Geology and Soil Mechanics
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Lecture - 19
Problems on Permeability

Welcome back. So, as we have decided in the last lecture that we will be taking one more example problem on permeability in soil.

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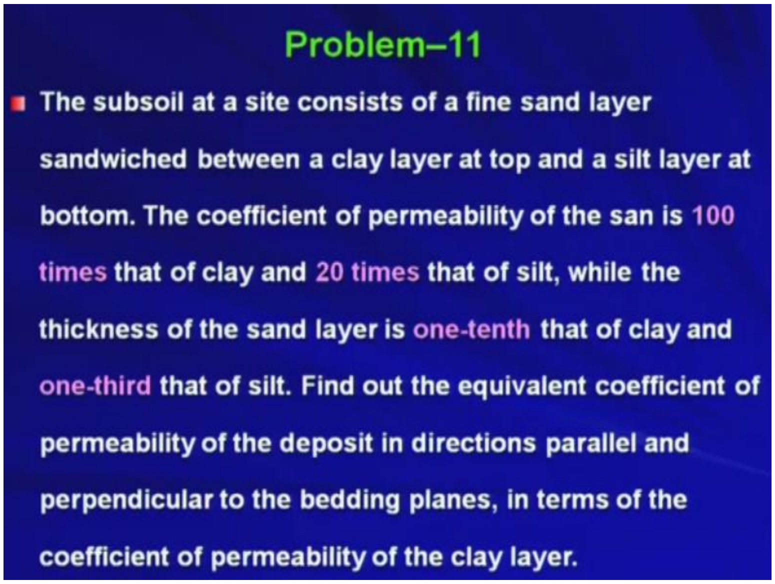

So, the problem 11 says the subsoil at a site consists of a fine sand layer sandwiched between a clay layer at top and a silt layer at bottom okay. The coefficient of permeability of the sand is 100 times that of clay and 20 times that of silt while the thickness of the sand layer is one-tenth that of clay and one-third that of silt. Find out the equivalent coefficient of permeability of the deposit in directions parallel to parallel and perpendicular to the bedding planes, in terms of the coefficient of permeability of the clay layer okay.

So, I hope that you have understood the problem. You have the sand layer okay and you have the clay layer at the top and silt layer at the bottom. That means the sand layer is getting sandwiched between clay layer and the silt layer. So, the information about the permeability of different layers are given that is permeability of sand layer is 100 times that of clay and 20 times that of silt and the thickness of the layers are also given.

So, we need to find out the equivalent permeability or the equivalent hydraulic conductivity when the flow in happening in the parallel or the horizontal direction, when the flow is happening in the vertical direction. So, that means the theory whatever has been covered to find out the equivalent hydraulic conductivity, now that will come into picture. So, let us start this problem.

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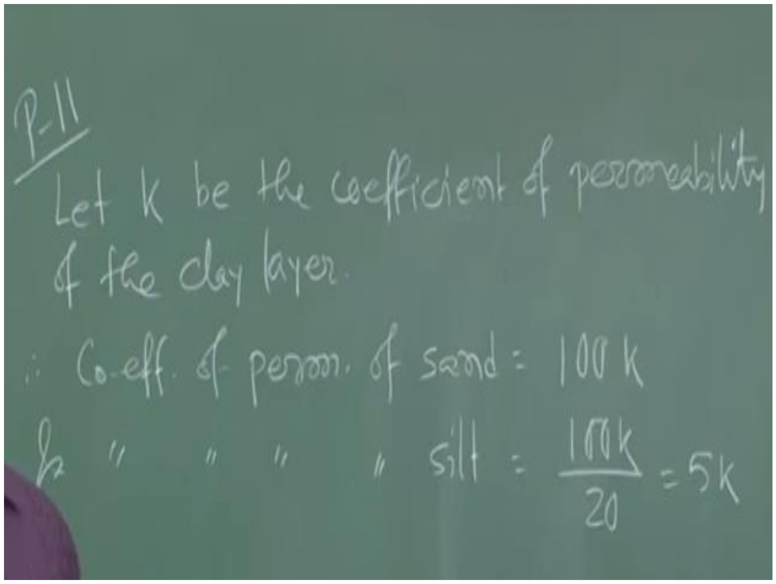

Let k okay be the coefficient of permeability of the clay layer okay. Therefore, coefficient of permeability of sand will be how much and that is given in the problem right. The coefficient of permeability of sand layer is 100 times that of clay layer. So, it will be 100 K fine. Similarly, coefficient of permeability of silt layer will be how much? That means the coefficient of permeability of the sand layer was 20 times that of silt layer. So, that means this is the coefficient of permeability of the silt layer. That means 120th of the sand layer permeability okay.

