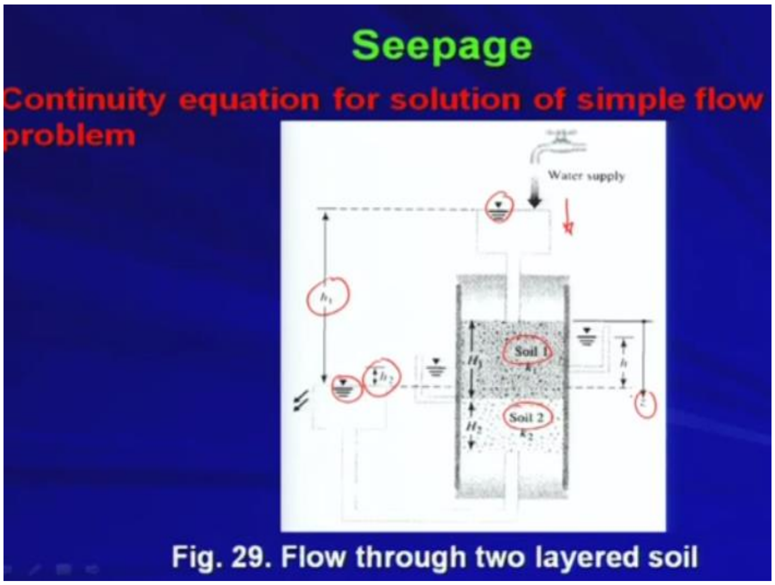
Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture - 20
Seepage

Welcome back. So, in the last lecture we just started another important topic in soil mechanics that is seepage in soil. So, there we discussed what is seepage and how could we derive the continuity equation and then basically we started this part continuity equation for solution of simple flow problem. So, we took this problem.

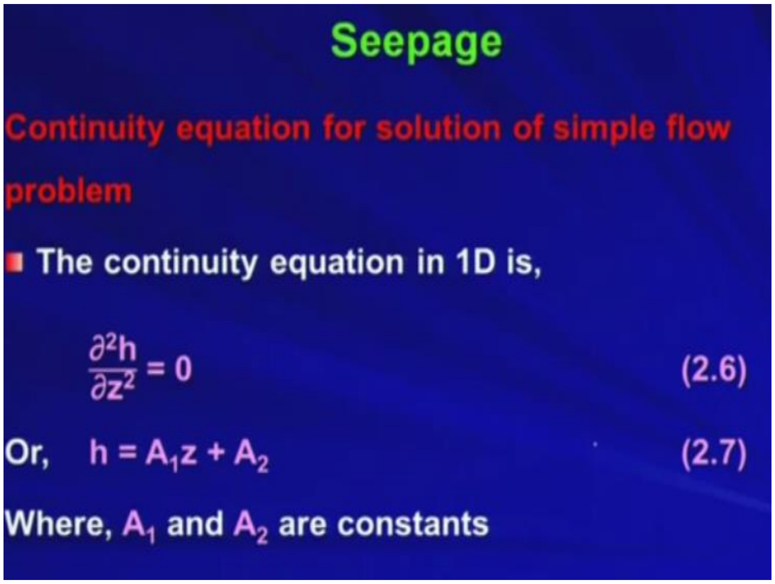
(Refer Slide Time: 00:42)



So, in this problem basically you have 2 different soil layer, soil 1 and soil 2. Soil 1 is having the height of the sample say capital H and whereas soil 2 is having the height H 2; k 1 and k 2 are the hydraulic conductivity of soil 1 and soil 2 respectively. So basically, what we are doing, we are putting water okay and basically that is at the entry level this is the head can be measured from this and at the exit level the head is measured here.

