Surveying
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Module - 2
Lecture - 3

Basic Concepts of Surveying

Welcome again, because we are here in basic surveying video lecture and today, we will be talking about, again, module number 2 and ours will be lecture number 3 in module number 2. The module is 'Basic Concepts of Surveying'.

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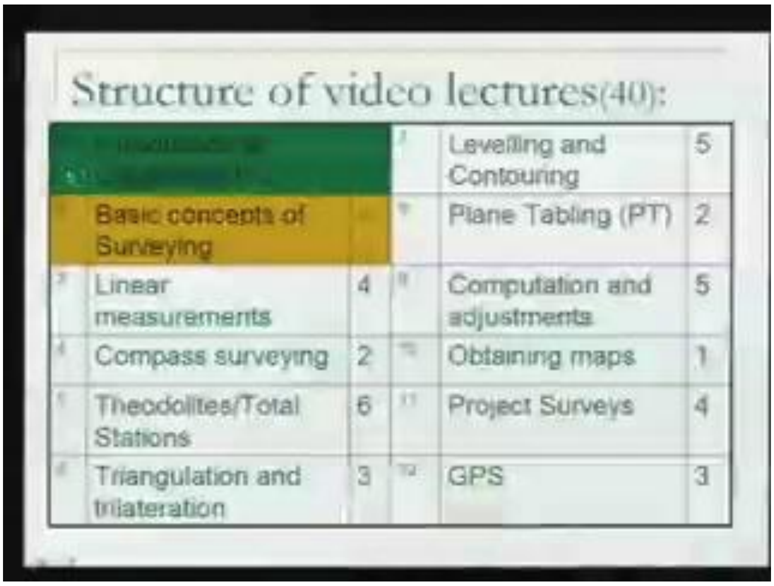

Well, we have already done module 1; we are in module 2.

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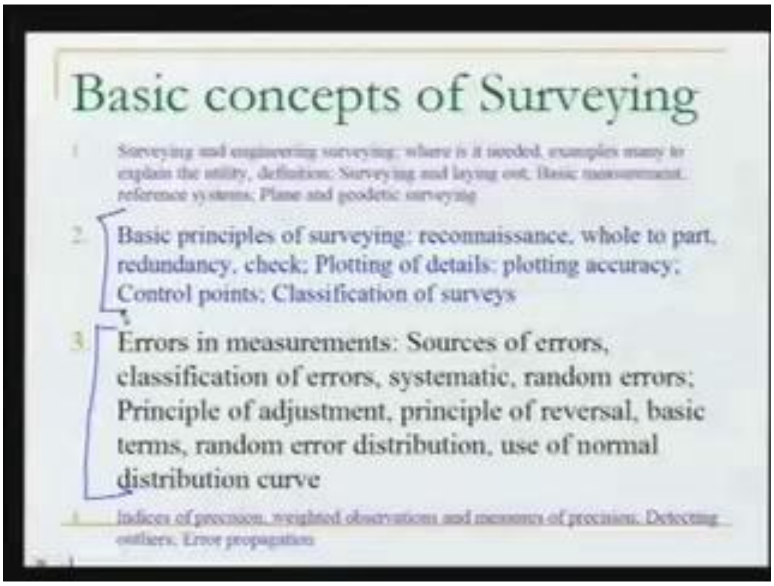

In module 2, what we will be discussing today is on chapter 3, or rather, the lecture number 3. What we did in our last video lecture was very important, and let us recapitulate that. We talked about the basic principles of surveying. These are the principles which we should always keep in our mind whenever we are going for surveying because they are very, very important points, and they control the kind of the work which you ultimately produce at the end of your job. Well, number one is the reconnaissance. What we did in our last video lecture, we started with an exercise - an exercise in which we started - well, there is a little ground, and we want to make a map of that ground.

So what all steps are involved? The very first one was recy survey or reconnaissance - we go to the ground, observe it - this is very important point in any surveying job. Then the second one; we decided ‘Okay, we are going to survey this particular area by a particular method’. In our last video lecture we chose the method of triangulation. For this triangulation, we need some points in that area so that we can form the triangle. Now, why we are forming the triangle? Because we wanted to bring the skeleton of the area in our plotting sheet. So, that idea is called ‘working from whole to part’ or ‘establishing the control network’. We want to establish first a very accurate control network, and we will be using that control network later on for our other surveys. So, we do not work from part to whole, rather, we work from whole to part. First we make the entire skeleton; we are not bothered about the details initially.

What we are bothered about initially is the skeleton - it has to be accurate, and the details can be filled in later on. Then another thing - we must always go for the redundancy. Redundancy in the observation; the meaning is, we should have multiple number of observations. Then we also talked about the check - we should have something, some observations, which are not used in our general plotting work, but these observations we will use later on. At the end of this final map, when the map is produced, when the final results are there with us, we want to check those results, so we should have something called a check.

Then, we are talking about plotting of the details. We can plot the details by various means. We saw that, if two points are fixed (Refer Slide Time 03:31) - we know this length - so these two points are fixed, a third point is to be fixed. What we need to do in order to fix this point: either we measure both the lengths, or we measure both the angles, or we measure one length and one angle, or maybe one length and one angle this way, or maybe - one more possibility - we drop a perpendicular from this point on this line (Refer Slide Time 03:52). So here, we know the distance here, along the lines, which we in our last lecture said X, and this particular distance - perpendicular from the point - we said this as offset. So by measuring this perpendicular and this distance, we can plot this position or this point with reference to these two points. So, we can plot our details that way.

Then, we were talking about very, very important thing, that is, the plotting accuracy. Plotting accuracy - in our map, the minimum point or the minimum thickness of a point that we can make is, generally, we take it as 0.25 mm. What is the meaning of this? Within that 0.25 mm we cannot see anything, because it is a single point. A line - thickness of the line - we cannot see any detail inside. Well, if that is the case, the corresponding distance in the ground - how do you find it? We find it by multiplying plotting accuracy by the scale so we know the corresponding distance there in the ground. So, the meaning is: because we cannot see anything within that point in our map, so we cannot plot any detail which is within that distance in the ground; the corresponding plotting accuracy distance. So we should always keep in mind whenever you are deciding about your scale: 'Okay, my scale of survey is this'. So you know what kind of details, how accurate you should observe in the field. So plotting accuracy is an important thing.

Then finally, we are talking about classification of survey - how we classify. There were different classifications: based on the instrument used, the area - the type of the area, the functionality - for what purpose you are using it - so there are different classifications. So this is all we have discussed in our last lecture.

Today, in our this video lecture, we will be talking about errors in measurement, and we will be looking into sources of errors, classification of errors, systematic; random errors, principle of adjustment, principle of reversal - maybe, then basic terms, random error distribution, use of normal distribution curve and all that. So, we begin now with our this lecture - we are going to talk about errors in measurement. We said that in surveying, the measurements are the basic things we need to measure the things. So, we need to also talk about the errors in measurement.

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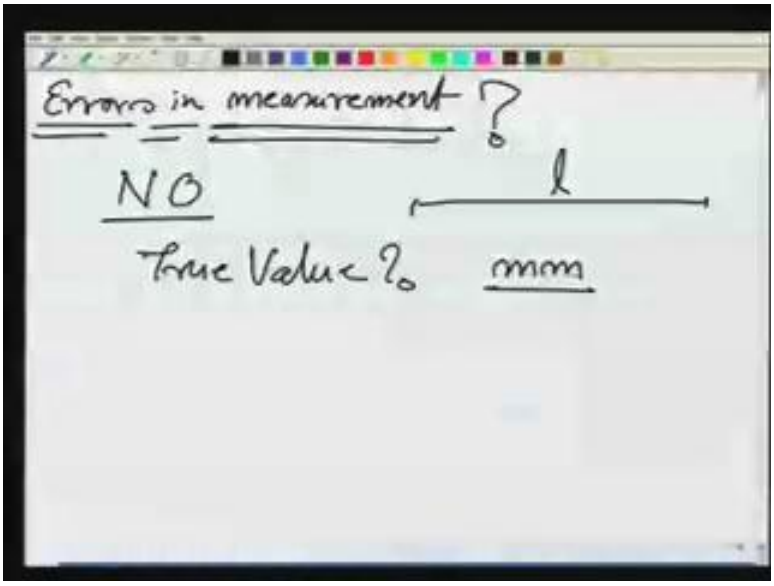

What is the meaning of this? Let us start explaining this. This is a very, very important area and we must know about this whenever we are doing the surveying. Well, let me give you an example. Here (Refer Slide Time 06:51), I have got a scale and in this scale we have some graduations. Any length; for example, let us say I want to measure a length here using this scale. So, what I will do? I will put this scale along this length and I say, 'Okay, what is this length?' So, this is a procedure of taking the measurement. We are taking an observation; we are measuring our length here. Now, in measuring this length 'l' (Refer Slide Time 07:19), what do we measure? Are we measuring the actual value, the true value of the length? Now, what is true value? True value is the value which is actually there. Are we able to really measure it with this scale? Now, the answer of that is: no, we cannot measure true value. Now, why? One very simple answer of that: this scale has got a least count, and the least count of this scale is in millimetres. We can measure up to 1 millimetre, or we can estimate within a millimetre; we can approximate within a millimetre. Well, when we are approximating, how good we are in the approximation? Are we really measuring the true length ‘l’? No, because we are limited by the least count of the instrument. So never, in practice, we can measure the true value of any length, angle or any geoinformation.

