

**Application of Soil Mechanics**  
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**Lecture - 29**

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**COMMON FEATURES OF  
SLOPE STABILITY  
ANALYSIS METHODS**

- Safety Factor:  $F = S/S_m$  where  $S$  = shear strength and  $S_m$  = mobilized shear resistance.  $F = 1$ : failure,  $F > 1$ : safety
- Shape and location of failure is not known a priori but assumed (trial and error to find minimum  $F$ )
- Static equilibrium (equilibrium of forces and moments on a sliding mass)
- Two-dimensional analysis

Last class we have discussed this about this slope stability analysis different methods, and factor safety a factor safety is equal to 1, then it is set to be the slope is failure if factor safety is greater than 1, then we can say that slope is safe.

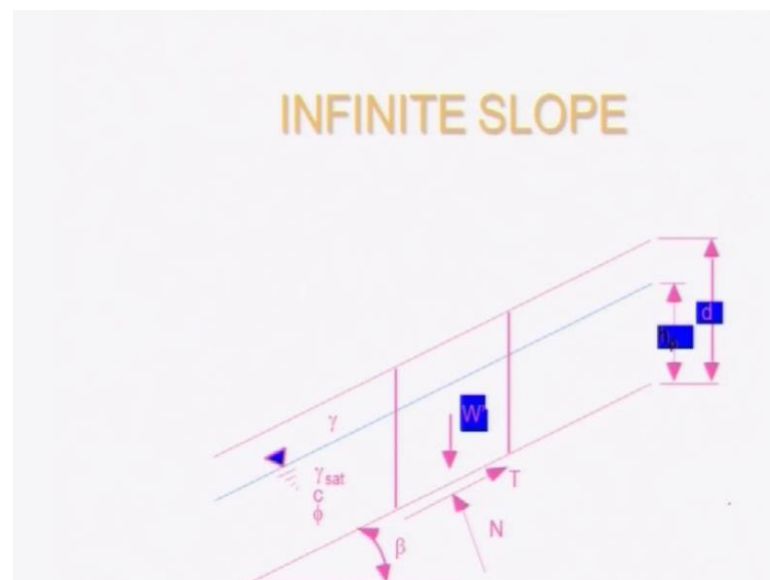
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## INFINITE SLOPE ANALYSIS

- Translational failures along a single plane failure surface parallel to slope surface
- The ratio of depth to failure surface to length of failure zone is relatively small ( $<10\%$ )
- Applies to surface raveling in granular materials or slab slides in cohesive materials
- Equilibrium of forces on a slice of the sliding mass along the failure surface is considered

Then there are two types of slopes as we have earlier, discussed on is your infinite slope analysis or other is infinite slope analysis.

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We have finish for these infinite slope analysis, it is a function of factor safety.

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## INFINITE SLOPE ANALYSIS

- $F = f(c', \phi', \gamma, \beta, d, u)$
- $F = (c' / \gamma d) \sec\beta \operatorname{cosec}\beta + (\tan\phi' / \tan\beta)(1 - r_u \sec^2\beta)$   
where  $r_u = u / \gamma d$  (different  $r_u$  for seepage parallel to slope face, seepage emerging, seepage downward, etc)
- For Granular Soil:  $F = (\tan\phi' / \tan\beta)(1 - r_u \sec^2\beta)$   
Dry Granular Soil ( $r_u = 0$ ):  $F = (\tan\phi' / \tan\beta)$
- For Cohesive Soil:  $F$  decreases with increasing depth to failure plane; if  $c$  is sufficiently large,  $d_c$  for  $F = 1$  may be large and infinite slope failure may not apply.

For  $c$ ,  $\gamma$ ,  $\beta$ , angle of this slope  $d$ , and  $u$ .

