Surveying
Prof. Bharat Lohani
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Module No. # 9
Lecture No. # 2

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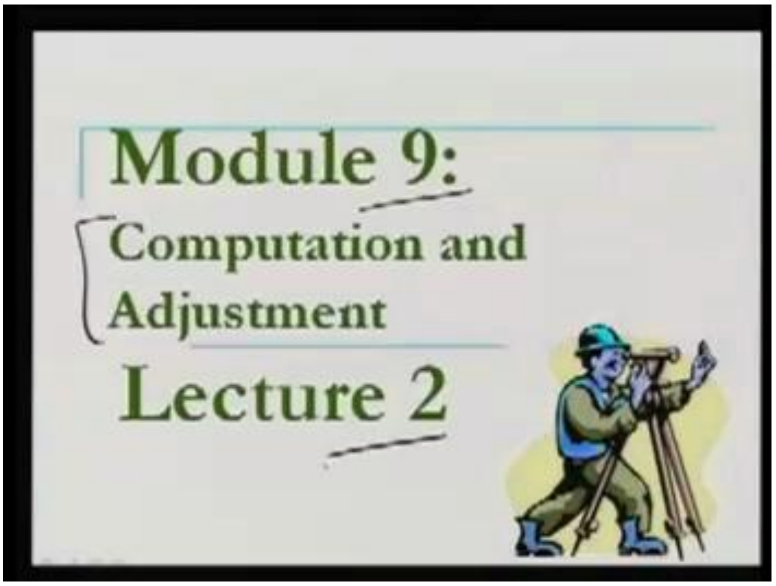

Welcome to this lecture series on basic surveying. Now, today, we are on module 9 which is about computation and adjustment. And we will be talking about the lecture number 2.

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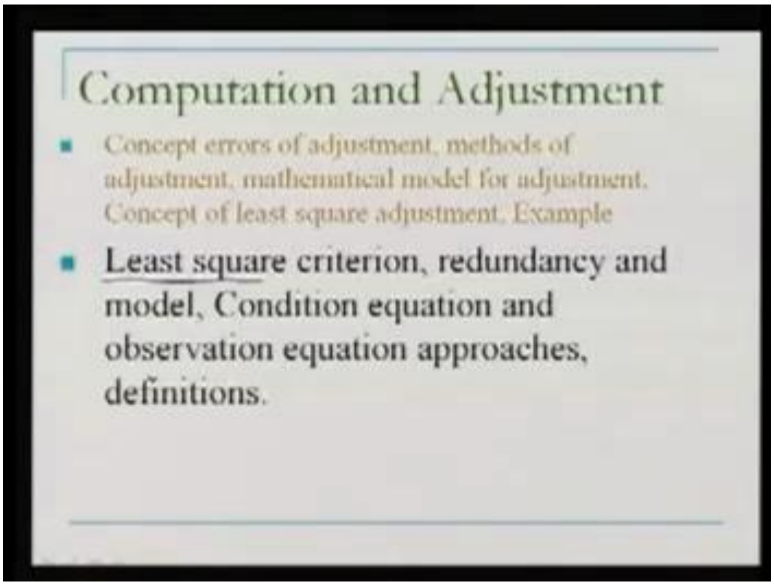

In this, we will be covering as you can see here again the concept of least square. Then how do we define the least square? Then we will be looking into the redundancy of a model. What is the redundancy? Why do we need it? How to plan for it? How to compute it? Then the adjustment can be carried out by 2 methods like condition equation method and observation equation method. There are other methods also, but we will be basically concentrating on these methods. So, we will start about these methods today. Next, we will start with again the concept of the least square. We have done it in the last lecture also, but again to revise it. For example let us say, we have a line and we want to take the measurement of line. So, what we can do?

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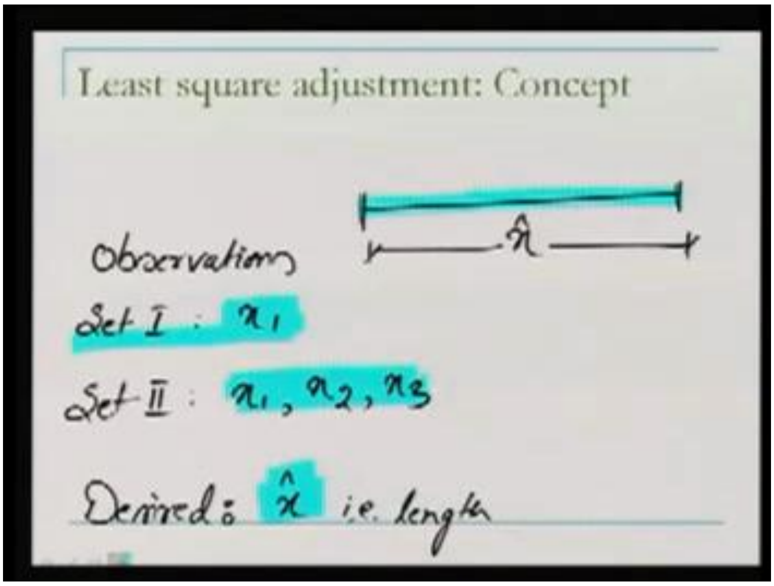

As we can see here in the slide over here this is the line and this line is to be observed. We can do 2 things we can go for only 1 set means we take only 1 observation and the second set we can take 3 observations. So, the same line is being measured 3 times. Now, what is desired here? The desired is the estimate of this line. Are you want to know what is the, you know length or the estimate of this line which is the best estimate from our available data. How do we proceed?