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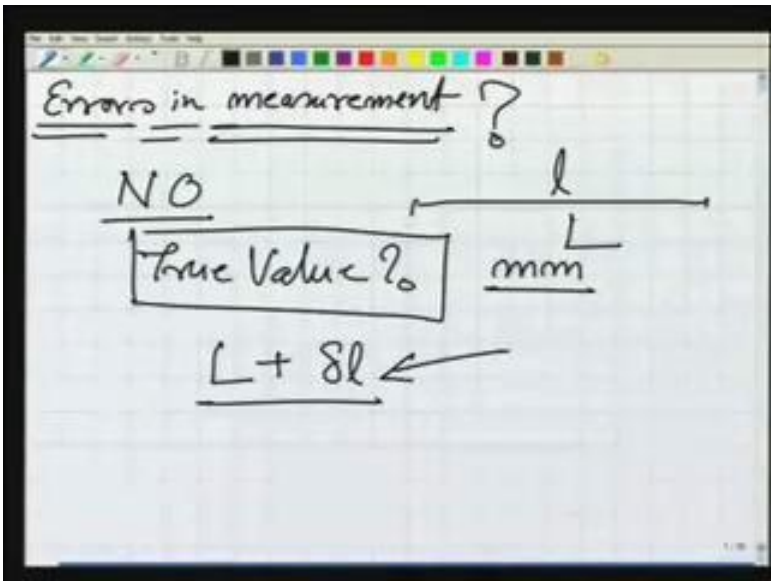

So, what we measure? We measure the true value - let us say we are writing it in capital for ‘l’ - plus some error term, because we are introducing some error here. So, this is the error term which we are introducing (Refer Slide Time 08:47). I gave you just one example of the error which is because of the least count; there may be many more sources of the error.

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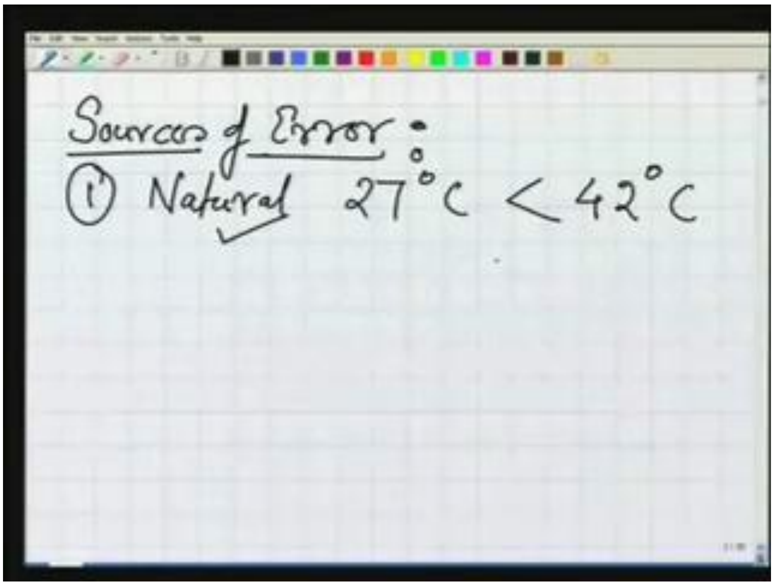

Well, let us start talking about those - what are the sources; the sources of error? Sources of error in measurement - well, let us talking that. Number one: are there any natural sources? Just think about that - let us think about this scale only .This scale says that this is one meter, which is written over here - I can read; from here to here (Refer Slide Time 09:34), it says this is 1 meter. So, when this scale was manufactured, when this length was given - 1 meter - it was given at a certain temperature. Let us say the temperature at which this scale was given 1 meter was 27 degree Celsius, and right now, when I am working in the field, the temperature here is not 27 degree Celsius - rather, more than that 42 degree Celsius. Now what happens because of this? If the material of this scale is such that it expands - all the materials will - it should expand with the temperature. Still, I see it is written 1 meter here, isn’t it? But because of the temperature, the actual length of this scale is not 1 meter, rather, slightly more than 1 meter. Now, if I measure any length with this scale which has expanded, our length measurement will not be accurate, rather, we are introducing some error. So this is one example of natural source of error.

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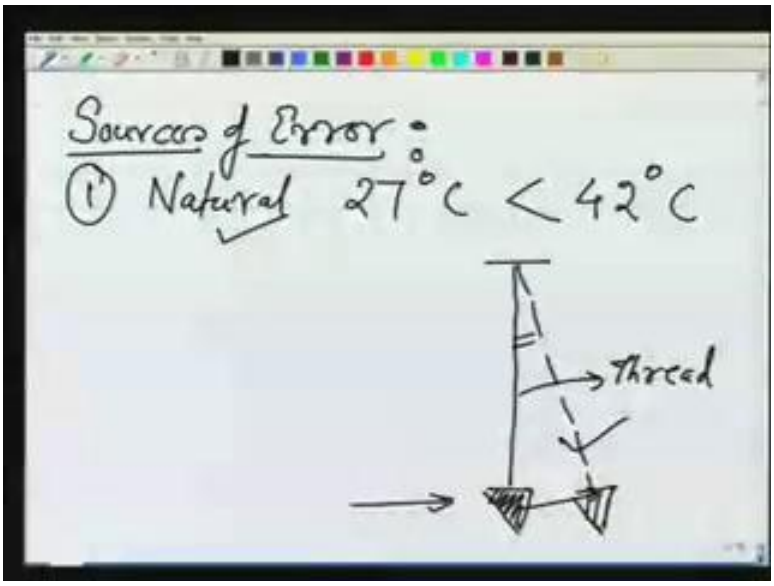

There could be many more - I can give you one more example. For example, let us say, many times you must have seen, as I am drawing here in the diagram (Refer Slide Time 10:57) - well, what I do, I suspend a heavy plumb-bob. What this plumb-bob is: here is the thread or twine (ddd 11:05), and here is a heavy bob, a very heavy material. We make use of this in order to find the direction of the gravity; obviously, because of this weight, it will align in the direction of the gravity. So, whenever we want to measure the direction of the gravity, we can make use of this simple device. We want to ensure whether a particular wall is vertical or not - you must have seen many masons using this plumb-bob to ensure the verticality. Now, how measure is coming into the picture? If there is some wind is blowing - it is not appreciable, but the wind is blowing there - because of the wind blowing, what will happen? A constant force will be acting on the plumb-bob and the plumb-bob will be now slightly displaced (Refer Slide Time 12:01). I am drawing slightly exaggerated one; it will not be this big, but it will be slightly swayed from its vertical position. So, what we are going to do? We are not able to measure the exact perpendicular or the vertical direction; the direction of the gravity; rather, we are measuring something else. So again, the nature is playing a role here. So, whenever we take the measurements, we have sources of the errors which are because of the nature.