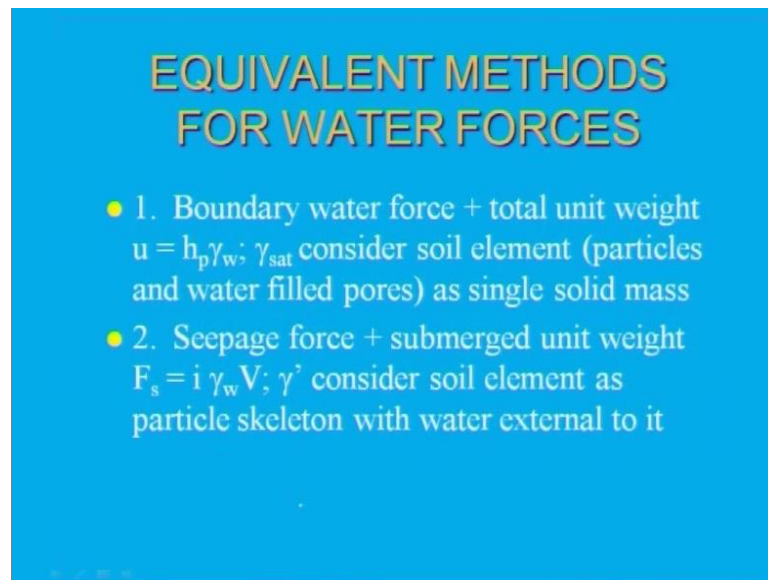
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## WATER FORCES ON SOIL

- Water fills voids and increase weight which increases driving forces
- Water also exerts pore pressures which decrease effective stress and therefore strength
- There are mathematically two equivalent ways of taking water forces into account in stability analyses

Then there are water forces, on soil consider.

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### EQUIVALENT METHODS FOR WATER FORCES

- 1. Boundary water force + total unit weight  
 $u = h_p \gamma_w$ ;  $\gamma_{sat}$  consider soil element (particles and water filled pores) as single solid mass
- 2. Seepage force + submerged unit weight  
 $F_s = i \gamma_w V$ ;  $\gamma'$  consider soil element as particle skeleton with water external to it

So, of or this there is two methods 1 is equivalent methods for water forces case one is boundary water forces side water forces or add the boundary, it has been consider plus your total unit weight has been consider case two it is a seepage force plus the submerged unit weight has been consider we have discussed this boundary water force case as well as the this seepage force cases, then come back to your finite slope we have discussed this of plane failure surface, and in this plane failure surface were have the cases the planar failure surface of or block slides occurs that also we discussed, then your finite slope of circular failure surface by means of rotational slides method of slices, and particularly cohesive soils, and ordinary method of slices by means of felonious method bishop's simplified method, and spenser's method we have discussed, and after that now, we are going to start this finite slope. Of noncircular failure surface finite slope of noncircular failure surfaces.

So, case one in this case of apt is not circular failure surface case one is your wedge method second is given by janbu this is called janbu simplified method, and third one is your morgenstern price method. So, it is called simplified that by means of price method let me come back to wedge method in case of wedge method failure surface consist of two or more planes, and applicable to slope containing several planes of interfaces, and weak layers; that means, failure surface consist of two or more planes, and applicable to slope containing several planes several planes of interfaces, and weak layers generally force equilibrium has to be stratified.

In case of wedge method, and the assumption is that resultant of side forces on each slices either acts horizontally or at varying angles from the horizontal typically up to 15 degree typically up to 15 degree, then in this case if you look at this wedge method failure surface consist of two or more planes, and applicable to slope containing several planes of interfaces, and weak layers. So, failure surface consist of two or more planes in the case of wedge method that it will look at the layer 1, and layer two or I can say layer a, and layer b; that means, if you look at here this is one wedge, and other wedge third like I name it 1 2 3 4 there are 4 wedges if you take out this number of wedge let us consider any of the wedge.

So, this resultant of the side forces it will angle theta, and below this normal, and normal, and tangential force normal forces at act as an angle theta m. Now, in wedge analysis equilibrium of force in each slice is considered to adjust, and can consider to adjust the inter slice forces, and balance them resulting in a correct solution, if you look at here a typical example has been consider. So, for layer 1 this is a four feet height layer one of soil is a four feet height layer two layer two soil is a ten feet height with these the properties has been given gamma c, and 5 prime for layer 2 c is 0.