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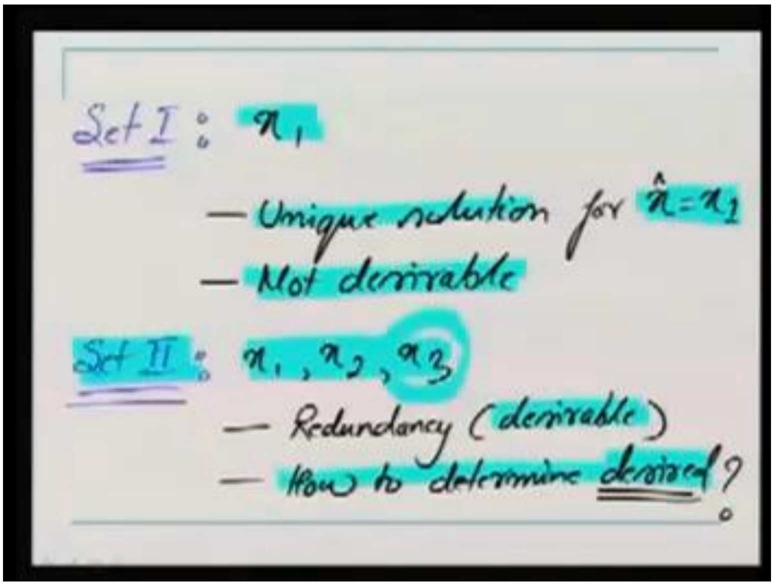

In the first set we have taken only x 1 and what we know? We shall taking only x 1 well our solution will be unique means our estimated value is equal to x 1 only 1 observation we have taken. You know we just observed it once. So, we cannot say anything else, about that length of the line except that observation. So, our estimate of that observation or we can say adjusted value the final value is equal to the observation. But this is not desirable as we have seen, because we do not know, how much error was there in that 1 observation? And whatever is the error amount we are going to take it we do not have any check. So, this is not desirable.

Now, in set 2, we are going to observe the same line 3 times now this is very much desirable, because if in x 3 there is some error. The chances are in x 1 and x 2 the error will be less. The errors are always there, but we have a way to check for the errors. For huge amount of errors as well as you know we are carrying out multiple observations. So, that we have more set of data the redundancy is there. And this redundancy is very much desirable whenever we do surveying or any observation. But the problem in this case is, how to determine the desired? Our desired is estimate or the length of that line, but we have now 3 set observations x 1 x 2 and x 3 out of these 3 which 1 or average of all these 3 or average of any 2 of these. So, what we see? We have now, many possible solutions. So, if I write all these possible solutions the solutions could be.

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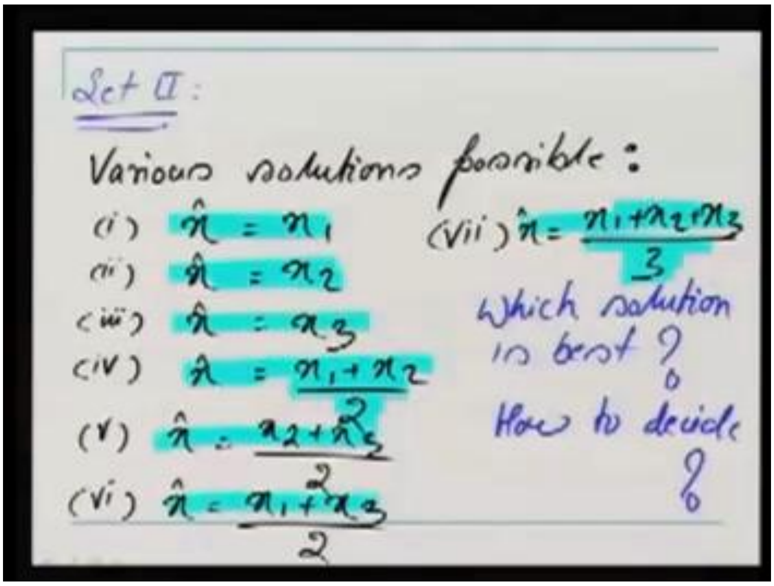

Well, any one of this observation. The estimate is equal to any one of this observation or may be average as you can see here estimate is average of first 2 observations. Similarly, here and here or may be average of all 3 observations. So, what we end up with? We end up with a situation where, if we are not applying a proper technique. You know we end up with a problem that out of all these possible solutions which one is the right solution. Which one should we take? We should have the way to say that.