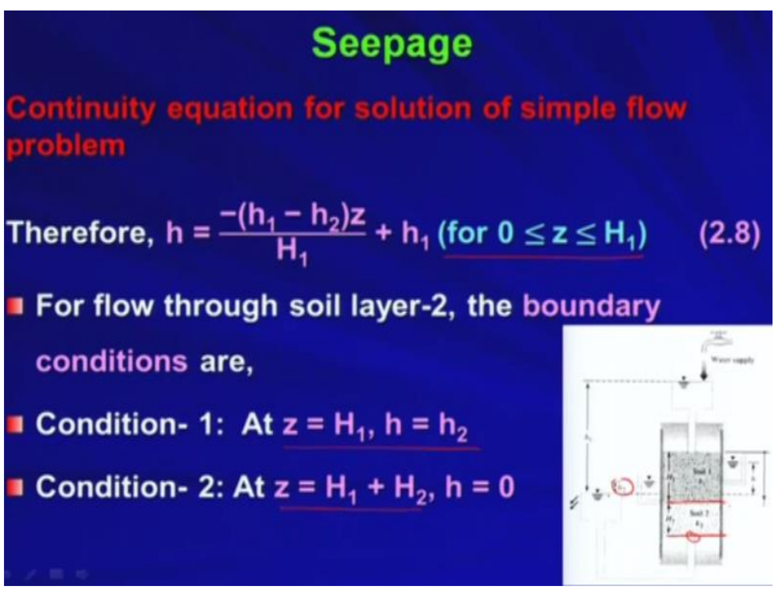
So now basically if you consider the level, water level, at the exit level is my datum then basically the head difference between the exit level and the entry level okay is say small h1 whereas the head difference between the exit level and the head at the interphase of 2 different soil layer is say small h2 and so basically, we are defining one coordinate system say z starting from the top of the soil layer 1. So basically, this is the arrangement. Now we are going to find out the continuity equation for this kind of problem.

(Refer Slide Time: 02:03)



So, in 1D the continuity equation whatever we have derived for 2D right. So, if you recall so that was del 2 h by del x 2 + del 2 h by del z 2 equal to 0. So that was the continuity equation for 2D problem. However, the continuity equation in 1D that will boil down to del 2 h by del z 2 equal to 0 so if I consider the flow in the z direction. So, the solution of this equation is small h is equal to A 1 z + A 2 where A 1 and A 2 are the constants. Now I need to find out this 2 constants A 1 and A 2.

(Refer Slide Time: 02:42)



So, for that I need the boundary conditions right to obtain A 1 and A 2. For flow through the soil layer 1 the boundary conditions are, first condition is at z equal to 0, if you recall the figure at z equal to 0, so this is the figure. So, at z equal to 0 that means you are here that means you are at the top of the soil layer 1. At that time, the head is nothing but h 1 because if you consider the exit level water level at the I mean whatever you are considering that is your datum so small h is equal to h 1 at z equal to 0, there is no issue for that.

So, second condition is at z equal to H 1 because when you are considering layer 1 basically your domain is starting from z equal to 0 to z equal to H 1 right. So, within this domain or within this region you are having the soil layer or soil sample 1. Now at z equal to H 1 what should be your small h? That is nothing but h 2 whatever we have defined right. So that is your h 2 that is the head obtained at the interphase of 2 different layers right.

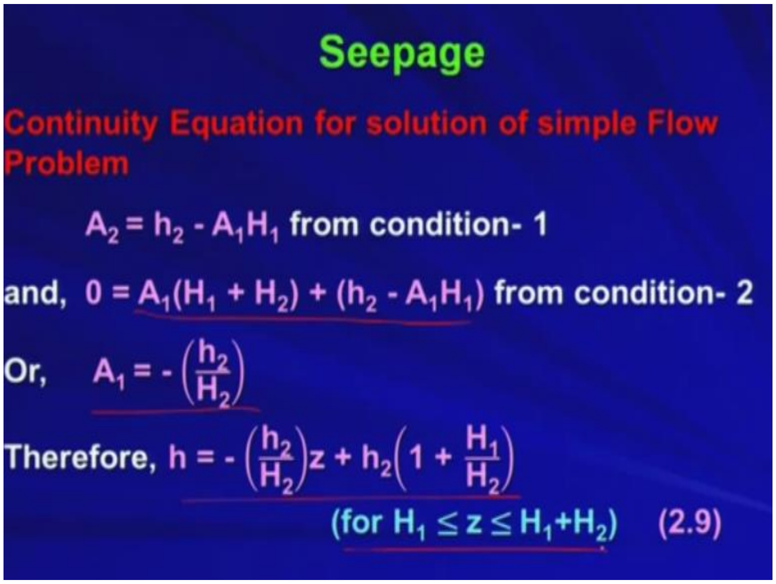
So, if you put these boundary conditions in the previous equation so you will be getting A 2 is equal to h 1 and A 1 is equal to - h 1 - h2 by H 1. So now these 2 unknown constants are now known to you. So, if you put that in the equation then you will be getting small h is equal to - h 1 - h 2 into z by H 1 + h 1 okay. So, I am putting the values of A 1 and A 2 so I will be getting the expression of h that is the head.