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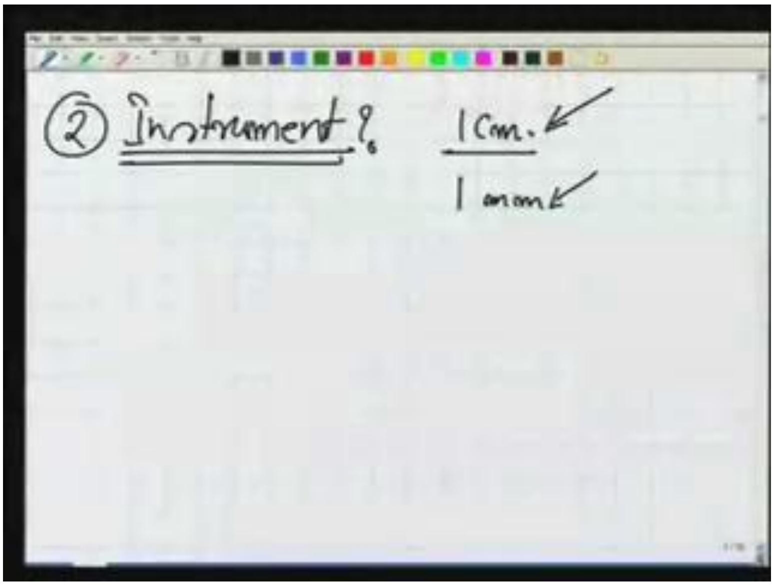

Then, the second one: the second source of the error is because of the instrument. Well, how - how come? I gave you one example that in this scale, if the least count - there is a limitation of the least count (Refer Slide Time 12:54). Let us say the least count in this case is 1 centimetre -in one case. In other case, the least count is 1 millimetre. Now can you start thinking the limitations? In this case (Refer Slide Time 13:13), we can measure more precisely, more closely, while in this case (Refer Slide Time 13:19), our measurement would not be that close. So, what is happening? Because of the instrument itself, there are some errors being introduced. I will give you one more example: well, this says '1 metre' (Refer Slide Time 13:30) - is it so? The question is, is it really 1 metre? Because this has been manufactured by someone; someone has manufactured an instrument, which may be the scale - the simple one like this - or maybe a sophisticated instrument. And this instrument says this is 1 metre, but is it really so? That is the big question. So, if it is not really so, it says it is 1 metre, but actually, if you measure it with some standard, you will find it is 1.01 metre. So we have some error included in our instrument, and whatever the length, we will measure with this (Refer Slide Time 14:14) ruler. So what we are doing? The actual length of this is 1.01 metre, but we are considering this to be 1 metre. The meaning is: if, there in the ground - let us say this length - if this length (Refer Slide Time 14:30) is 1.01 metre, and if you are measuring it with this scale which has actual length of 1.01 metre, then this length, which is actually 1.01 metre will be shown as 1 metre, isn't it? Because our scale is showing 1.01 metre as 1 metre. So this is one source of error which is coming because of the instrument.

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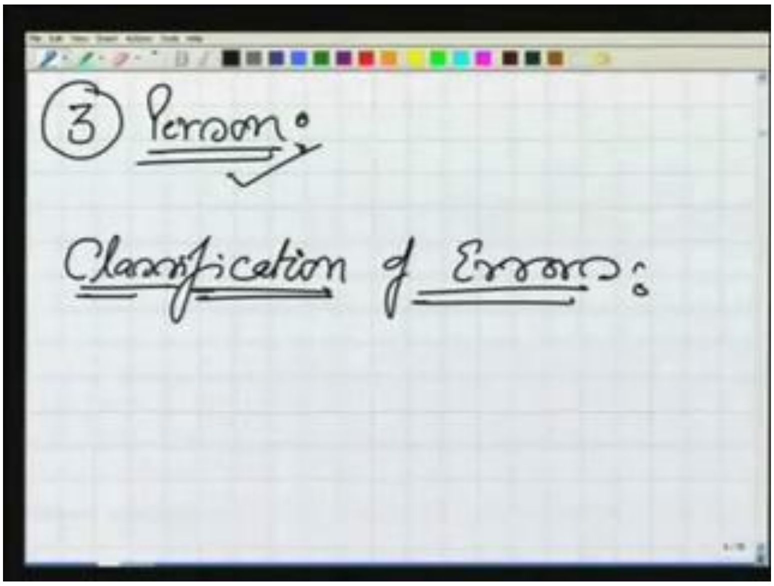

Well, the third one - third one you can guess very well. What is the third source of the error? It is because of the person, or 'personal error'. The person who is working there - well, I do not want - I am a bit tired; I am not happy today, and I am taking the measurement. The weather is not very good; it is very hot, so I am not very, you know, happy about working. So what do I do? I start committing a mistake when I take the measurement. Also, you know when we take the observations over a scale, where our eyes should be - our eyes should be exactly over the graduation (Refer Slide Time 15: 37) or exactly over the mark for which you want to measure the length, and then we see the graduation corresponding to that. Well, one source of error could be if I displace my head here or here - the parallax. So what is happening? Now, the person who is working with the instrument is introducing some errors. So, there are errors because of the person also.

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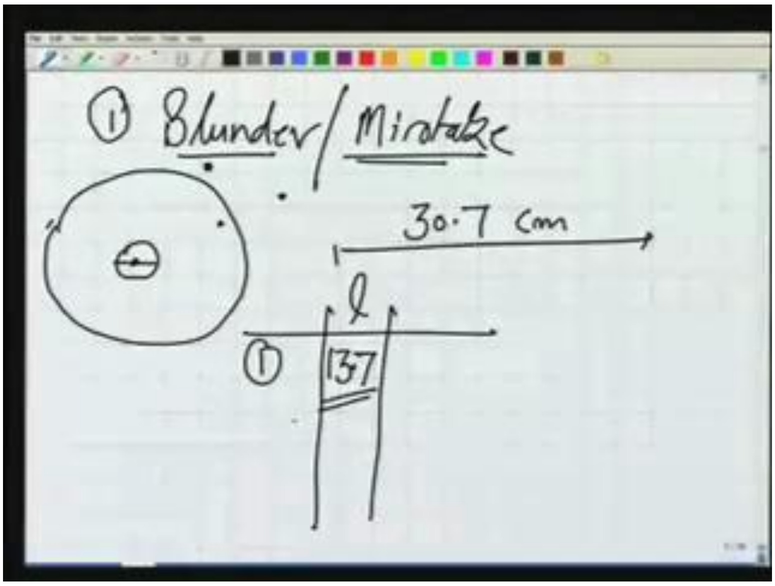

Well, having seen all these types of the errors, what we will do now, we will go into another classification which we say, 'classification of errors'. So far we are talking about the sources of the error; now we will see classification of errors. What are their types? What I will do, I will start with one example, let us say, the number one case, because it is the number one type of error we say to be blunder or mistake. Well, the example in this case is: let us say, this is the target (Refer Slide Time 16:48) - many of you must have done the firing in your NCC classes or somewhere. So, when you are firing, what you have try to do, you have a little bulls eye here (Refer Slide Time 17:00). Your aim is to fire at the point here in the bulls eye. Well, if you are careless - careless means, you are not even targeting the bulls eye - what you might end up, you might end up firing your bullet somewhere here (Refer Slide Time 17:21) or outside or very much outside, isn't it? So what you are doing? Actually, the person who is taking the observations is committing some mistakes; there are some blunders. So that kind of thing, we say the 'blunder'.

Now, how it can happen if I am measuring this length? Well, I measure this length, I find this to be 30 - actually, the length there in the ground is 30.7 centimetre (Refer Slide Time 17:48). Well, I record it, I say, 'Well, I have observed it 30.7' Now, when I write this length in my notebook (Refer Slide Time 18:04), length measurement number one - I write it as, not 30; 13.7. It might happen; it does happen, because whenever you are working in the field, you know, if the conditions to work in the field are not very, very good - which is normally so, because you do the survey out there in the ground; the conditions are not very favourable always. So, your mind is a bit tired, so you end up writing a length which is 30.7, which you also measured, as 13.7. So what you are doing actually, you are introducing error, which we will say blunder or mistake.

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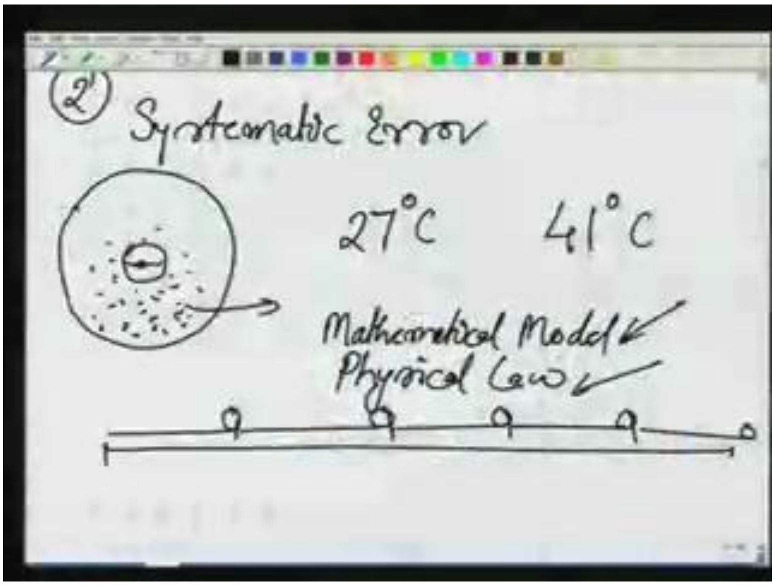

Well, the second classification of the error: the second one of this, we say 'systematic error'. Systematic error - now what happens in this case? The same example of the target here. Well, the bulls eye (Refer Slide Time 19:08); you are trying to fire here. Let us say you are not aware about the gravity - you are a very good fireman; very good fireman, but you do not know that the gravity exists. So, what you are doing, you are trying to target at the centre, but because of the gravity, all your bullets are coming here (Refer Slide Time 19:28) - down. Maybe a little one will go there (Refer Slide Time 19:32), but most of the bullets will be coming here. Is it or not? What is happening now? The gravity is playing its role; so a systematic force in the nature is bringing your bullets down, and your bullets are firing here, not at the target. So, this is a systematic force which is working; so the error is, we say, systematic error.