That means it is a cohesion less soil for layer 1 5 is equal to 0 this is a cohesive soil. So, considering 1 2 3 4 that core wedges let, us start with this start with this part one number one ways. So, this is your number 1. So, it is weight component mean the resultant of normal, and shear force on plane a b let this plane resultant of normal, and shear force on plane a b if you look at here resultant, and normal forces will act in the plane a b now, go back to you slide to if you go back to your wedge two this is the wedge two the forces coming here wedge two will be if you go back to your normal force slice will act, and the plane in the plane b c, and this is your plane b c, and this 1 if I take it out if I look at here this 1.

Is your side force between slice 1, and slice 2 it this my slice force between slice 1, and slice 2 this is slice one, and this slice 2. So, in this case this is you slice forces whatever the slice forces coming in slice 1, and slice 2, and this part is your cohesion force this part is your cohesion force this is your cohesion force acted on the force acted on the plane b c along the b c what is the cohesion force, because here 5 is equal to 0, and c is that; that means, this is purely cohesion soil in this case this force equilibrium that polygon has been drawn. So, this is a trial solution trial solution for assume factor safety

of 1.5 1.5.

Now, if I come back to slice, third slice look at here third slice. In case of third slice is if you look at here this part your this part your side course between slice three, and four slice forces side force between slice three, and four this is my three, and this is the four this is your side force these are only your side force acted on the this, and this will be there, then this is your  $w_3$  if I complete this equilibrium condition has to be satisfied the force colygon polygon has be completed. So, this your weight  $w_3$  acted I put it here,, but practically, and here it is you side force acted of, and this part this force if you look at here from here to here it says side forces between slice two, and three slice two, and three if you look at here there are two side forces come in to picture for slice three one is your side force.

It for slice two, and three acting in this plane similarly for slice three, and four side forces acting in this plane. So, partic this part this is your representing side forces acting between the slice two, and three between the slice two, and three, and this is representing this is representing particularly this part if you look at here this part this is representing in your side force between three, and four between three, and fourth, and this is your weight of your slice free, and straight line forces normal force on ct normal force on c d it acted on this normal force has been brought it, and this part this part is your cohesion force cohesion force on c d, because here if you look at the this layer to there is a cohesion force c is equal to 5 point.

For square 5 is equal to zero. So, this force polygon has to once this this is a in equilibrium condition; that means, this force polygon has to be completed from there you can find it out what is your factor safety or it to get your factor safety people look at here side force between three, and four, then the for four polygon for case one slice case two slice case three slice case four this is a case of case four means slice four slice four if I take this slice four, and slice number four, and this is your weight this component is your weight component weight on of this slice, and this is your cohesion component on the four on the phase de in this phase de this is your cohesion component this is your normal force normal force in de this your normal force, and this part is your.

As if you look at this once I start from slice 1 2 3 4 the moment I go to the slice four, and this part of your this is the, because this is not a complete force polygon. So, whatever

this were getting this is your on balance force of on slice four this balance force on slice force; that means, slice four; that means, whatever factor safety you've assume that is not correct means the methodology how we are going for ways analysis if you look at here once again, I am explaining for example, you consider this is a two layer soil, and this your slopes, and take it into wedge number of wedges or number of slices 1 2 3 4.

This slices has been assumed this slices has been assumed you can take it also more number of slices in between also. So, then what what will happen you start with one by one slice slice one slice two slice number three slice number four, and each slice this equilibrium condition has to be satisfied to satisfy the equilibrium conditions look at the forces acted on this considering this side forces in these method which method they have considered the courses between, this slices that mean slice 1, and slice 2 slice forces has been consider not like earlier the side forces has been cancelled. So, each slices this course polygon has been drawn, and it trail assume you take it trail assumption of your factor safety you start with a factor safety assumed value of factor safety let us say assumed value of factor safety.

Let, us we have trail with these factor safety is equal to 1.5, and start with these slice one two three four once you start taking the this assume factor safety 1.5, and we've started from the top to bottom we find that this is our on balance force; that means, this force polygon is not completed; that means, equilibrium condition has not been satisfied, then what will happen, because you assumed a factor safety of one point 5 depending upon these what is your on balance force accordingly whether it is a positive or negative side accordingly or larger or smaller accordingly you can take your factor safety either you can take one point three one or may be 1.6 depending upon that you assume this trial factor safety till this force polygon will complete till this force will complete; that means, equilibrium condition satisfied.