So that is valid so this boundary conditions we applied the boundary conditions for the soil sample or the sample layer 1 right. So, this head, the expression of this head is basically valid for the soil layer 1 right. So that means this is valid within this region. That means z greater than equal to 0 to less than equal to H 1. That means within that domain basically your head can be expressed by this expression right.

Similarly, for flow through soil layer 2 the boundary conditions are so this is your first boundary condition right. At z equal to H 1 that means you are at the interphase of soil layer 1 and soil layer 2, already we have seen the head is h is equal to h 2 whereas in at condition I mean condition 2 that means at z equal to H 1 + H 2 that means at the exit level your head is 0 because you are considering that is your datum right.

So, at z equal to H 1 + H 2 that means at the exit level at the end point of the soil layer 2 is nothing but 0 okay. So that you can see here. So now we are basically starting from here and we are ending at here. So here your h is h 2 whereas your, at this level h is 0 because that is at the exit level.

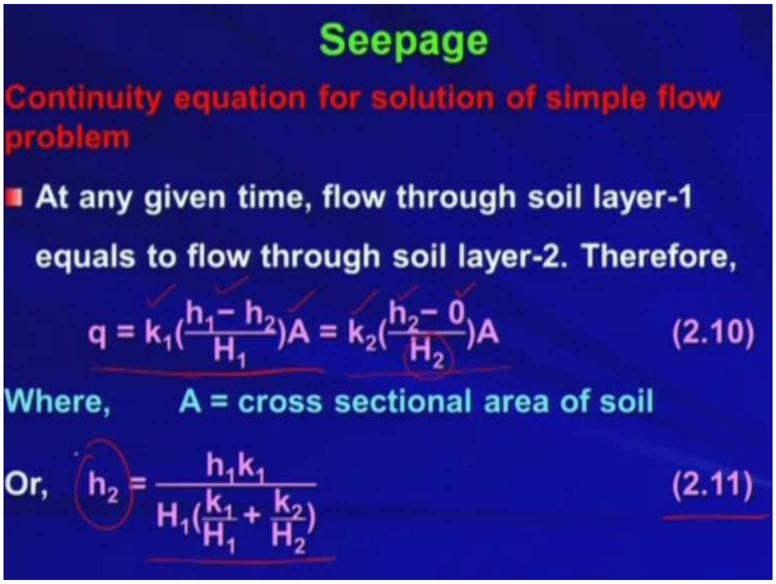
(Refer Slide Time: 05:59)



Now so we can find out, so the solution will be remaining same so h is equal to A 1 z + A 2 so whatever we have seen in the last equation so only thing is that we are now going to find out the constants based on the boundary conditions. Now if you put this boundary conditions whatever are applicable to soil layer 2 so basically you will be getting A 2 is equal to h 2 - A 1 H 1 from condition 1 which gives this expression okay from condition 2.

Now from this I can write A is equal to - h 2 by capital H 2 and therefore we can write h small h is equal to this right where h 2 by H 2 into z + h 2 into 1 + capital H 1 by H 2 and you must remember that this is this expression of small h is valid within this range that is H 1 I mean z is greater than equal to H 1 that means you are just crossing the interphase of soil layer 1 and soil layer 2 and you are entering the soil layer 2 and it will be valid till z equal to I mean z less than equal to H 1 + H 2 right. So, you are within the domain soil layer 2.