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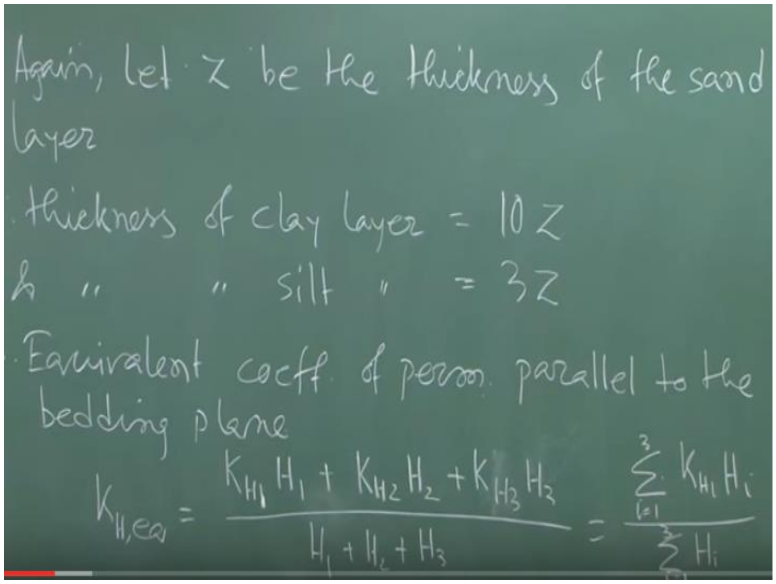

Again, let z be the thickness of the sand layer okay. So, let us assume that z is the thickness of the sand layer. So, therefore the thickness of clay layer will be how much, that is given in the problem, that is 10 times the thickness of the sand layer and the thickness of the silt layer is 3 times that of sand layer, that is given in the problem. So, we are not doing anything new okay.

So, therefore equivalent coefficient of permeability parallel that is horizontal direction parallel to the bedding plane is equal to K H equivalent is equal to K H 1 into H 1 + K H 2 into H 2 + K H 3 into H 3 divided by the total depth of the bedding plane H 1 +H 2 + H 3 right. So, this can be simply written as summation of i equal to 1 to 3 K H i H i by summation of i equal to 1 to 3 H i okay. So, this is the expression already we have derived for equivalent hydraulic conductivity for parallel or the horizontal flow right.

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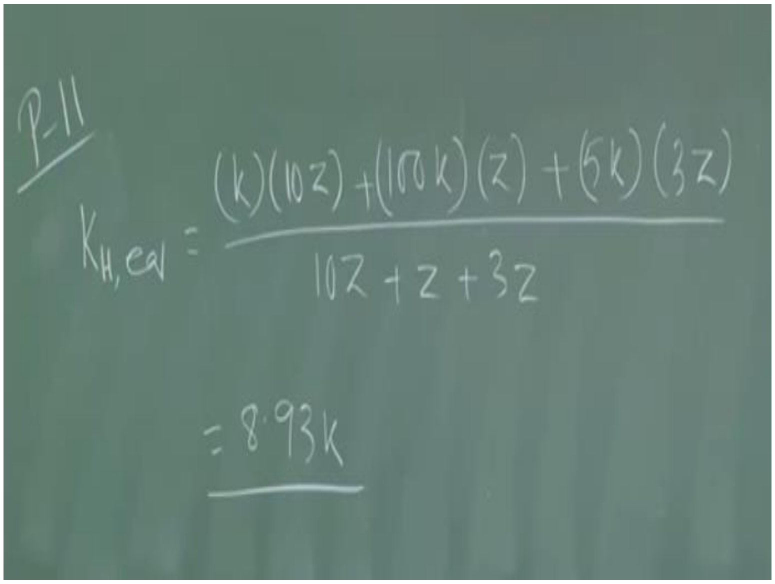

So, now if you put the values so you will be getting K H eq is equal to k into 10z right that is the permeability of clay layer multiplied by the thickness of the clay layer plus 100k that is the permeability of the sand layer that is the second layer basically into and what is the thickness, z plus 5k, that is the permeability of the silt layer and the thickness of the silt layer is 3z divided by 10z + z + 3z which comes as 8.93 k. So, if you know the permeability of the clay layer that is nothing but k okay you can find out the equivalent permeability of the soil deposit in the horizontal direction okay that is K H eq is nothing but 8.93 K okay. Now we will go to the second part that is the equivalent hydraulic conductivity in the vertical direction.