So, this is the we have we solve the this particularly ways analysis if it is a non circular case, then how you are going to solve this case, then second method is Janbu simplified method. So, Janbu has given a method of slices applicable to circular as well as non circular failure surfaces Janbu as given both for circular as well as non circular failure surfaces, and factor safety is a function of  $c$   $b$  weight, and  $f_0$  is a correction factor.

If you look at the this factor safety is a function of it is retain in terms of  $f_0$   $f_0$  is a

correction factor that with depth with length ratio depth of the slope, and length of the slope ratio that depth to length ratio of sliding mass, and type of soil. So, it is based on what Janbu has simplified method given a simply correction factor has been given correction factor has been given based on your length of this lighting mass as well as depth of the sliding mass, if you look at here this is my depth of this sliding mass as well as the this is the length of the sliding mass length, and depth  $h$  sliding mass of each has been you can intro based on that, because it has to satisfy your equilibrium condition a correction factor has been applied at this method can be applicable for both circular as well as non circular failure surfaces. So, Janbu has given particularly in this case if you look at here as I say if this is my non circular.

Failure surfaces this is the depth  $h$  this is the depth  $h$ , and from here to here, because these are all non circular these are all non circular failure surfaces or, then this is the depth  $h$ , and from here to here top to bottom from here to here this is your length best on the length, and depth they have given your correction factor  $f_0$  of different kind of soil for guess one this is the case of ratio of  $d$  by  $l$  depth to length ratio it is very this is the case of your factor safety of not factor safety it is your correction factor with these for  $f_0$  is equal to 0 this is your case  $c = 0$  for  $c$  is equal to 0 in this case once  $f_0$  is equal to 0; that means, this case is soil, and this is for  $c$  edge  $f_0$  as well as frictional cohesion less soil this is your purely cohesion less soil.

This correction factor chart has been given, then third one is your price method in this case no assumption is made the advantage of the both this methods over the this price method over the this last two method no assumption is made regarding inclination or point of application resultants, and if you go back here if you go back to your wedge method what is your assumption. If you look at this assumptions resultants of side force on each slice either acts horizontally either acts horizontally or at a varying angle from the horizontal typically up to 15 degree; that means, if this my slice the resultant of side force.

The resultant of side force it will act either the resultant of the slice force it will act either horizontally look at here, this way it will act or attend varying angle with horizontally if this is my horizontal at in varying angle maximum varying angle will be 15 degree; that means, if I make this angle if this make this angle more than 15 degree it may possible that the resultant may at an angle more than 15 degree this is the limitation this is the

limitation in this case of this which method as well as Janbu's method. So, to overcome that Morgenstern's method, and Price.

They have not turned any assumption they have not made any assumption particularly for non-circular cases of the failure surfaces. So, no assumption is made regarding inclination or point of application that means, there is no assumption whatever this inclination here accordingly the point application point of application of result, and will be acted, and these are determined as the part of the solution, and these are from determined from part of the solution there is no assumption the actual result, and forces where it is acted with respect to horizontal this has to be determined as a part of the solution, and because other methods are very simplified it requires computer programming. So, solve the basic equation this basic equations this basic equations it is somehow or somewhat you can say that it is exact, but not practical it is more exact, but it is not practical.

So, these are all basic summaries of these slope stability analysis, what are the methods, and reference has been given reference has been taken from this Duncan 1987, and your *Soil Mechanics Design Manual* by means of your departmental of navy in United States US in nineteen eighty two one more thing I want to show it now, if I go back to there are other cases also if you go back to previous one what I have discussed about rotational slip analysis or rotational analysis for untrained frictional frictionless failure total stress.