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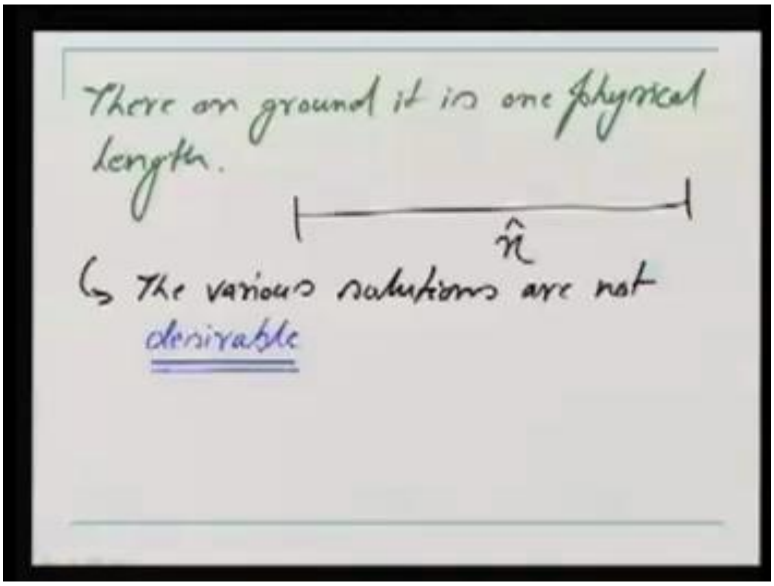

Now, to answer this question, we will go for the technique of the least square, because you know there in the ground is one physical line it has a length between 2 points. It cannot have multiple. You know lengths our multiple estimates as you have seen in the previous slide. It is 1 line so; it should have only 1 value. So, what will you try to do making use of all these observations which are desirable, because we want to have redundancy in the observations? We will try to estimate the best possible solution for the client a best possible length, how to do that?

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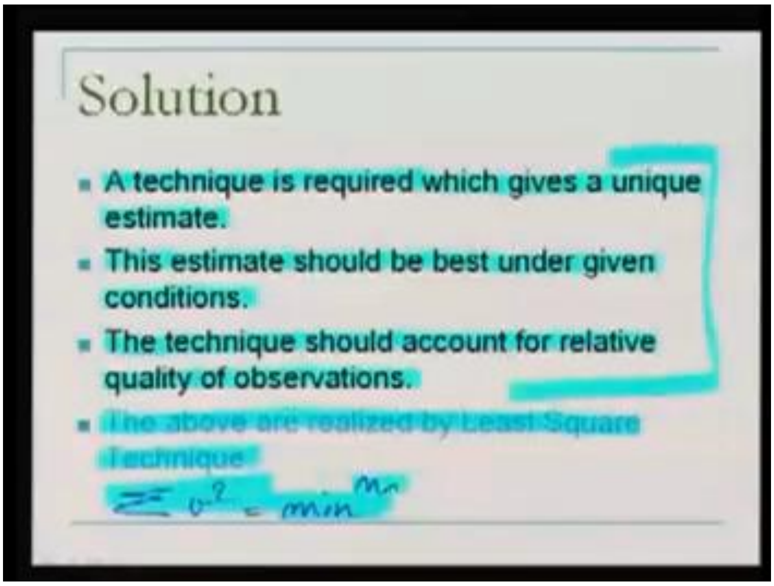

So, we are looking for a technique which gives a unique estimate obviously. Now, this estimate should be the best under given conditions further the technique should account for relative quality of observations now, this also very important. We know about that stochastic model. Depending you know we took 3 observations x 1 x 2 and x 3. If we have some idea about the precision of these observations, the weight that we can give to these observations we should take that weight also into account. If we know x 3 was observed when the weather was very poor you know and we it was very difficult to observe in the field, but still we have an observation x 3. So, what we would you like to do you we would like to give x 1 and x 2 more weight and x 3 less weight.

So, we should have some method of giving weights and these weights should be accounted for when we are computing the final estimate out of our adjustment process. So, this entire thing all these can be realised by least. And we have all already seen. How do we write the least square? The sum of square of residuals we can take the weights also into account should be minimum. We have seen that also by using you know line fitting example, we wanted to make or the best fitting line was the one for which or rather which deviates least from all the sets of points. All the points there that are the best solution which deviates least in overall stands and that can be done by the least square. Now, what is this criterion of the least square? How do we actually do it? So, what we will do.

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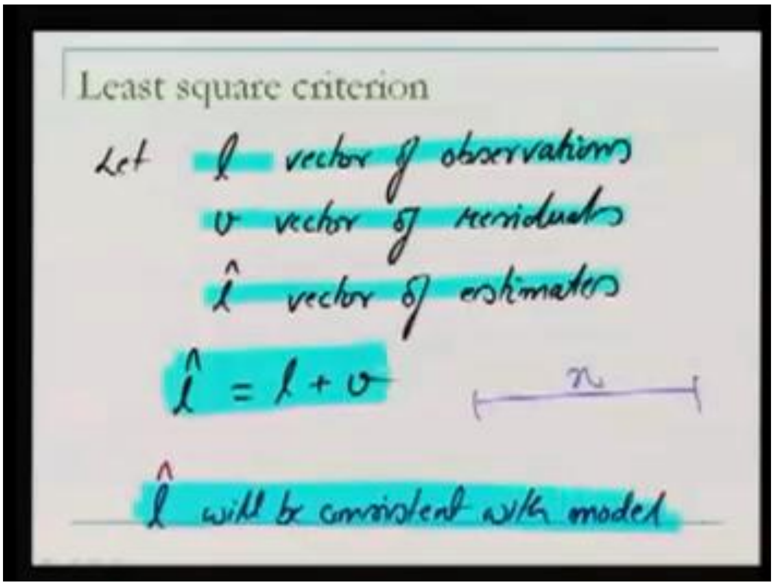