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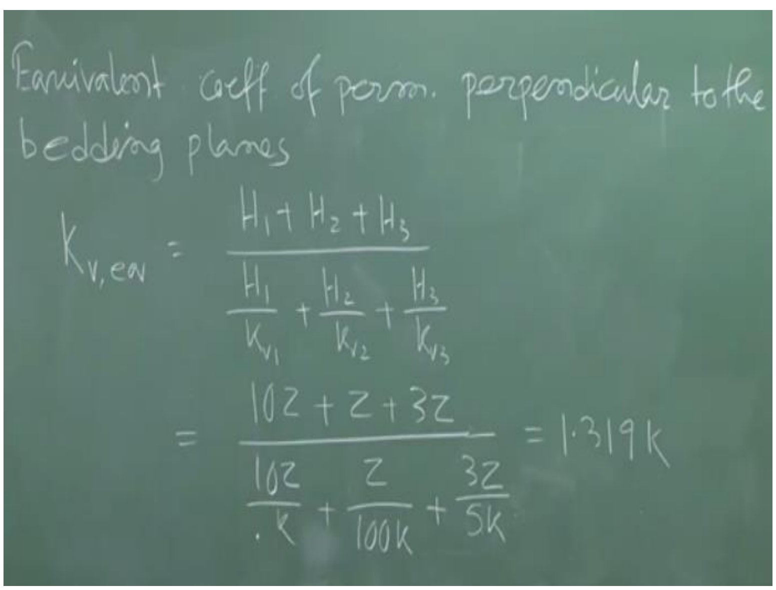

So, equivalent coefficient of permeability perpendicular to the bedding planes perpendicular means vertical direction okay, so that will be expressed and you know the expression so I need not to go through in much detail, K v equivalent was equal to H 1 + H 2 + H 3 divided by H 1 by K v 1 + H 2 by K v 2 + H 3 by K v 3.

Now here it is worth noting that the permeability in the horizontal direction for individual layer is equal to the permeability in the vertical direction in individual layer. We are not considering different permeability in horizontal as well as vertical direction okay. So, that means you can consider the soil is completely isotopic soil okay.

So, if that is true then we can put the values here 10z that is H 1 + z + 3z by 10z by k + z by 100k + 3z by 5k which gives me 1.319 K okay so that is the equivalent conductivity equivalent hydraulic conductivity in the vertical direction that means perpendicular to the bedding planes okay. So, that these 3 problems in the last lecture we have taken 2 problems in permeability and we have taken 1 problem in this lecture in permeability. I hope that it will cover the concept or it will it will try to generate the background in permeability of soil.

Now we will start a new topic that is nothing but seepage in soil. So, this concept is basically based on the concept whatever you have learnt just now that is the permeability in soil and now basically seepage is very important to obtain I mean if you want to find out the uplift pressure below some hydraulic structure like dam, concrete dam or say how much water is seeping through from the reservoir to the downstream side. So, those kind of information if you want okay so you need to know or you need to develop the background in the seepage in soil. Otherwise it will be very difficult to understand the concept okay.

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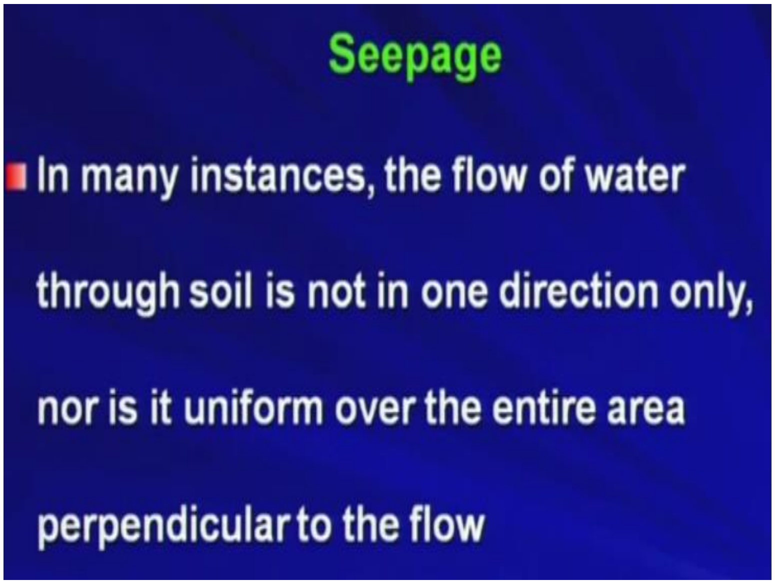

Now in many instances the flow of water through soil is not in one direction right. So, already if you see that water flow is happening, already you have seen from the permeability chapter that water flow is happening in the horizontal direction as well in the vertical direction. That means the water will find its own path. So, wherever it gets path it will try to follow in this direction. So, now depending on the I mean flow barriers or the flow resistance in different directions it will flow in respective direction.