Generally for fix, and frictionless failure; that means, if it is a untrained frictionless failure generally what happens we follow total stress analysis this is for cohesive, and frictional failure method of slices as well as Bishop's conventional method has been used we have discussed, this method of slices also Bishop's conventional method also we have discussed now, if I go back to this for rotational slip for total stress analysis to look at this total stress analysis for total stress analysis, it is assumed that  $c = 0$  is your frictional resistance in undrained conditions is equal to 0. So, strength parameter those in undrained soil particularly strength parameter are those in undrained soil this is your strength parameter for those undrained soil if we look at this total stress analysis this is your circular.

Slip surface in this circular slip surface you identify the center where the failure surface

has been made, and this center with radius are, and find it out the weight find it out weight of this weight of this enter soil mass. So, within this failure surface that is your weight  $w$ , and it will acted in the center of gravity  $cg$  how far this weight component is far away from your center of rotation from point  $o$  this distance is your point  $e$ , if I take considering the equilibrium condition taking in taking moment at the point  $o$  here factor safety is coming about to be  $c r \text{ square } c r \text{ square } \theta$  by  $w e$  if you look at here factor safety is your resisting or retaining moment divided.

By disturbing moment divided by your disturbing moment if you come back to your restraining moment in this case  $c$  are square  $\theta$  this unit. If you look at here this cohesive force  $c$  this is acted on by  $c$  with these the total length we can get it from, because this cohesive force from where it will act this unit cohesion or cohesive force it will act along the very ferry of your failure surface. So, if you look at a the failure surface what we have assumed is your rotational or circular slip surface this is acted along  $a$ , and  $b$ . So, unit cohesion in to length  $a$   $l$  has been found it by  $r$  in to  $\theta$  this is your radius it will acted on this angle is your  $\theta$ . So, this restraining movement will come out to be  $c$  are  $\theta$ .

It will be acted over if I take this it will multiplied into  $r$  above point  $o$ . So, this will a  $c r \text{ square } \theta$ ; that means, resisting moment, then which  $l$  is your disturbing moment this is, because of your weight component of the soil mass along, this slope along the failure surface that will be at a at a distance this is your weight component at a distance  $e$  from your center of rotation  $o$  point  $o$ . So, this will be  $w$  into  $e$  in this case a  $w$  is your weight component this is your  $w$  is  $o$   $l$  compo  $o$  at component it acted at the point of center of gravity  $cg$  here it is your  $c g c g$ , then  $c g$  is distance is at a distance  $e$  from point  $o$ . So, if I put it  $w$  in to  $e$   $w$  into  $e$  this is your clockwise, and  $c r \theta$  into  $r c r \theta$  into  $r$  that means,  $c r \text{ square } \theta$  it will be anti clockwise.

So, factor safety is your restraining moment divided by disturbing moment which is equal to  $c r \text{ square } \theta$  by  $w e$  were  $cg$  or cohesive strength either you can it generally Pascal or kilopascal, then  $r$  is as I said slip circle radius this is your slip circle radius are is equal to slip circle radius in meter  $\theta$  is equal to slip sector in radius this is your  $\theta$  this is in terms of radian  $w g$  or weight of sliding sector,  $w$  is equal to weight of sliding sector  $e$  is equal to eccentricity  $e$  is equal to this weight component how far from there is equal to eccentricity this is your in moment this particularly this has been a total

space are analysis.

In this case the assumption is in case of total stress analysis  $\sigma_u$  is equal to zero  $\sigma_u$  is equal to 0. Now, in this method of slices there are two cases one is your of slice does that has given by swedish circle method. So, it is use for cohesive soil as well as end also frictional soil  $c$  as well as  $c$   $\phi$  soil if you look at here this is my circular failure surface circular failure surface assume circular failure surface  $r$  is your radius, and  $\theta$  this failure surface make an angle  $\theta$ , then it has been made into finite number into slices. So, as I said last class  $n$  number of slices you can go up to 1 2 3 4 5 up to  $n$  number of slices depending upon that how many number of more slices you'll take it more number means your result correctness of the result will be good.