We will assume l let us say is a vector of observations anything it could be the observations for a line, there is a line. And we are observing the length of this line x. x 1 x 2 x 3. So, l is the vector of observations we have taken 10 20 30 100. Whatever number of observations? Next we take v as the vector of residuals. We know what the residuals are? If in observation I add the residual we get the estimate, l is the vector of estimates.

And these they are related that this. That l had is equal to l plus v as we are saying. If we add residual to the observation we get the estimate. Now, this estimate should be consistent with our model. We have seen now, that our observations should satisfy some model. We have seen those geometric conditions observation conditions we have seen those. So, our observations should satisfy those. So, by the least square approach, what we will do? We try to find the estimates which will also be consistent with our model.

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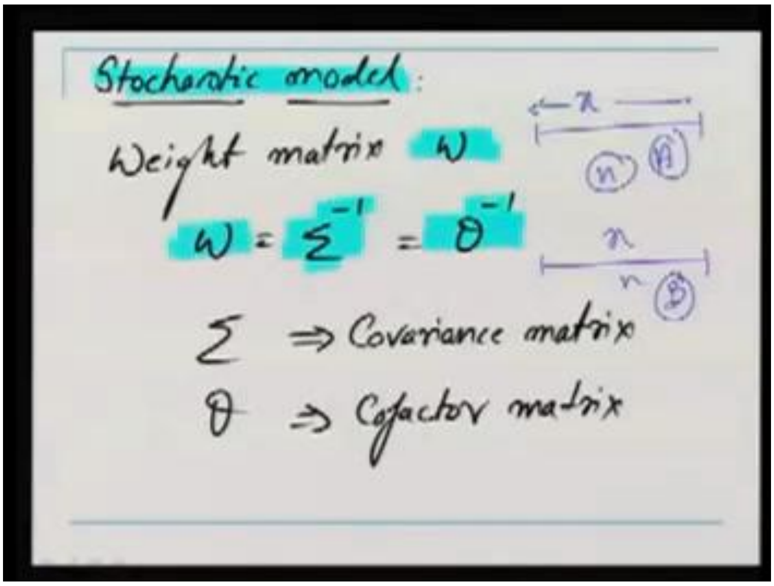

In addition, we already know that we should take into account the stochastic model. Where, we account for it by the weight matrix. Now, over here how do we decide the weight there are various criterion. One of them is that the weight is inversely proportional to the variance in any observation. We are taking for example, let us say for length. A length is being observed and it has been observed let us say n number of times. Now, the other day also the same length was observed x and again observed. Let us say n number of times this was observed by a person A this was observed by a person B.

Now, out of 2 sets of these observations, what we can do? We can find the mean; we can find the variance standard deviation. And more is the standard deviation of these observations. What is the meaning of it? They deviate more from the mean. So, the deviation of the observation the variance is more so; that means, the precision of the observation is less. So, if the precision is less we would like to give less weight to the observation. So, because of this only we know it we have already discussed this previously that weight matrix is inverse of covariance matrix or more appropriately the cofactor matrix.

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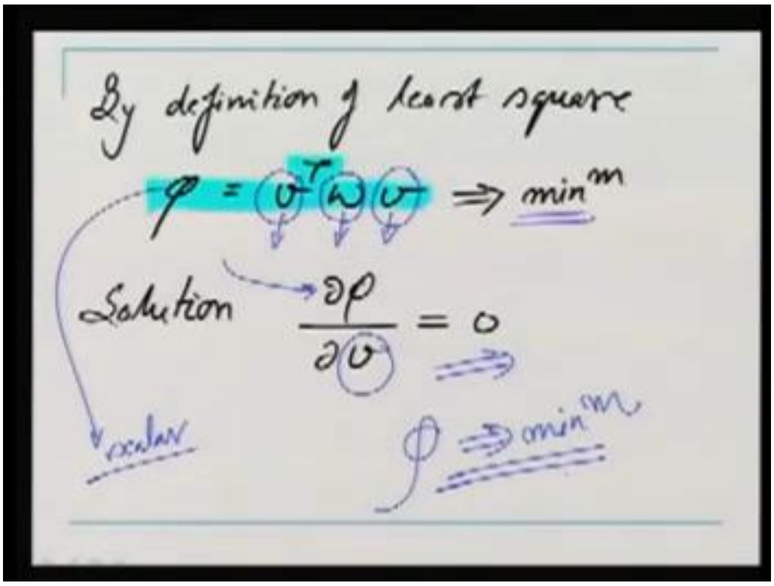

Fine, we know by the least square method, how do we apply it? As you can see over here that phi is v transpose w v and v phi is not scalar here phi is a scalar. And these are the matrices where w is for weight v is for residual and v make this minimum. How do we handle it? We handle it by finding the partial derivative of phi with respect to residuals. And equating to 0this is how we make phi minimum. So, that is the way we carry out the solution.