So, if it I mean if your permeability in the horizontal direction is very less and if your permeability in the vertical direction is very high so depending on that it will adjust right, water will adjust the its own flow. That means in many instances the flow of water through the soil is not in one direction only nor is it uniform over the entire area perpendicular to the flow right. It is not uniform through the I mean on the perpendicular through the entire area perpendicular to the flow. That means if the flow is happening in this direction, if this is the perpendicular plane say it will not be remaining constant right it will not be remaining uniform. So, it will be non-uniform flow along the same cross-section okay.

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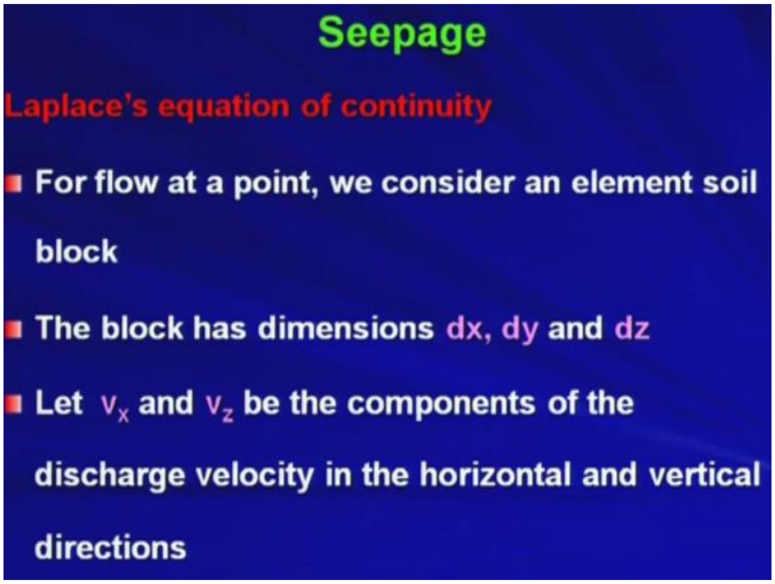

Now before going to the detailed analysis or the detailed concept for the seepage analysis we need to know the Laplace equation for continuity. At least we will develop the equation of Laplace equation for continuity for 2D system. You can extend that thing very easily for the 3D flow okay. So, for flow at a point we consider an element soil block so we will consider one element soil block. I will come to the figure later on.

First, I am going I am talking about those parameters. Now we are we will consider one small soil element okay. The block has dimensions of dx, dy, and dz along the respective x, y, z direction. So, you have dx in the x direction, dy in the y direction, and dz in the z direction. So, this is a kind of cube okay. Let v x, so we are considering 2D flow to develop this Laplace equation of continuity; however, it can be extendable to the 3D flow with very ease right.

So, let v x and v z be the components of the discharge velocity in the horizontal and vertical directions. As of now, we are very familiar to what is discharge velocity and how we can find out the discharge velocity from our previous topic in permeability right. So, v x and v z are the components of the discharge velocity in the horizontal direction that is so that means we are considering x direction as the horizontal direction and z direction is the vertical direction. So, v x and v z are the discharge velocity components along the x and z directions.

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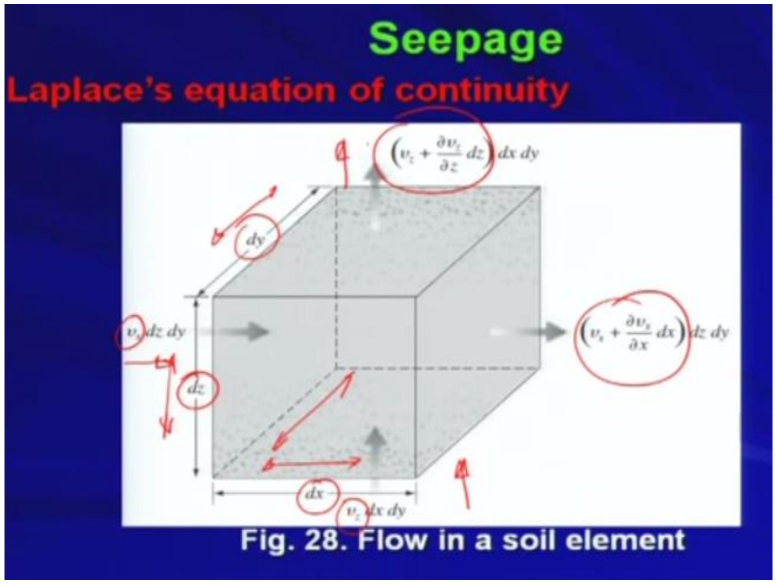

Now this is the basically soil block. We are considering soil element, small soil element. So, as you can see that this is the block. Along the x direction you have dx dimension, along the y direction you have dy dimension, along the z direction you have dz dimension and the flow is happening, that is inflow is happening from this direction, from left to right. So, that is the entry, that is the velocity at the entry level or the inflow that is v x is the velocity whereas when it is exiting the soil block it will be you will be having some increment because of your mathematics you know that incremental flow.