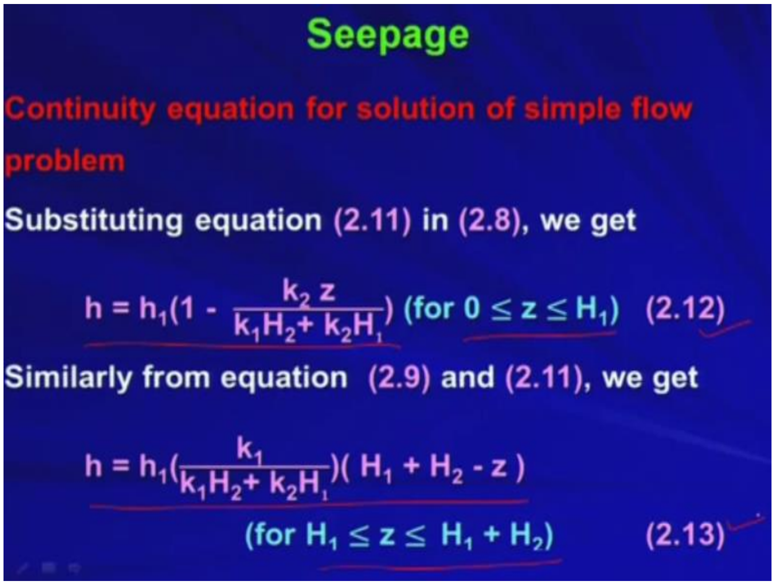
(Refer Slide Time: 07:17)



So, at any given time flow through soil layer 1 equals to the flow through soil layer 2 because you know from the continuity equation that whatever flow is happening through soil layer 1 the same amount of flow is passing through the soil layer 2. Therefore, I can write so q this is coming from soil layer 1 what is this is k 1 that is the property of soil layer 1 h 1 - h 2 by capital H 1 that gives me the hydraulic gradient for soil layer 1 and that is the cross-sectional area and cross-sectional area for both the layers are same right okay and so this is the flow through, this is the flow through your soil layer 1.

Now this is the flow through your soil layer 2. They must be same right. So, this is k 2 is nothing but the property or the hydraulic conductivity of soil layer 2 and h 2 at the starting point 0 at the end point or the exit point and capital H 2 is the length of soil layer 2. So, this must be equal. So, from this we can get small h 2 is equal to this right. So, if you solve this then you will be getting one equation, equation 2.11 which gives me the expression for small h 2 in terms of small h 1, k 1, capital H 1, and capital H 2.

(Refer Slide Time: 08:49)

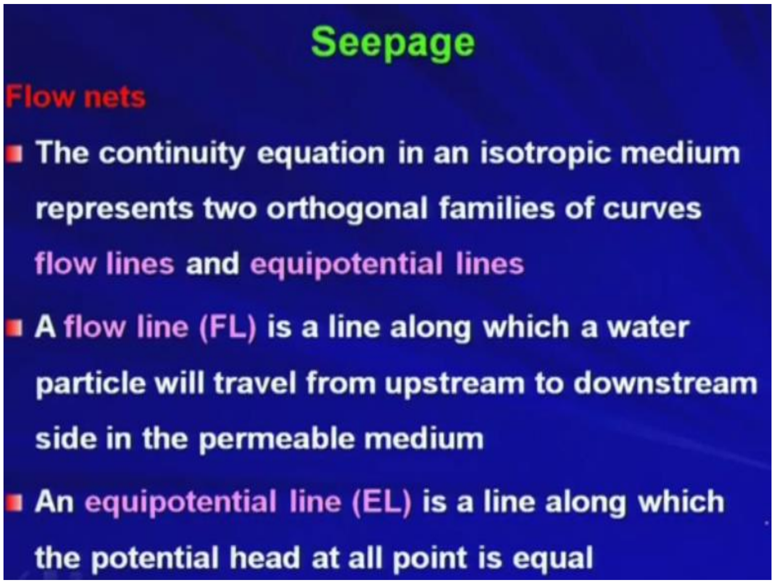


So, substituting equation 2.11 in equation 2.8 we get h is equal to, so you can remember the equation 2.8 was giving me the head okay for the soil layer 1. So now we are putting the value or putting the expression of h 2 in terms of h 1 small h1, capital H 1, capital H 2, in that equation and you will be getting the head, the expression for head in soil layer 1 like this and this is valid as I told before, it is valid within 0 to capital H 1 that means within the soil domain 1 or the soil layer 1.