Similarly, for example, let us say we are measuring with this scale, and we know this scale was made at a temperature of 27 degree Celsius, while the temperature now is 41 degree Celsius while I am using it. Obviously, this scale has some expansion in its length (Refer Slide Time 20:21). So, what is happening in all the measurements? Let us say I use this scale to measure this length, and I keep this scale once, twice, thrice, four times, and so on (Refer Slide Time 20:33). So, what I am doing? I am introducing some errors always (Refer Slide Time 20:38), everywhere. So this kind of error, for which - you saw it in the case of the gravity, in the case of the temperature - for which we can write the mathematical model or, we can say, those errors which follow some physical law, as in the case of temperature. There are many, many cases where, you know, you can write a mathematical model for the error, because it follows some physical law. So you can eliminate that error. Those errors we say 'systematic error'.

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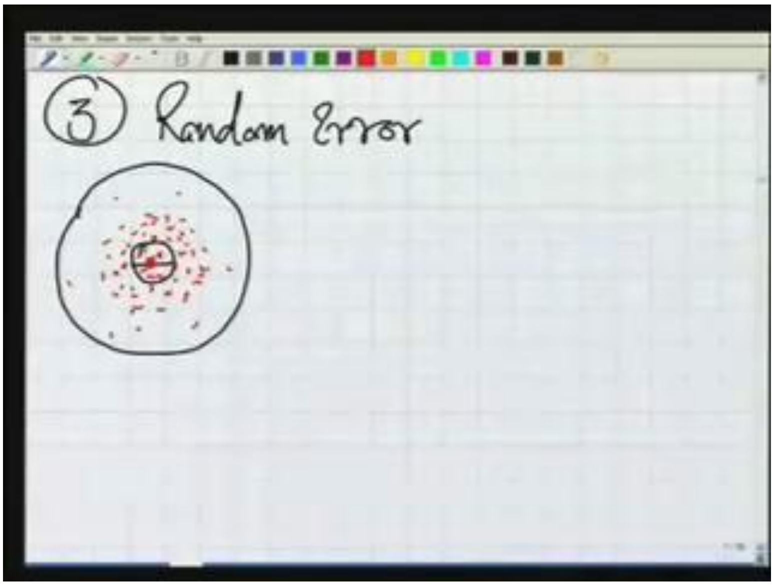

Well, then number three: this is very interesting error, now, and we say this error to be random - random error in our observation. For example, let us say - again, I am going back to our target. You are trying to fire at the bulls eye, which is this point (Refer Slide Time 21:51). Your aim is to fire at the centre here. Well, what we do, we ensure there is no blunder; no mistake. Your firing instructor is very, very strict; he is ensuring that there is no mistake or no blunder. You are not firing here and there; at the same time you are also aware that the gravity is there. So, what you are doing? You are taking some precautions and you are raising your gun a little bit, so that instead of firing at this point (Refer Slide Time 22:21), you fire somewhere here (Refer Slide Time 22:25), so that you are taking care of the gravity also. Now, if this is the case, what will happen? All your measurements, all your bullets, will now fire like this (Refer Slide Time 22:33). Maybe some are farther also, but most of them will be nearer. Now this is very interesting observation; very, very interesting observation: what we see here, these bullets are spread throughout my bulls eye, while more number of them are concentrated in the bulls eye and less are in the outer periphery.

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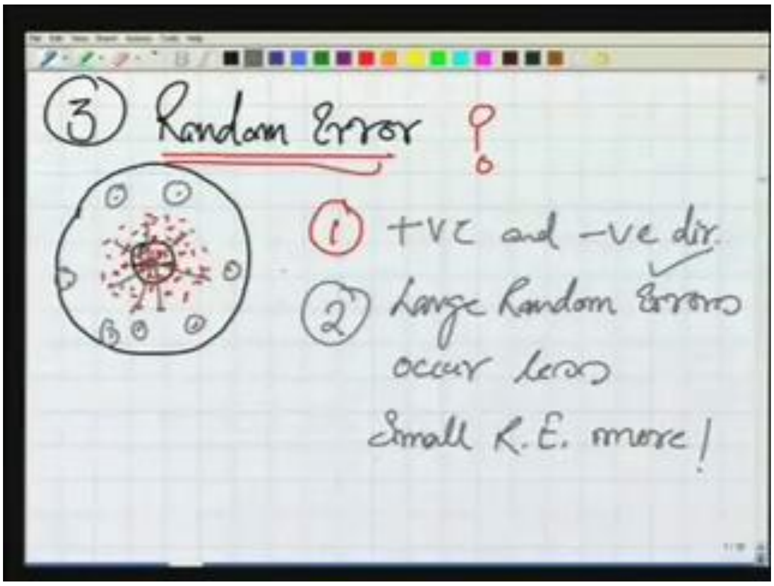

Now, one thing: why we could not fire in the bulls eye exactly? Can we? No, still there are many sources of errors which we do not know. You are breathing, some little turbulence in the air, some little problem with the gun, some little problem with the bullet itself - it does not go in the straight path. So, there are many sources of errors which you cannot account for, and these are the sources of errors which may go in positive direction or in negative direction; we do not know how they varying. So, that kind of errors; those sources of errors, they contribute the error which is called random error. What we observe here? The random error, their distribution is - number one point about them - they are equally distributed in positive and negative directions. What is the meaning? Over here, there are random errors in all the directions. It is not that they are in only one direction; they are in all the directions. So, their probability of occurrence in positive direction and negative direction is same. Well, the example here will be the ruler; the scale. While I am measuring this length, I am trying to keep my scale exactly over the point here and here (Refer Slide Time 24:32). What is the guarantee that I am keeping exactly over it? Again, there is some subjectivity involved; the human judgement is involved. I may be keeping it slightly here (Refer Slide Time 24:40) or slightly here, so that introduces the random errors. So, what will happen? The error amount - now number two, point number two about it - what we see here: random error which are large in magnitude - for example, this (Refer Slide Time 24:56) - occur very less. There is less possibility, of the random errors which are large in amount, of occurring, but the random error which is small, for example, here (Refer Slide Time 25:11) - here, we have a huge concentration - the chances of their occurring are more. So, we can say: large random errors, they occur less, while small random errors, they occur more (Refer Slide Time 25:20).

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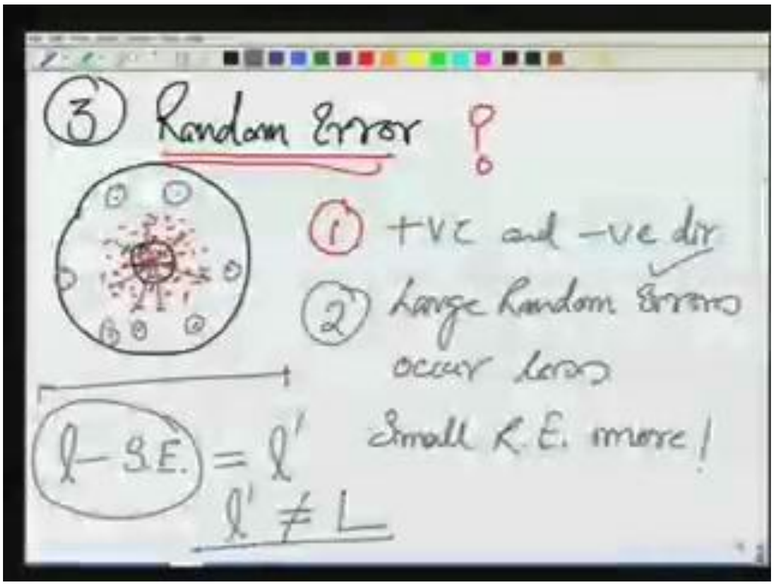

We will see in a moment some further things about this random error, but this is one error, you know, if you eliminate from your observation - let us say, your observation is ‘l’. For a length which you needed, you measure it as l. You eliminate that this is not blunder or mistake - fine. Then, you eliminate some systematic errors - systematic error ‘SE’ - from your observation (Refer Slide Time 26:04). You know the model how this error has occurred. So, you eliminate this systematic error also. Now, in your observation, what about the outcome of this (Refer Slide Time 26:17)? For example, let us say the outcome is l’. Still, this l’ is not equal to true length (Refer Slide Time 26:25), because your l’ has still some errors, and those are the errors which we say random errors. So, we can say the random errors are those errors which we made in observation even after eliminating systematic errors from the observation.