So, v x plus del v del v x del x dx. So, that is your incremental velocity when it is exiting the soil block. That means your inflow velocity is v x whereas outflow velocity is v x plus del v x del x dx okay and now that is happening on the cross section of this block that is v x into dz this side into dy. So, that is the cross sectional plane through which the inflow is happening. Even in the outflow the same cross section is remaining that is dx and dy.

Similarly, your flow is happening from bottom to top so that is v z that is your inflow velocity and that is your outflow velocity v z plus del v z del z dz and it is happening over the cross-sectional area dx into dy right. So, dx in this direction and dy in this direction right. So, along this cross section your inflow is happening in the vertical direction and along and your outflow is happening in this direction okay. So, have you understood this flow, flow is happening through the soil element.

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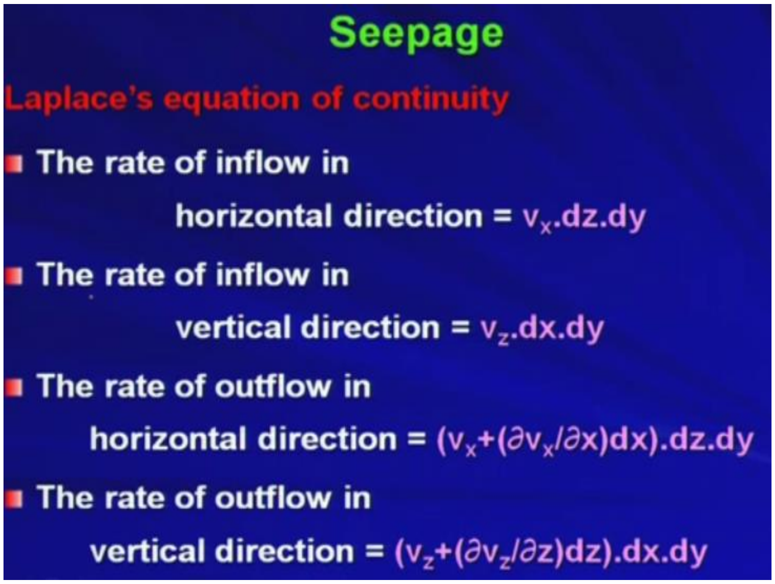

Now the rate of inflow in the horizontal direction is equal to v x into dz into dy as we have seen v x into dz into dy so that is your the rate of inflow in horizontal direction. Similarly, rate of inflow in the vertical direction is v z into dx into dy, dx and dy was the cross-sectional area through which the inflow is happening in the vertical direction and vertical direction the discharge velocity component was z, so that is the inflow.

Similarly, we can find out the rate of outflow in horizontal direction is v x plus that is the incremental velocity v x plus del v x del x dx into the area will be cross sectional area will be remaining same that is dz into dy. Similarly, we can find out the rate of outflow in the vertical direction that is incremental velocity v z plus del v z del z dz into dx dy. So, that will be remaining same in the vertical direction.

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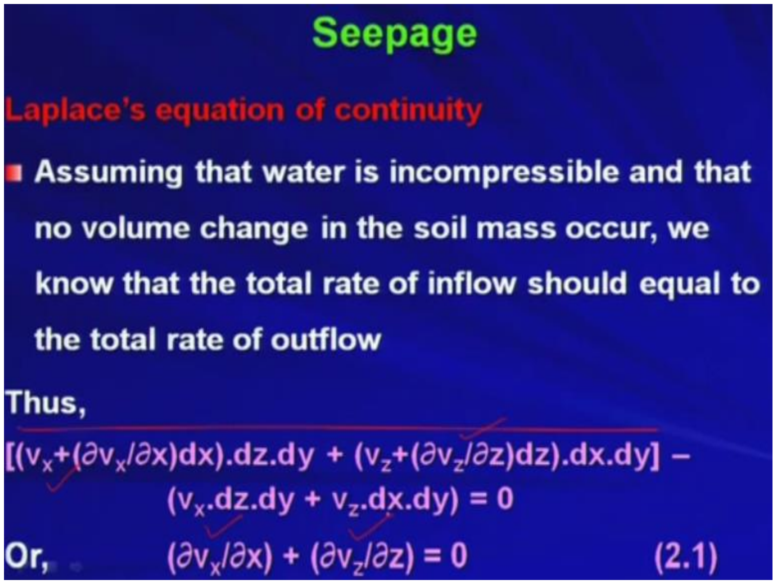

So, assuming the water is incompressible and that no volume change in the soil mass occur, we know that the total rate of inflow should equal to the total rate of outflow because if we consider that the water is incompressible that means the water is not showing any volume change during the flow because it is completely incompressible and you cannot expect any volume deformation or the volume change in the incompressible material as well as if we consider the there is no soil I mean no volume change in the soil mass is happening that means the whatever volume you started with and the volume will be remaining same even after flow then we can say that whatever discharge, whatever I mean rate of inflow was happening or was there the rate of outflow will be remaining same because you are not experiencing any volume change inside the soil element neither from the soil volume nor from the water volume.