Similarly, from equation 2.9 which was giving me the head expression for soil layer 2 and I am putting or I am substituting the expression whatever I have obtained in 2.11 in 2.9, I will be getting the expression of h that is the head like this right and this head this expression is valid within this domain that means you are within the soil layer 2.

So both small I mean if you want to find out head at any point starting from the entry point that means the top of the soil layer 1 and at the and between the exit point that is the bottom of soil layer 2 within this region if you want to find out the head you have to use equation 2.12, 2.13 based on where exactly you are that means whether you are in soil layer 1 or whether you are in soil layer 2 based on that you use this expression to find out the head by satisfying the continuity equation.

(Refer Slide Time: 10:35)



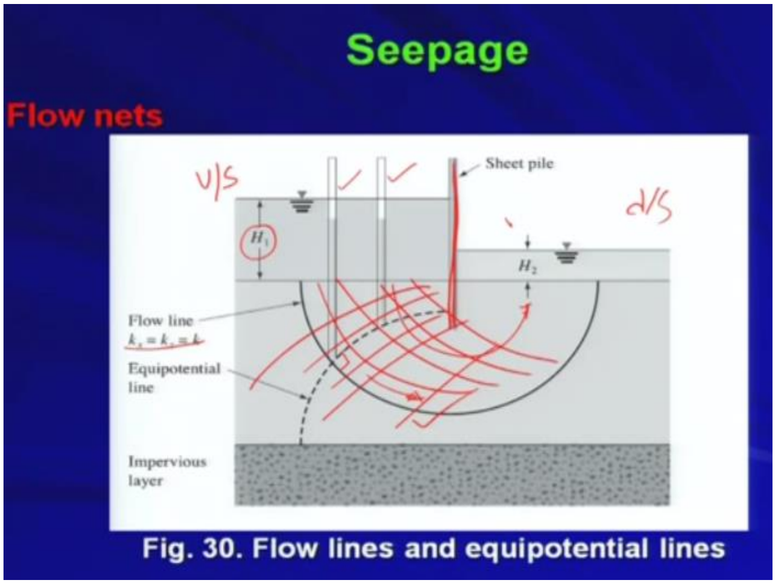
Now coming to some new concept so whatever we have leant so far from the continuity equation and how the flow is happening how the seepage is occurring and how basically you are finding out the head so based on those things now we are going to find out or going to determine some new thing that is nothing but that is known as flow nets. Now the continuity equation in an isotropic medium okay isotropic medium with respect to flow right.

That means your kx must be equal to kz. If I consider the two-dimensional flow so represents 2 orthogonal families of curves okay that is one is known as flow line and another one is known as equipotential lines. Now if you recall the continuity equation that was giving me del 2 h by del x 2 plus del 2 h by del z 2 equal to 0. Now that continuity equation is valid for isotropic soil, already we have that or isotropic medium right.

Now this equation or this expression if you remember and if you recall your mathematical background so basically this will give you 2 different families of curves okay which are orthogonal to each other and one family will be known as or one family will be termed as flow line, another family will be known as equipotential line. Now we will see I mean what are the I mean importance of these 2 lines and what are the I mean major say issues associated with these 2 lines.

Now a flow line is a line along which a water particle will travel from upstream to downstream side in the permeable medium. So, if you have the permeable medium and if the flow is happening from upstream side to downstream side then the flow line will represent that along this line basically the flow will occur okay and whereas and equipotential line is a line along which the potential head at all points is equal right. So along that equipotential line your head okay potential head is same so that means that is that is why it is equipotential line. So, one I mean between 2 equipotential lines basically you will be getting some equipotential drop.