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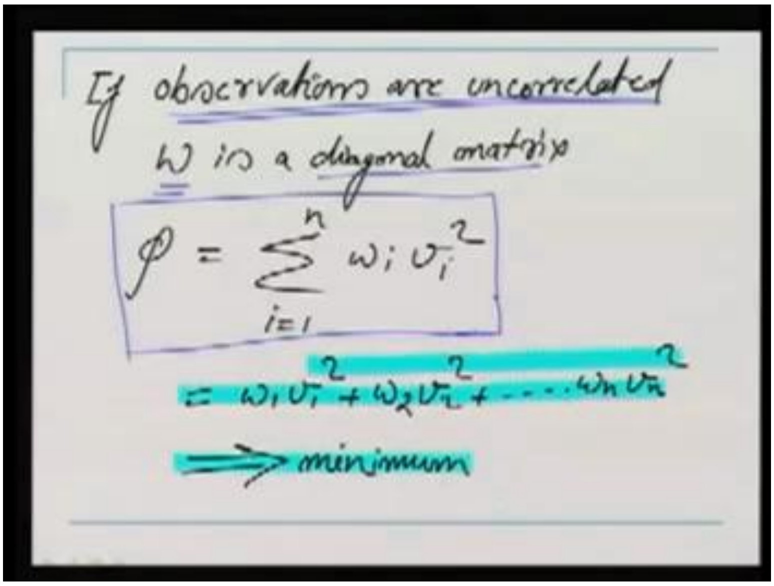

What we will do? We will look into some examples in a moment. But if depending our w if our w, w is the weight matrix. If the observations are uncorrelated observations were the sets of observations and they are uncorrelated. There is no all observations are totally independent no correlation between then if it is. So, the w matrix will be a diagonal matrix and our least square solution appears like this which we can further write as. So, we are taking weights into account here. w 1 v 1 square plus w 2 v 2 square plus So, on and w n vn square. And what we would, we will try to do? We will try to make this minimum in our least square approach.

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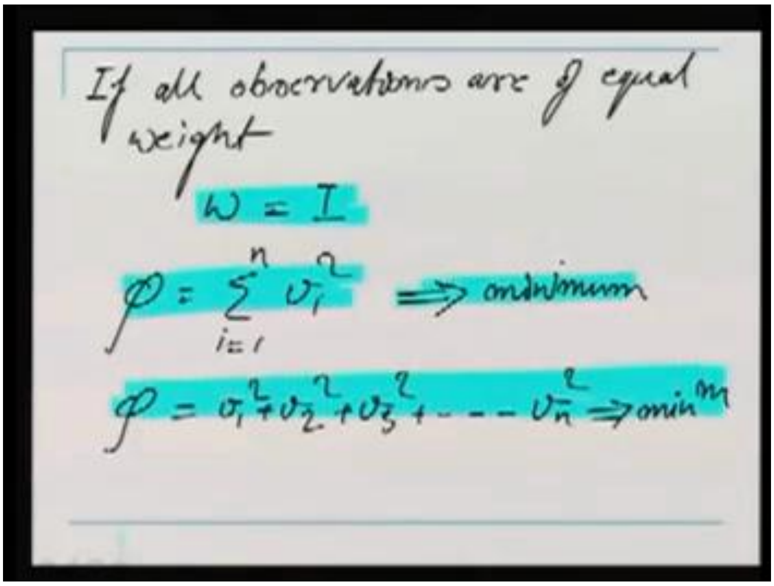

Further if the w all the all the observations you know there may be situations. When all the observations have been taken with the same weight if we know that the weights are same. Then in that case this w matrix will become an identity matrix if it is. So, we end up with that this phi is equal to sigma i varies from 1 to n v i square. And we will try to make that minimum all right and we can expand it further like this.

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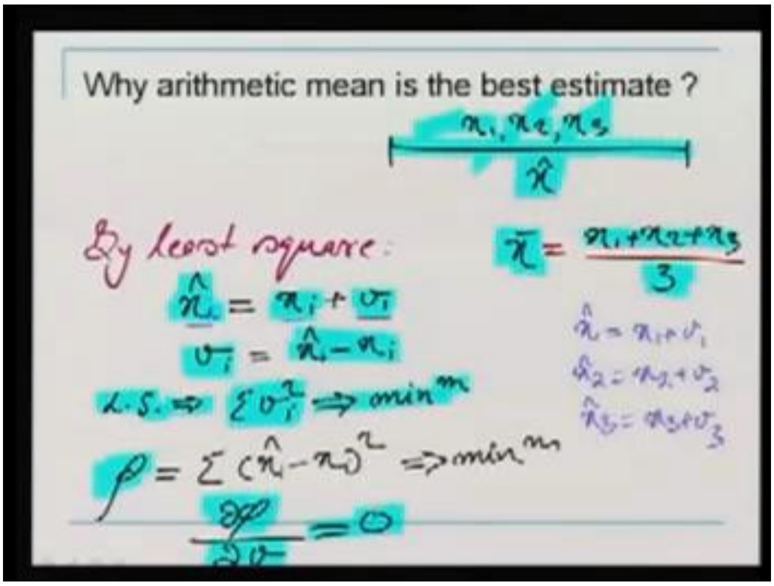