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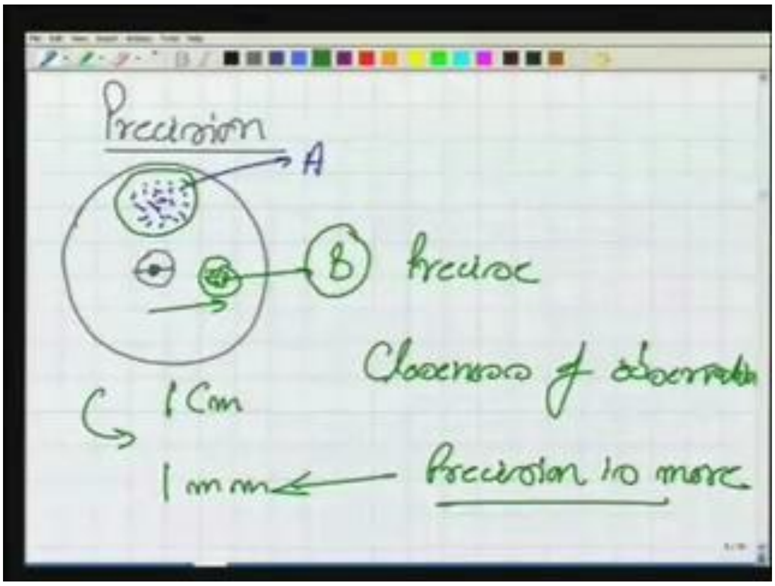

Now, what we will do, we will discuss two more terms here. One term is called 'precision'. What is the meaning of that? Again, I am going back to our example of target. That is the bulls eye (Refer Slide Time 27:59), we are trying to target the bulls eye somewhere here (Refer Slide Time 27:04), but you are not able to do that. Precision means, let us say, when we are trying to target here, there are some systematic errors also working, and because of the systematic errors, most of your observations, or we can say, the bullets, are clubbed in this area (Refer Slide Time 27:22). It might happen; let us say some force is working upward now - not like the gravity - and most of the observations are now in this area. Well, this is one case - this is the - a person A fired these bullets. Person B also fired these bullets, and for person B, what he gets, he gets all the bullets, let us say, like this (Refer Slide Time 27:53). Well, what would we like to say about person A and person B? Who is better firer; who is better fireman; who has got better aim? Despite there is some systematic error - which is, maybe, your barrel is slightly tilted; though you are targeting the bulls eye, all your bullets are being fired here (Refer Slide Time 28:20), slightly swayed, but this person has all his bullets very close to each other; while this person has his bullets, you know, widespread. So, definitely, we can say person B is more precise (Refer Slide Time 28:38) in his firing than person A. This explains the term precision: we mean by precision the closeness of observation (Refer Slide Time 28:49); how close your observations are. Another example: well, in this scale - the least count of this scale is 1 centimetre (Refer Slide Time 29:02). You are trying to measure up to 1 centimetre and then you are approximating within 1 centimetre - that is case number one. In case number two, the least count is 1 millimetre (Refer Slide Time 29:18). So, now you are confident to measure up to the millimetre, and you are estimating only within 1 millimetre. Obviously, in this case, the precision is more (Refer Slide Time 29:31). Then, compare to the other case.

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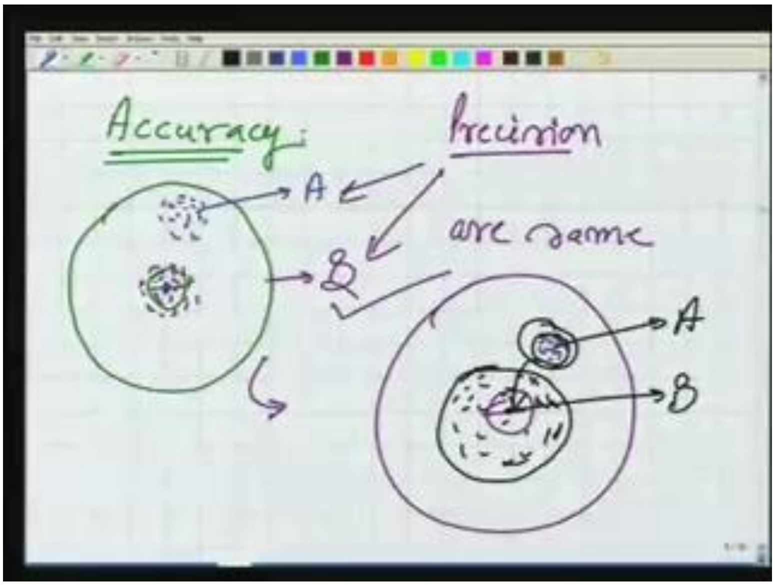

Another term which is related with the precision is accuracy. Well, what is accuracy? Let us go back to again our target - the bulls eye here (Refer Slide Time 30:01); you are trying to fire at the centre. Now, in one case, you fire here (Refer Slide Time 30:14) - let us say the fireman A, he fires here. In the second case, there is another fireman, and he fires here (Refer Slide Time 30:26). Now, out of A and B, who is more accurate? What is the meaning of accuracy? Obviously, you will say B is more accurate because he is firing nearer to the target, nearer to the bulls eye, while A is not that accurate. Now, in terms of precision, both A and B are same; the precision spread is same, but A is not accurate. So what is the meaning of accuracy? The meaning of accuracy is, closeness of your observation to the true value; how close your observations are to the true value. But, there may be a very interesting case; in that case, in this bulls eye - now, there are two persons. One person has got his bullets here (Refer Slide Time 31:31), the other person has got his bullets here (Refer Slide Time 31:37). You should take the mean of these - the mean is here somewhere (Refer Slide Time 31:48) - that is for observer A, and for the second person B, the mean will be somewhere here (Refer Slide Time 32:53). Even if the precision is more in this case (Refer Slide Time 32:01), it is not accurate. Here (Refer Slide Time 32:06) the precision is less - your observations are far apart - but your accuracy is more. But if you eliminate the systematic error from your observations, if the systematic error is eliminated, then what will happen? All these bullets (Refer Slide Time 21), they will come to the target or the bulls eye. So, if the systematic errors are eliminated in that case, the precision of the observations becomes an indicator of the accuracy.

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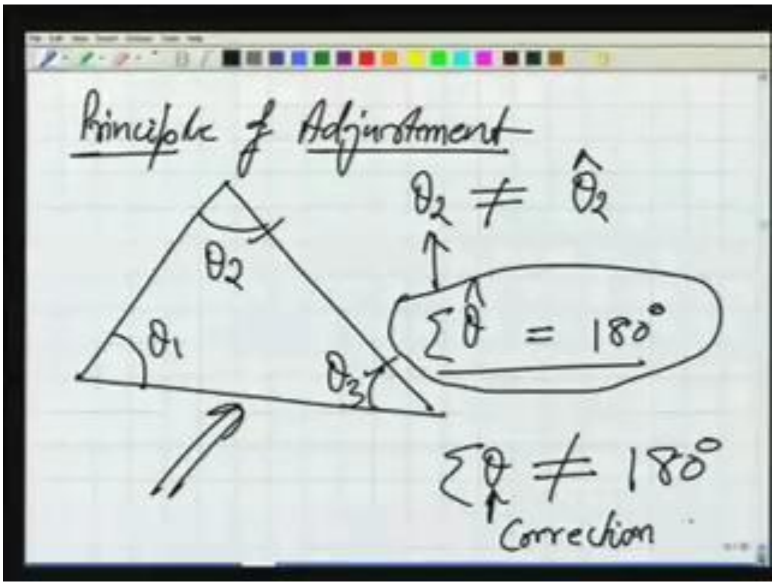
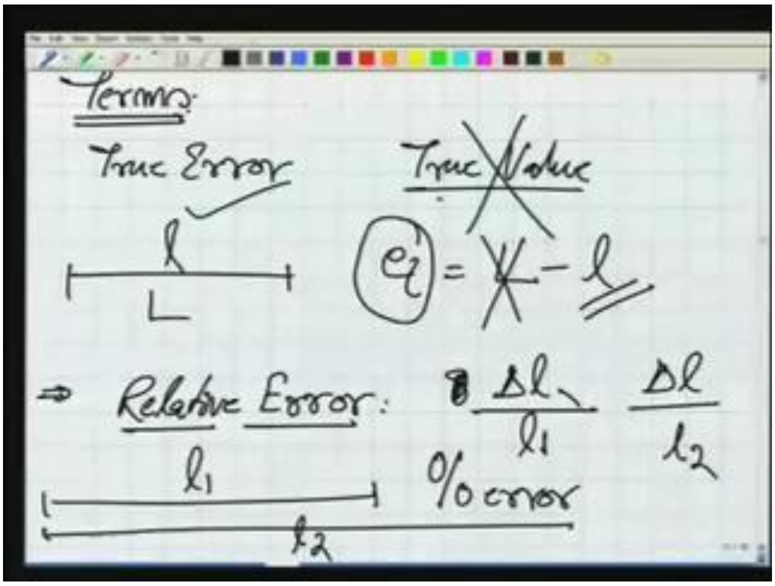