(Refer Slide Time: 12:56)



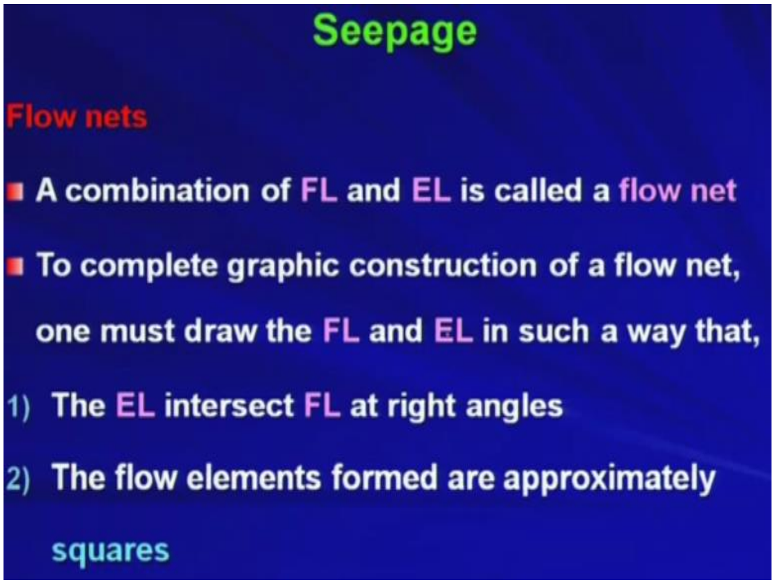
Now if you look at this figure then it will be very clear to you. Say suppose this is my sheet piles that means some barrier okay. Now the you have the head on the upstream, this is your upstream side. You have the head on the upstream side as capital H 1 and this is your downstream side because flow can happen something like that. So not through this sheet pile rather the flow will be happening below the sheet pile like this.

Now basically you see this line is basically the flow line in your isotropic medium where k x is equal to k z equal k right. So, the flow will be happening along this line whereas this dash line is known as equipotential line. So that means if you put some piezometer at different points on the equipotential line you will be getting the same height of water. So that means your I mean your head is remaining same along the equipotential line and they will be orthogonal okay.

So, the flow line will intersect the equipotential line in a normal direction or the perpendicular direction okay. So now this is the thing. So now if you have this kind of structure so basically by drawing this kind of so now you can have n number of flow lines, n number of equipotential line, and basically you can form some net right some grid kind of thing and if you solve each grid basically you will be getting the flow information through the body or through the soil deposit as well as the other information's you will be getting that how much pressure you are getting at the base of the structure and all those things.

We will come to those things later on. So basically, what I mean to say by getting or by drawing the flow lines and equipotential lines number of flow lines and equipotential lines basically you will be getting one flow net that is say grid kind of thing and that flow net will help you to get several information about the flow.

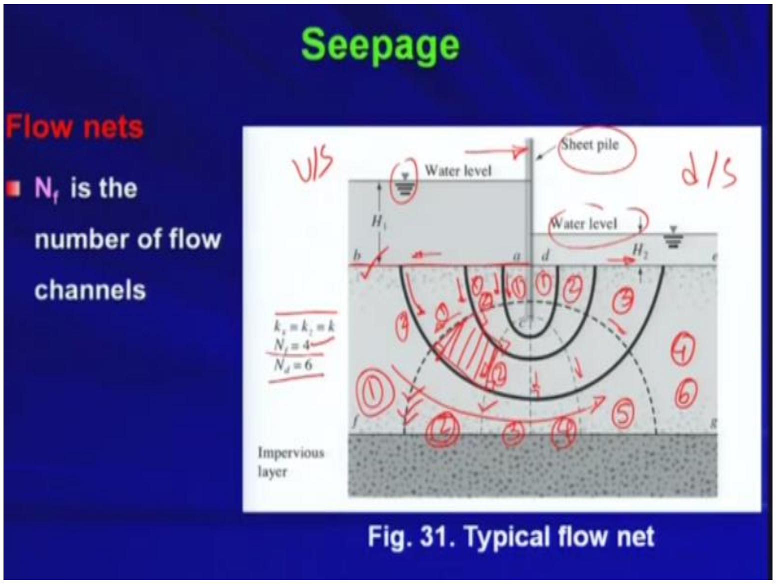
(Refer Slide Time: 15:04)



Now a combination of flow line flow lines and equipotential lines is called a flow net. As I told you that that means you are getting n number of flow lines, n number of equipotential lines and that will form a grid kind of thing and that grid is known as flow net and this is very essential for analyzing some hydraulic structure. To complete graphic construction of a flow net one must draw the flow line and equipotential line in such a way that the equipotential line intersects flow line at right angles right. So that is the basic requirement because based on that basically you have got these 2 lines equipotential lines and flow lines so they must be orthogonal.