Fine now, one question over here. Since our high school days or even before that, we know that whenever we are taking any observation. We are taking the weight we have some chemical and we are weighing it. What we do? We weigh the chemical let us say 20 times and then finally, because now, we got 20 of values. We take arithmetic mean of that we have been asked to measure know the length of a line. We measure it 100 times 10 times 30 times as desired and we take the arithmetic mean of that.

Now, what is this arithmetic mean? Has it got any least square sense is it the best estimate in the least square sense. That is the question something which we may have been doing since our high school days. We ((refer time: 14:32)) compute for any sets of observation. We straight away compute the arithmetic mean. So, let us look at that now. Now, here in this again we have a line as you can see the line is over here. And we want to take the observation of this line and we have taken the observations as x 1 x 2 and x 3. What we want? We want to find the most valuable value of this line or we should estimate of that line. The adjusted value, this is what we want, how do we do it conventionally? We find the arithmetic mean arithmetic mean is x 1 plus x 2 plus x 3 divided by 3, because we have got 3 observations.

So, we can find the arithmetic mean, but is this really the best estimate if we carry out the same problem by the least square. This ((refer time: 15:37)) we are going to check. Well, in the case of the least square what we can do? As we always write we can write here x i hat is x i plus vi. So, what we are doing here? x i is observation v i is the residual. So, if we apply the residual on the observation what we get? We get the corresponding estimate. So, we are getting the corresponding estimate here. So, this thing can be written for all the observations, because over here the observations are x 1 x 2. So, we can write it as x 1 hat is x 1 plus v 1; x 2 hat is x 2 plus v 2; x 3 hat is x 3 plus v 3 we can write it like that. Now, further as you can see I am bringing these v i on left hand side. So, the right hand side becomes x i hat minus x i x i hat minus x i.

Now, the least square what is the least square? In the case of the least square right now, we are considering all the observations. x 1 x 2 and x 3 all the observations had been taken with unit same weight. So, there is no w here no weight here. And then the least square says sigma of v i square it should be minimum. So, this is what we will try to do now. Now, if sigma of v i square is minimum. What is sigma of v i square? Sigma of v i our v i is over here that is our v i its value is x i hat minus xi. So, we write it here sigma of x i hat I will write i for sometime then later on I will not use this i. We will talk about that why it is so? So, sigma of x i hat minus xi square this should be minimum. In order to make it minimum, what we need to do? We need to find the partial derivative of this phi with residuals and make them equal to zero. So, let us do it now, to do it.

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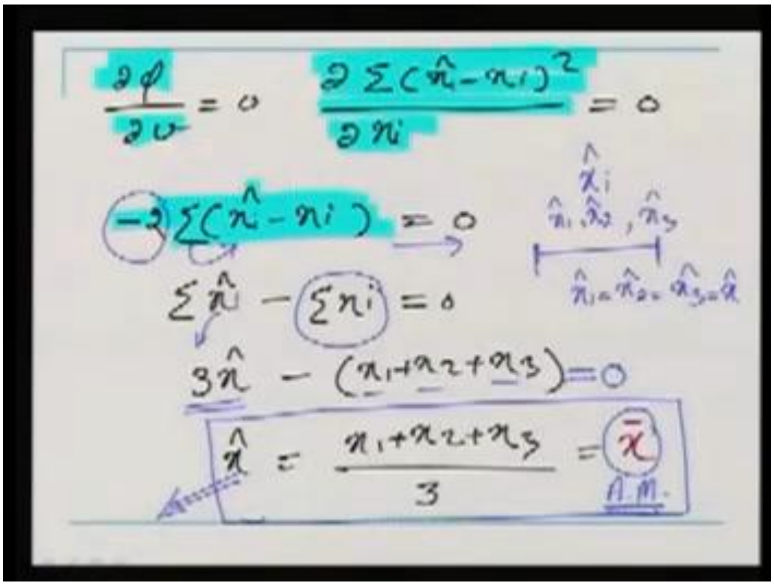

To do it ((refer time: 18:32)) that is our if you are doing it. So, del by del x i of sigma x i hat over here minus x i square this should be equal to 0. Now, if you differentiate this with respect to xi it becomes minus 2 sigma x i hat minus x i . So, this should be equal to zero. Now, please do it yourself also. Now, this 2 is eliminated and we take this sigma inside. So, we can write this as sigma of x i hat minus sigma xi is equal to 0. Now, we have been using this sigma x sorry this xi hat. So, far; however, it is a single line for each observation x 1 hat x 2 hat these are the for each observation these are the estimates x 3 hat well this is 1 line there. So, all these should be equal. So, x 1 hat should be equal to x 2 hat should be equal to x 3 hat and this we are writing now, as x hat.