Now, what we will do, we will look at one more important principle here that is called 'principle of adjustment'. Now, we know by this time whatever we are measuring - for example, let us say there is a triangle, and for this triangle we are measuring three angles: theta 1, theta 2 and theta 3 (Refer Slide Time 33:05). There are three points there in the ground - one point, another point, another point (Refer Slide Time 33:15), and using some equipment, some instrument, I stand at point 1 - number 1 - I measure the theta 1. Similarly, I measure theta 2, theta 3; all the angles. Now, we know that - this theta 2, for example - it is not really the true value. The true value, in this case, is 'theta 2 hat', let us say (Refer Slide Time 33:41). Let us say this is the true value, but theta 2 is not the true value - why? Because theta 2 has got some errors in it. We have discussed this thing that we cannot measure the true value. So, all our theta 2 has got the error.

Now, if that is the case - well, because this is the triangle, I can write now as ‘sigma theta head’ the true angle - the true angles between these three points should come to 180. Obviously, the true angles, if we can measure true angles - the ‘theta hat’ - they should come to 180 degree, but if you are writing ‘sigma theta’ (Refer Slide Time 34:25), the observations - will it be equal to 180? No, because all our observations have got some error also. Well, there in the field there are three points for which we have measured these angles. The angles have got the error, but the field does not. The angles which I have measured have got the error, so what I need to do, I need to correct my angles so that they satisfy the condition of the field. The condition of this field is that these three angles which I have observed should come to 180. So, what I need to do? I need to apply some corrections to the angles, and we do it by the procedure of adjustment. We will see in detail about this adjustment later on. So, now we will talk something more about the errors; some more terms.

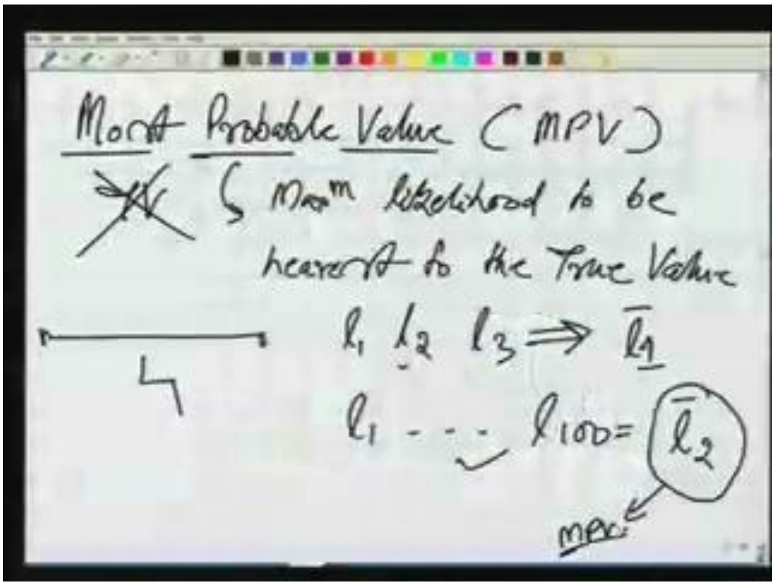
(Refer Slide Time: 35:27)



Well, the very first term I would like to see is true error. We have already seen the term ‘true value’ - we cannot measure the true value; there are limitations - so many errors come in to the observation, so we cannot measure the true value. Well, so, do we know the true error then? If I know the true value, then only, I will know the true error, because the true error - for example, let us say there is a length; its true value is ‘L’ (Refer Slide Time 36:02). What I observe is ‘l’. So, the error - if I write error as ‘e’, and the true error will be ‘L minus l’(Refer Slide Time 36:11). But I do not know

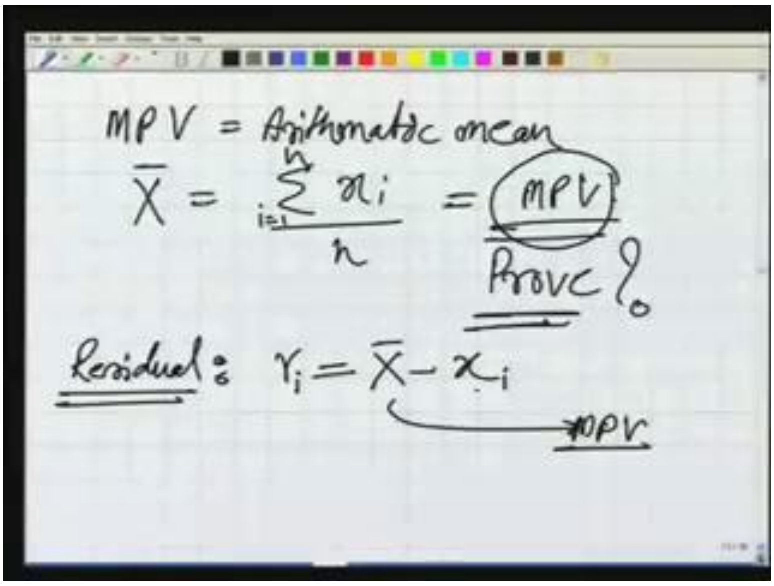
L (Refer Slide Time 36:18) - if I do not know L, just by using the observation, I cannot find the true error. Well, what we can find? We can find - I am going to give you another term that is called ‘relative error’ (Refer Slide Time 36:34). In some cases, for example, here in the case of the scale, I know what kind of error is being introduced because of the least count; I know that thing. Let us say, that error which is being introduced because of the least count is ‘delta l’ (Refer Slide Time 36:58). Now, if I observe a length of line which is here (Refer Slide Time 37:05) - l_1 - I can find delta l by l_1, and this we say as the relative error. Now, why do we need it? We need it because, using the same scale, I can also measure a length which is l_2 (Refer Slide Time 37:21) - in that case, the relative error will become delta l by l_2. So, we can compare our observations; compare the error. So this is how this relative error is used. Many times, this is also called ‘percentage error’ (Refer Slide Time 37:37), where you simply multiply this by 100 (Refer Slide Time 37:41), and that becomes the percentage error.

(Refer Slide Time: 38:00)