So, equipotential lines intersect flow lines at right angles at each and every point, each and every node in the flow net. The flow elements formed are approximately square. So that is in your hand. You are basically creating or you are just forming the flow net by drawing several equipotential lines and flow lines so you construct the flow net in such a way that the flow elements that means the region bounded by 2 equipotential lines and 2 flow lines okay so that region or that domain should be approximately in a square shape.

(Refer Slide Time: 16:31)



Now let us see so now we will see some typical flow net. This is another barrier what is nothing but sheet pile so it will not allow the flow from upstream to downstream like this, but however the flow can occur below the soil deposit okay. So now this is the water level on the upstream side, this is the water level on the downstream side. So now ab is basically the bed level on the upstream side whereas d is the bed level on the downstream side.

So, this is your downstream side, this is your upstream side. Now basically you are constructing the flow net. So here in this situation you have the isotropic soil that means your k x equal to k z equal to k. Now basically you are choosing the how many number of so that is basically based on your judgment. So, you can this is something like optimization kind of thing. You can go for several number of flow channels right.

So, flow channels means say now you first identify what are the different flow lines in this flow net and what are the different equipotential lines in this flow net. Now so this ab, along this ab if you want to measure the head or the pressure of the potential head what you will get? It will be remaining same. So, ab is eventually or virtually an equipotential line right. Similarly, you can draw another equipotential line which is shown by this dash line, then another equipotential line like this, and this and this this this and again de is another equipotential line because along de you have the same head.

So, this dashed line you are constructing, so these are nothing but your equipotential line, along this lines you will be getting same potential head. Now this thick lines, thick black lines they are nothing but the flow line, so flow will be occurring like that. So, you construct the flow lines and this is your one flow line this is another flow line, this is another flow line. So, you construct the flow lines and equipotential lines such that they are perpendicular at the intersection point okay.

So, and they form, so this is one zone okay which is covered or which is bounded by 2 flow lines say 1 and 2, 2 flow lines and 2 equipotential lines 1 and 2. So this zone must be square in shape okay, almost I mean approximately should be square. Now to make it square and to get the right angle or the 90-degree angle between the flow line and equipotential line you can construct or you can I mean draw n number of say equipotential lines and n number of flow lines right.

So now basically N f so here you need to find out the flow channel. So, flow channels are nothing but it is bounded by 2 nearby flow lines. So how many flow channels are here in this in this problem or in this typical flow net. So, this is flow channel 1, this is 2 rather I should say I should write here 1, this is 2, this is 3, and this is 4. So, there are 4 flow channels that is N f. So how many equipotential drops are here so that means that is the indication from one equipotential line to another equipotential line. So how many drops are there?

So, you will be getting 1 drop here, 2, 3, 4, 5, 6. So that means from this equipotential line you are coming to this equipotential line. So, you are getting some equipotential drop and that is nothing but N d that is defined as or that is represented as N d. So, from this equipotential line ab between ab and this equipotential line you are getting first drop. Then from this equipotential line to this equipotential line you are getting the second drop and so on right.

So, you are getting 6 number of equipotential drops that is N d equal to 6 and 4 number of flow channels that is N f equal to 4 right. I hope that you have understood how we are forming the flow net and how we are basically discretizing the whole flow domain with the help of several flow lines and equipotential lines. So, and then we will be getting some grid and we will be getting some flow channels through which the flow will be occurring and we will be getting the potential drops from one equipotential line to another equipotential line and based on this formation we can find out the flow across this structure or across this soil deposit that means from upstream to downstream.

So, in the next class we will be talking about these things and we will be seeing how we can use this flow net to get the information about the flow through the porous medium. So, thank you very much.