So, whatever inflow is happening or inflow is occurring the same amount of water will be going out from the element so inflow must be equal to outflow. If that is true then we can write down, so this was this total was your outflow right. So, v x plus del v x del x dx into dz dy plus v z into del v z del z dz into dx dy.

So, this is basically outflow happening in the x direction. This is the outflow happening in the z direction, vertical direction okay and this is your inflow in the x direction, this is your inflow in the z direction. So, outflow must be equal to inflow. So, that is this equation. So, if that is true then from this equation I can write down del vx del x plus del v z del z is equal to 0 that is equation say 2.1 okay.

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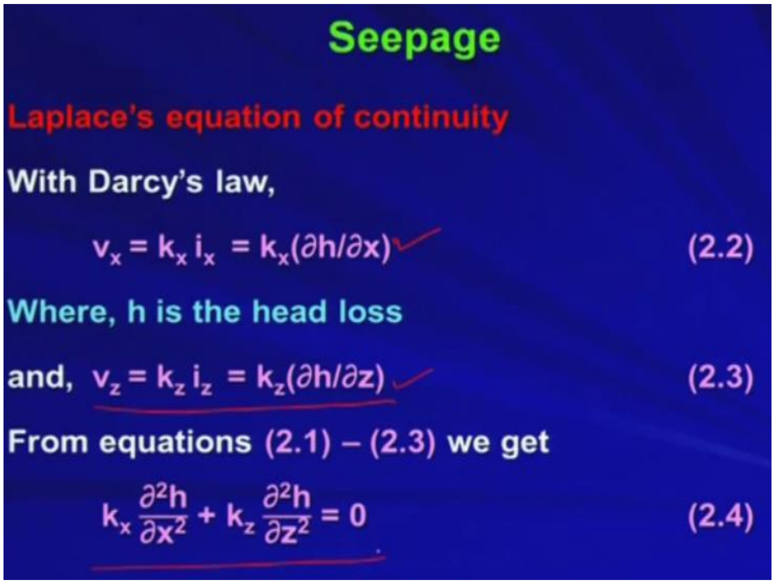

Now I can express v x as per Darcy's law whatever we have seen in the previous chapter that is permeability in soil. Through Darcy's law we can write down v x what is v x that is the hydraulic gradient in x direction say k x multiplied by sorry coefficient of permeability in x direction k x and hydraulic gradient in x direction say i x. So, k x i x. What is i x basically k x into del h del x where h is the head loss right.

So, along x direction you are experiencing some head loss and that is causing the velocity right from x direction from upstream to downstream or whatever is happening. So, v x is nothing but k x into del h del x okay. Similarly, I can write v z is equal to k z into del h del z. So, the same head loss is happening I mean this same kind of expression we will be getting through the head loss okay.

Now in place in equation 2.1 if we place this expression of v x v z and v x so we will be getting one expression very similar to that k x into del 2 h del x 2 plus k z into del 2 h del z 2 is equal to 0. So, this is your basically Laplace equation of continuity.

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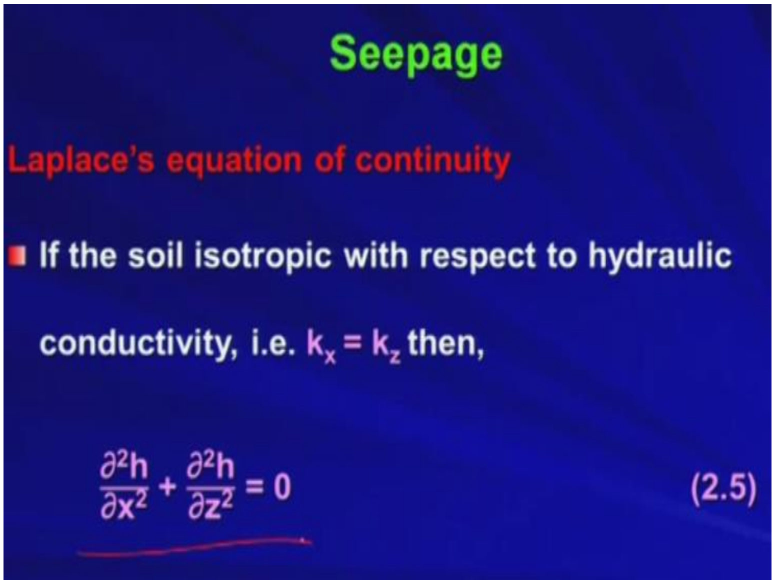

So, now if I consider the soil is isotropic that means in any direction, in x direction or z direction or y direction whatever direction you consider your hydraulic conductivity will be remaining same. So, if the soil is isotropic with respect to the hydraulic conductivity then I can say k x is equal to k z, am I right. So, in x direction as well as in z direction your hydraulic conductivity will be remaining same if I consider the soil is isotropic with respect to the hydraulic conductivity.