So, if you are doing it for only 3 observations hat I can do here I can write it as three times x hat this minus look at this sigma xi is sum of all the observations. So, x 1 plus x 2 plus x 3 and that is equal to 0. Well, from here we can write that x hat which is the estimate of this line from least square is x 1 plus x 2 plus x 3 divided by 3 and the same is also the arithmetic mean. So, what we have proved here. We started with the concept of the least square, starting with that concept we found the residuals we we made this sigma of sum of residuals square sum of residual square. We made it equal to the minimum and in doing that we arrive at the conclusion, that the estimate or the best possible estimate. That we can achieve for this set of observation by least square is same as the arithmetic mean.

So, or if our observations are of same weight that is important here you know we did not consider the weight here our weight matrix was identity. If our all the observations for example, the chemical that we are weighing if we are ((refer time: 21:33)) weighed it 10 times and all in all of these. Whenever we are weighing we are saying that well weight of the observation is the same the quality of observation precision of the observation is same. In that case taking the arithmetic mean is same as carrying out the risk for solution. Let us look at another concept that is redundancy and a model.

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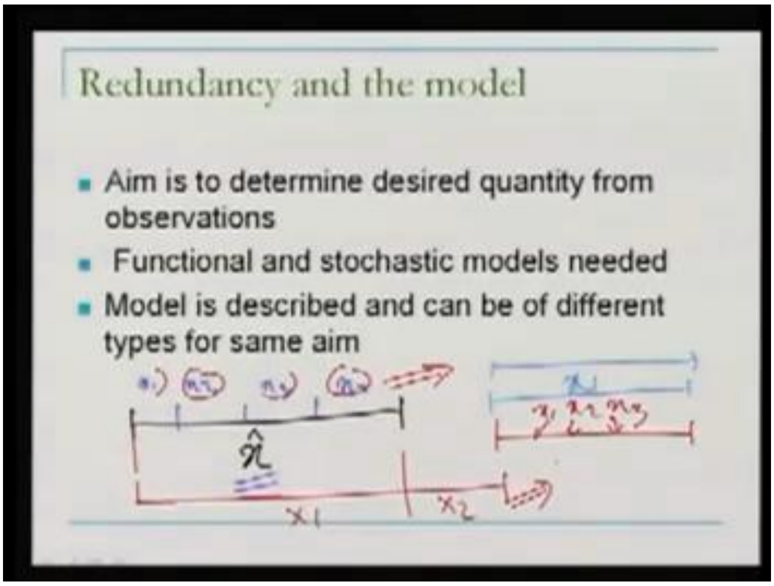

What we do in the field? We always desire to get something in the field for example, to determine the area of a rectangle to determine the length between 2 points, to determine angles between 2 points at an observation point. These are the things which are desired to determine for example, the shape of a triangle these are the desired things. In order to achieve realise this, what is desired? What we do? We carry out some observations. Now, in order to realise something which is desired we can decide the way we carry out the observations in different way. Person A can carry out the observations in 1 scheme person B can carry out the observations in a different scheme person C can carry out the observation in a different scheme.

So, I will give you 1 example here for example; let us say here is the length. We need to determine the estimate for this. What we can do someone measures this length let us say x 1 x 2 x 3 x 4 and he says well summing sum of all these will give me my final length. Someone decides to do it in such a way, because the instruments that he has do not permit this kind of observation. So, what he does? He takes a measurement of this land which is let us say x 1 and then he measures this one x 2.

So, these 2 persons he had a different model and he also has a different model. Similarly, someone says why to do these all observations. He does measurement of this land also it by a different colour. He measures this length for example, if this is the length, he measures this length only once for example, x 1. Someone says well I will not take only single measurement. I rather carry out multiple measurements x 1 x 2 and x 3. X 1 is also this full measurement x 2 is also full measurement and x 3 is also full measurement. So, what I am trying to say, in order to realise the desired thing we can define the model in different way now, because our aim is that model will help us later on to adjust our observations.

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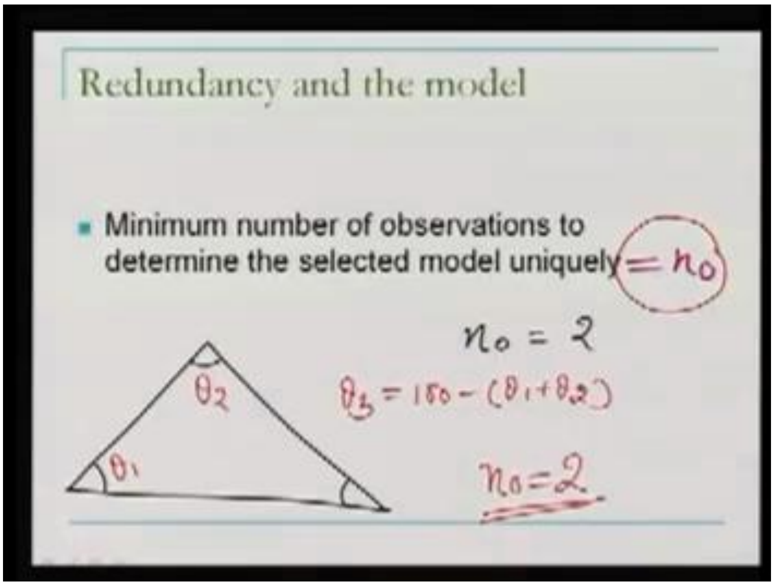