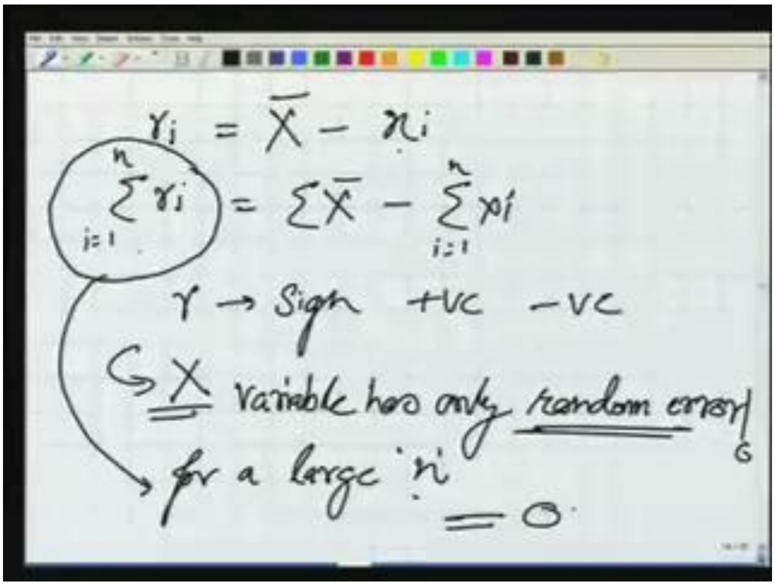
Now, we will talk about one very important concept that is called ‘Most Probable Value’ or MPV. We do not know true value; yes, we do not know that. What is most probable value? It is a kind of replacement for the true value which we use mostly in our cases. The most probable value is defined as one which has maximum likelihood to be nearest to the true value (Refer Slide Time 38:24). Well, let us explain this. A value which has maximum probability of being nearest to the true value - you know we cannot get the true value by any means. But, what we can do, we can go nearer to the true value. I will give one example here. Let us say there is a length L (Refer Slide Time 39:00) - L is the true value which we cannot measure. What we are doing? We are observing this: in one case, l_1, l_2, l_3 - we have taken three observations (Refer Slide Time 39:09). In another case, we are taking from l_1 to l_{100} - hundred observations of the same length (Refer Slide Time 39:16) - more redundancy here in this case (Refer Slide Time 39:23); less redundancy here (Refer Slide Time 39:24). Now, by taking the mean - arithmetic mean - of these, I will get an 'l bar' (Refer Slide Time 39:30) of case number 1, I will get - there also - an 'l bar' of case number 2 (Refer Slide Time 39:36); there are two cases. So, obviously, just by common sense right now, you can say that the probability of l_2 being nearer to the true value is more. So, we can say this (Refer Slide Time 39:54) is our most probable value or rather, the other way round: out of all these 100 measurements, which measurement is most probable? Is it l_1, $l_{2,}$ $l_{3,}$ l_{100} ? We do not know about it, but once we have the observation which is the mean, we can say this l_2 has the maximum probability of being nearer to the true value. So this l_2 is our most probable value. What we are saying in this case, we are saying that the most probable value is the arithmetic mean (Refer Slide Time 40:28) - this is how we take it mostly.

(Refer Slide Time: 40:29)



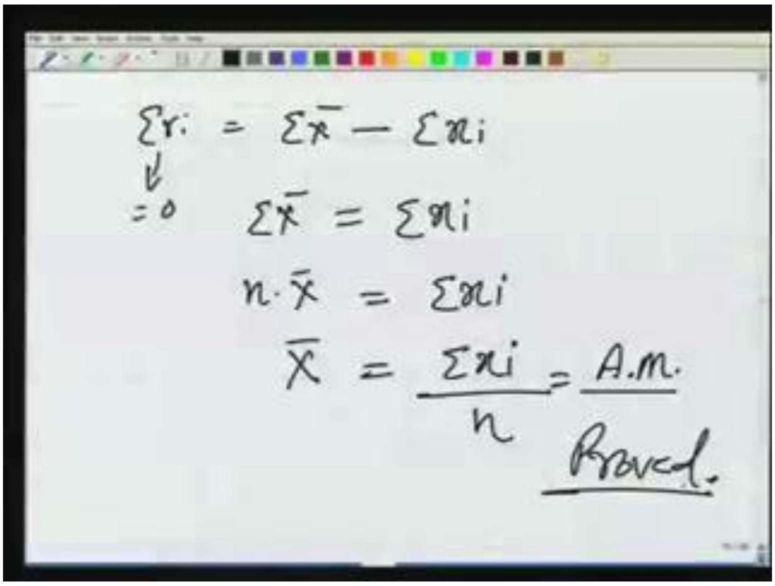
If I write the most probable value for some observation as 'X bar', so the meaning of that is, it is sigma x_i by n, where i varies from 1 to n (Refer Slide Time 40:40). For anything - any quantity, any variable - there are n number of observations - x_1, x_2, x_3 - which we are writing as x_i here, and that is the arithmetic mean. And we are saying that the arithmetic mean is the most probable value. Is it, really so - can we prove it? Now, to prove it, we will do a little exercise, and I will define one more term before that -the term is 'residual' (Refer Slide Time 41:24). I write the residual as 'r', and this is written as 'X bar minus x' for any observation I (Refer Slide Time 41:32). So, what is the residual? Residual is the mean of the observation or - sorry, X bar we are taking as most probable value; let the X bar be the most probable value which has maximum chance of being nearest to the true value. If that is so, in the case of the X bar, we define the residual as the most probable value minus the observation. So, for all the observations - i is equal to 1 to n - we can find their corresponding residual. So this is called the residual. Now, we will make use of this in order to prove that the most probable value is nothing but arithmetic mean of the observation.

(Refer Slide Time: 42:37)



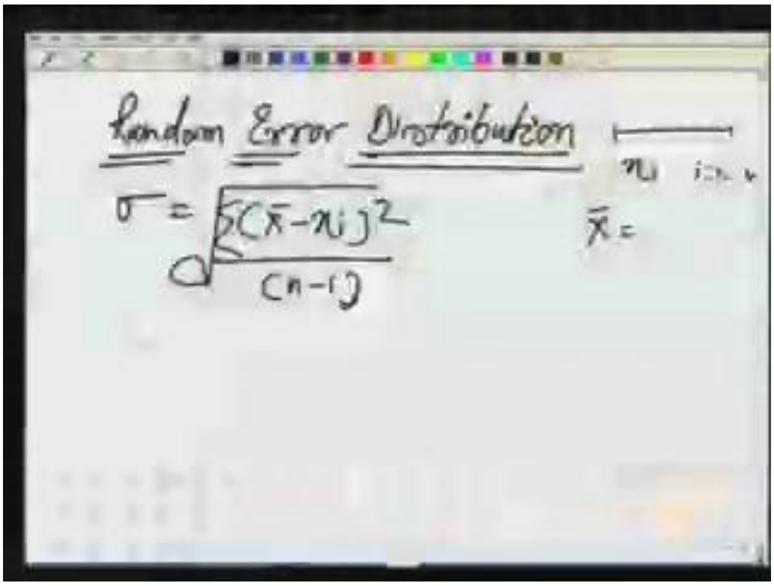
So, we had written that our r_i or the residual was X bar minus x_i. What I do, I sum it up for all the observations: let us say sigma r_i for i is equal to 1 to n will be sigma X bar minus sigma x_i; i equal to 1 to n (Refer Slide Time 42:47). Well, now look at this part (Refer Slide Time 43:07) - this is very important. The residual - what will be the sign of residual? We are talking here, now, only in the case when all our observations, our variable, has only random error (Refer Slide Time 43:26). Let us be clear about it - whatever we are talking here, our variable, our observation, has only the random error in it; we have already eliminated the blunder, we have taken care of the systematic error from the observations, and our observation have got only the random error. We are talking about that case, and only in that case we are saying that the most probable value will be the arithmetic mean. And this is what we are going to prove. And only in that case, we are defining the residual, which is the most probable value minus the observation. Well once that is clear, we are looking into this part (Refer Slide Time 44:18). What this part will be, now? For a large n - many observations - you know what is the sign of r, the residual? The r may be in positive direction as well as in negative direction (Refer Slide Time 44:34), which we saw. The random errors have got equal probability of going into the positive direction as well as in the negative direction - this is what we have seen already. So, the sign of r will be positive and negative with equal probability. If that is so, for large n, this particular part (Refer Slide Time 44:54) will be equal to 0.

(Refer Slide Time: 45:13)



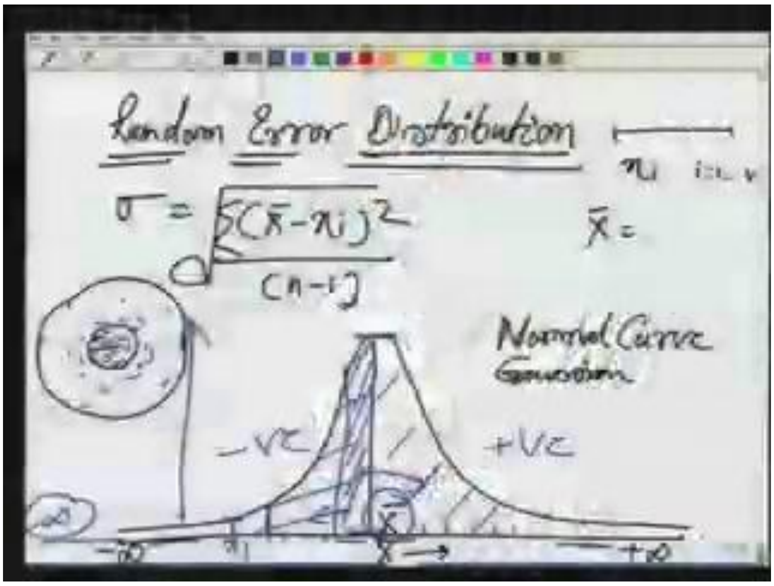
If that is so, what we can write now? Sigma r_i is sigma x bar minus sigma x_i (Refer Slide Time 45:13). What we can write if this becomes 0 (Refer Slide Time 45:14)? You can write it as: sigma X bar is sigma x_i (Refer Slide Time 45:18). What is sigma X bar? Sigma X bar is n times the X bar is being summed up (Refer Slide Time 45:26). So, that is equal to sigma x_i (Refer Slide Time 45:32), so the X bar - most probable value - comes out to be sigma x_i divided by n (Refer Slide Time 45:37), and this is nothing but the arithmetic mean. So, what we proved? We have proved that, if our observations have only random errors, in that case, the most probable value of the observations is arithmetic mean.