If that is true then from the previous expression or equation 2.4 will be boiling down to del 2 h del x 2 plus del 2 h del z 2 is equal to 0 and this equation is valid or this is nothing but the Laplace equation of continuity for isotropic soil okay. In an isotropic soil, we have seen the previous equation which is giving me the continuity condition.

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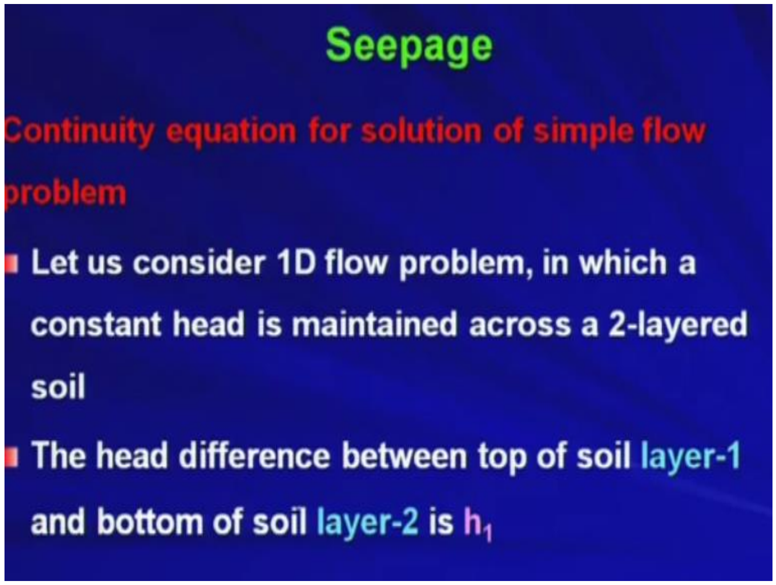

Now let us consider 1D flow problem in which a constant head is maintained across a 2-layered soil okay. The head difference between top of layer 1 and the bottom of layer 2 is h 1. So, we will see the figure in the next slide and you will see that, the figure is like this.

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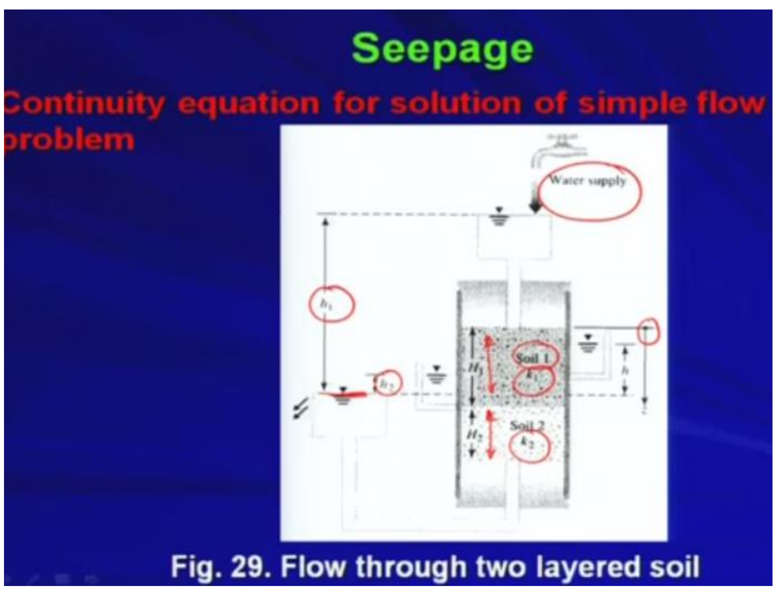

So, you are considering, this is your soil 1 okay and depth of say soil 1 is H 1, depth of soil 2 is H 2 and you are supplying water on top of the soil 1 okay and you are maintaining constant head basically and this is the head this is the water level okay water head at the exit of soil 2 that is the specimen 2 and the permeability of soil 2 is k 2, permeability of soil 1 is k 1 and this H 1 is the head difference as it is given in the previous slide.

The head difference between top of soil layer 1 and the bottom of soil layer 2 is h 1. So, h 1 is the difference between top of soil layer 1 and bottom of soil layer 2, the head difference okay and we are considering the z okay starting from the top of soil layer 1 and it is continuous say some coordinate axis.