So, it has been taken one two three four 5 six for example, six slices has been consider any slice you to take into take into consideration for any of this slice find it out four equilibrium, then find it out what are the forces going to act if you look at this forces going to act in this cases letter forces  $l$  one, and  $l$  two your weight component this is your weight component acting vertical down what normal component, and as well as well as your tangential component if I take into consideration; that means, what is your factor safety as I say this is your resisting moment by disturbing movement. So, in this case resisting moment is your chosen course it would be anti clockwise  $c r \theta$ , then plus plus your  $n$   $n$  is your normal component  $n$   $n$  into ten 5  $n$  is a disturbing moment for each slice.

If I take into  $t$   $n$  tangential component is coming that is your  $t$   $n$  that is your disturbing moment. So, summation of; that means, if I of  $n$   $n$  number of slice is one two three one two three four 5 six seven eight or maybe be said in this case seven number of slices; that means, of each slice you have to find it out normal component as well as tangential component. So, summation of normal, and tangential component you make it. So, you can find it out your factor safety so; that means, one two  $n$ , and here one two  $n$ . So, a factor safety is equal to for method of slices  $c r \theta$  this is for  $c$   $\phi$  soil  $n$  ten 5 divided by tangential component the  $n$ , then there is another chance it may happen that there will be a tension cracks, because for particularly cohesive soil if cohesion component is there is a possibility that there is a tension cracks what you mean by tension cracks that,

There will be a gap in the soil that will be a gap in the soil in this case you will look at

here this is the tension crack this tension crack you can find it out up to at what height up to what height tension crack can develop. So, what will happen, because of the tension crack what will happen it will reduce this angle of sliding sector it will reduce this angle of sliding sector this  $\theta$  will be reduce to  $\theta'$  sliding sector this angle will be reduce, because of your tension crack, then you'll have to find it out your height of tension crack frictional soil frictional soil you'll have to find it out cohesive soil you have to find it out your.

Height of tension crack you can get it  $2c$  by row is equal to unit bit of soil of  $2c$  by  $\gamma$  can put it  $c$  is equal to cohesive strength either it is Pascal or kilo Pascal 2 times of  $c$  by  $\gamma$  or row. So, you can find it out, then from there for this is for frictionless soil, and for cohesive as well as frictional soil; that means, frictional as well as frictionless soil; that means, particularly  $c$  pie soil typically if I say  $c$  pie soil this is for  $c$  soil this is for  $c$  pie soil the height of tension crack so you can find it out  $2c$  by row or  $\gamma$   $2c$  by row or  $\gamma$ , then 45 degree plus 5 by 2 from there you can find it out height of tension crack, then once you get the height of tension crack, then.

Next part is your as I discussed what is the effect of the height of tension crack, because of the effect of height of tension crack this angle will be reduced why this angle will be reduced, because once you draw the failure surface the failure surface will go from here here to here it will end here earlier case it is what happen once there is no tension crack the failure surface will go propagate, and it will end up to the surface look at here. So, in this case what will happen this, because there is a tension crack there is a tension crack the surface will go off to the tension crack; that means, what you have to find it out you have to identify what is a height of tension crack once you get a height of tension crack, then you can assume failure surface you can you can draw your failure surface, then your  $\theta$  will be modified to  $\theta'$ , because of tension crack this  $\theta'$  will be reduced one this  $\theta'$  will be reduced.

So, your weight as well as  $c$  component also, that resisting moment also will be reduced, then second part second concern your first concern in this case tension crack second concern is your location of slip circle center this is my slip circle this is my slip circle how do I locate were this slip circle center is line were exactly, it is line I do not know where is the center where is the radius. So, that I can draw the slip circles. So, this is there is no simple way this generally determine by means of trial in error by means of

trial in error. So,  $f$  is in this case  $f$  is more sensitive to horizontal movement than vertical movements factor safeties more sensitive.

Horizontal movement than your vertical movements horizontal movement than your vertical movements. So, it has been the center has been found out by means of a trial, and error method. So, by trial, and error you will get your slip circle of this center with a critical failure surface; that means, every you draw the failure surface one two three four, and find it out your center, then trial, and error you'll identified for critical failure surface were exactly your center will be located this is your second concern pass concern is your tension crack second is your location of slip circle, then will discuss little bit more about your effective stress analysis may be next class I will stop it here.