(Refer Slide Time: 46:07)



Well, having said that now we will see another important thing, that is, ‘random error distribution’. We have already seen some of the things, but would like to see it in detail now, how the random errors are actually distributed, and in doing that for any set of observations - for example, let us say this (46:38) is a variable or a length which we need to measure. We measure it as x_i - i is 1 to n - and we find the most probable value which is the arithmetic mean. Also, we can find one more term that is called sigma (Refer Slide Time 46:53), that is, the standard deviation, which you find by X bar minus x_i - sigma of that - divided by n minus 1 and a square here - that is called the standard deviation. What does the standard deviation show you? The standard deviation shows you the spread of the point. We will talk about this standard deviation also later on, in detail, how we will make use of this.

(Refer Slide Time 47:26)



Well, the random error, the way we are talking about them, they are actually distributed in a particular way (Refer Slide Time 47:29). Here, it is our variable X, and this particular value is X bar here, while here, it is from positive infinity, minus infinity (Refer Slide Time 47:45). Now, this curve - what does it show? We will talk about it - this is basically called ‘normal curve’. Also, it is called Gaussian curve - many times, it is called ‘bell-shaped curve’ because it has the shape of a bell. Now, what is the meaning of this? We saw earlier also that our random error, how they are distributed - just think of the example of our firing of bullet (Refer Slide Time 48:24); how those were distributed. If I do it again here, we are trying to target here (Refer Slide Time 48:31, but you end up firing somewhere here also (Refer Slide Time 48:44), but more observations in this area, more as you go nearer. Then how the bullets were there; we saw the random error is distributed. And the same thing is seen here in this curve. There is an equal probability of an observation going below your most probable value on this side (Refer Slide Time 48:56) - that means the error is negative - or on the positive side, the error is positive, because the most probable value is one which is nearest to the true value. Any error which is negative into that will take its value towards negative side (Refer Slide Time 49:13) or the less than the most probable value, and any error which adds to it with the positive sign will take this value of the variable towards the positive side (Refer Slide Time 49:27).

So, what we see? Our observations or our errors are equally distributed (Refer Slide Time 49:36) in both positive and negative sides; the chances of getting negative error and positive error are same. Now, here in this curve, this is our variable, while this is the probability of occurrence of any variable (Refer Slide Time 49:49). For example, if a variable is here, x_1 (Refer Slide Time 49:59) - so, what is the probability of occurrence of x_1? The probability of occurrence of x_1 is this value (Refer Slide Time 50:06). The probability of occurrence of your most probable value X bar is maximum - obviously, there is also a probability of occurring an error (Refer Slide Time 50:17) which will be infinite in size, on negative side as well as in positive side, but its probability will be very less. So, this is how the curve is: those errors which are large in magnitude - for example, here (Refer Slide Time 50:35) - these errors are large in magnitude, so their probability - sorry, they are small in magnitude, and their probability of occurrence is very high, while the errors which are large in magnitude on this side (Refer Slide Time 50:47), on this part, on this part - their probability of occurrence is less. So, this is how estimated random error is distributed. So, what we will see, we will look further on this distribution of the random error.

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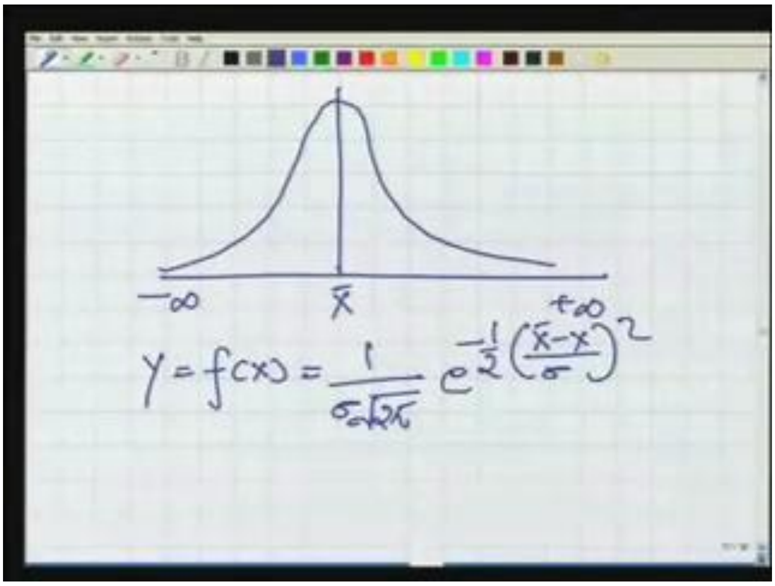
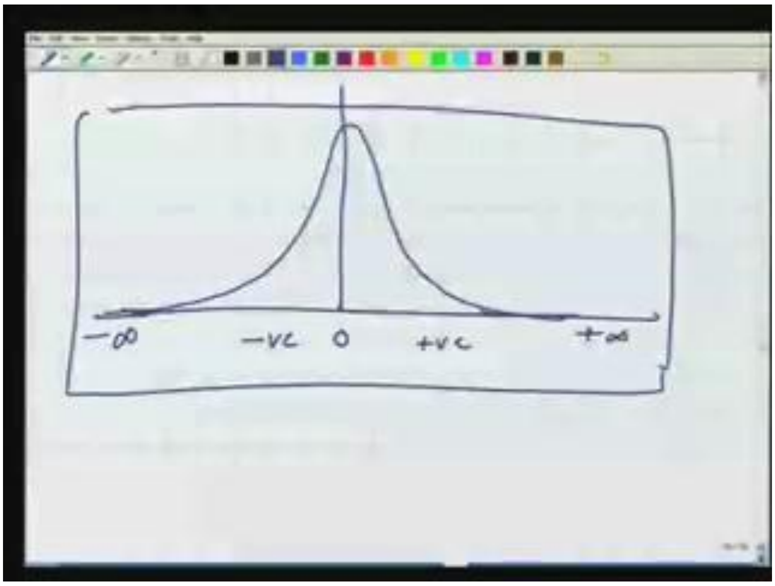


I am going to draw the same error distribution or normal curve again, from minus infinite to plus infinite, and that is our X bar, the most probable value, and the distribution, the probability density function of this is given as ‘f(x)’ which is 1 by sigma and root 2 pi, e - exponential - raised to power minus 1/2 into X bar minus x

(Refer Slide Time 51:23) - ‘x’ is an observation and a variable, while ‘X bar’ is the most probable value - divided by sigma square. This is the probability; f(x) or the Y gives the probability of occurrence of the variable x. So, this is the equation for this curve.

(Refer Slide Time: 52:14)



Now, what we can do, we can draw the same curve for the error also. This will not touch here, they will not meet. Plus infinite, minus infinite (52:26), and in the case of the error, we will have zero error here (Refer Slide Time 52:32), because it is the most probable value, and the negative values of the error, and positive values of the error - and the error will be also distributed in the similar way, the way the normal distribution is. And in most of the observations in surveying, unless the observations are biased because of some systematic error component, we will have all our errors generally distributed in this form.

So, what we saw today, we saw about the errors in the observation, errors in the measurement, and this is a very important concept - you should know about these errors, because in surveying, you go the field, you measure something there. So, you should know what kind of errors may creep in. These errors, they come because of the nature, because of the instrument, because of you - the person. Then second thing we talked about: the types of errors - there could be mistakes, there could be the systematic errors; we can eliminate the systematic errors provided we know the mathematical model or the physical law which they follow. We can also do the corrections for all the systematic errors. Then, after eliminating all the errors, if still, our observations are not free of the error, we have another kind of error here, which is random error. So we saw today a lot about the random error. What it is; how it is distributed - it is distributed in a Gaussian way or the normal curve ; we saw the characteristic of this, that the positive error and the negative error have got equal likelihood of occurring; the smaller errors have more probability of occurring while the large errors have got less - so, this is what we have seen. What we will do in our next lecture, we will try to make use of these concepts and we will try to, you know, take the normal curve, standard normal curve, and we will try to make use of that, so that we understand that what we have measured, how we represent it, how you present it, how confident we are about a particular measurement. Well, thank you very much.