So, z is 0 means the top of soil layer 1 z is h 1 means the intersection or the interphase between soil layer 1 and soil layer 2 whereas when z is equal to capital H 1 plus capital H 2 is nothing but the bottom of soil layer 2 okay and here we can see that h 2, h 2 is nothing but the head difference between this soil I mean when, we will be discussing that thing, so when it is happening at the bottom of from the bottom of soil layer 2 and the middle of soil layer 1 okay.

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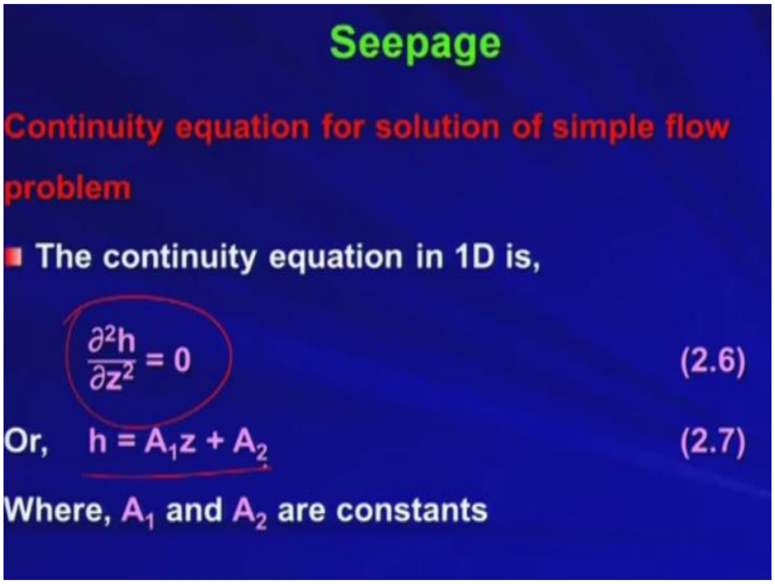

So, the continuity equation in 1D it will come down to because we are considering the flow in the z direction say so that will be nothing but del 2 h by del z 2 is equal to 0 okay. So, if you solve this differential, partial differential equation, it will be taking this form where small h equal to A 1 plus z plus A 2 where A 1 and A 2 are the constants right.

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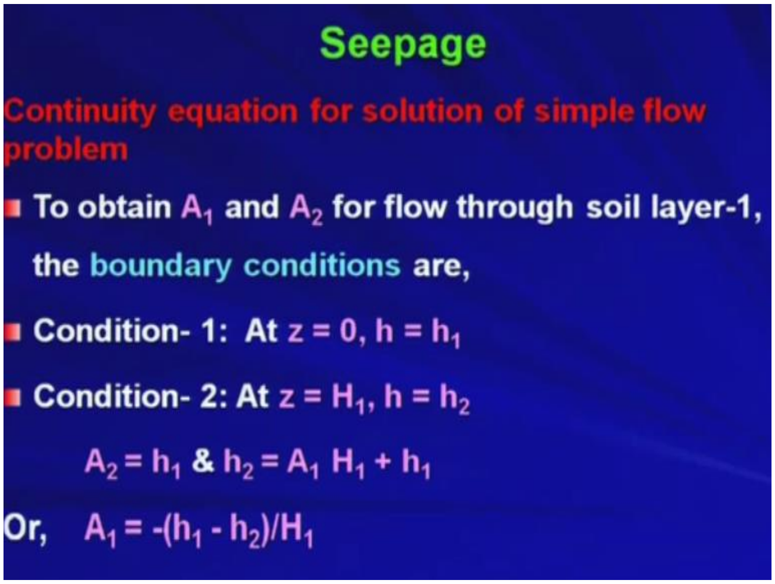

So, to obtain A 1 and A 2 for flow through soil layer 1 you need the boundary conditions. Now the boundary conditions are first condition is at z equal to 0, h is equal to h 1 right. The head was h 1. At z equal to 0 means at top of your soil layer 1 and at z equal to h 1 that means at the interphase your small h was equal to h 2 right as we have decided as we have seen in the figure that small h that is the head I mean if you consider the bottom of I mean the water level at the bottom of the soil layer 2 is your datum then small h 1 is the head at the top of soil layer 1 and at the end of soil layer 1 the head will be h 2 which is measured from the if you go back to the figure you will see that yes in this figure you see this is your h 2 okay as we have marked here okay. So, now if you put these boundary conditions you will be getting A 2 equal to h 1 and h 2 is equal to A 1 plus H 1 plus H 1. So, from there I will get A 1 is equal to minus h 1 minus h 2 by H 1 okay.

So, I will stop here today. So, we will be I mean continuing this thing in the next class and we will be seeing that you can put some different boundary conditions when the flow is happening through the soil layer 2 and based on that we can find out the final expression. Thank you very much.