Further in any model we should have some redundancy that we have also seen. For example here, in this triangle the desired is to determine the shape of the triangle. We can determine the shape of this triangle only by observing theta 1 and theta 2, because the theta 3 can be realised by we can realise the theta 3 by this also. So, the shape of the triangle is known, but here in this case we do not have any redundancy in the model. So, before we begin our survey, before we go to the field we rely or rather we try to plan it out. What will be our model of for the computation, whether our model will be able to have some redundancy? Because the redundancy is required in order to eliminate the errors or in order to have a check on the error.

Whether it will be possible to carry out least square solution for our estimate, for our desired quantity whether it is possible or not? Whether our observations whether the way we have decided that we are going to take the observations will be able to fulfil our requirement. I will give you one example of that. Well, anyway we will start by some notations we write n naught n naught is the minimum number of observations which are required to determine our selected model. Now, here in this case the minimum required is 2 n naught is 2, because if both observations are very correct no errors. We can know the shape of the triangle. So, the minimum required are 2 in this case.

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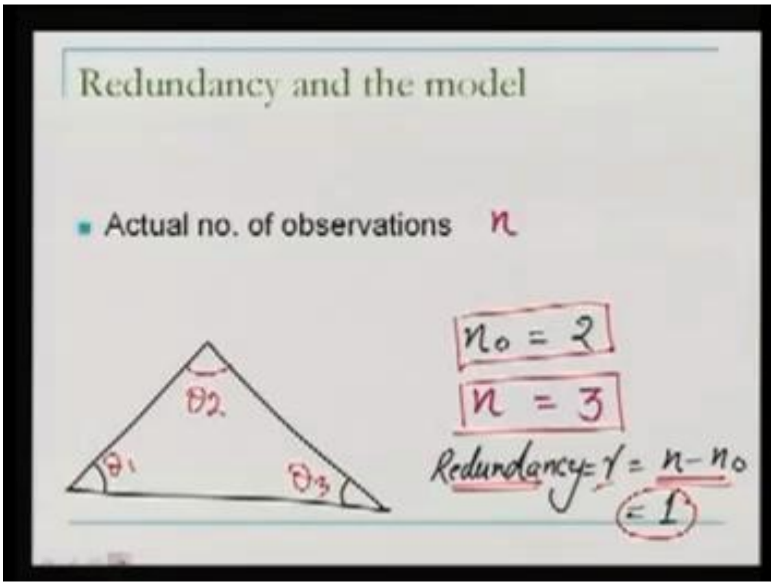

But, if we have done let us say theta 1 theta 2 and to include redundancy. We are doing theta 3 also we have taken 3 observations. If this is the case though the desired is two our number of observations is 3. We define the term redundancy as r is n minus n naught; that means, the extra observations we need only n naught observations to define our model, but we are taking n number of observations extra observations. So, 1 in this case r is equal to 1 thus the redundancy in our model.

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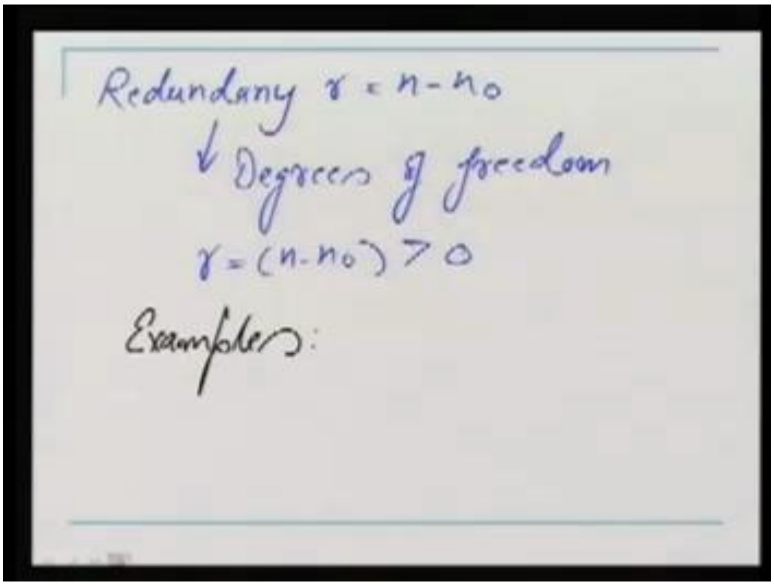

This is also many times called degrees of freedom and will be more than equal more than 0 now, one more example here.

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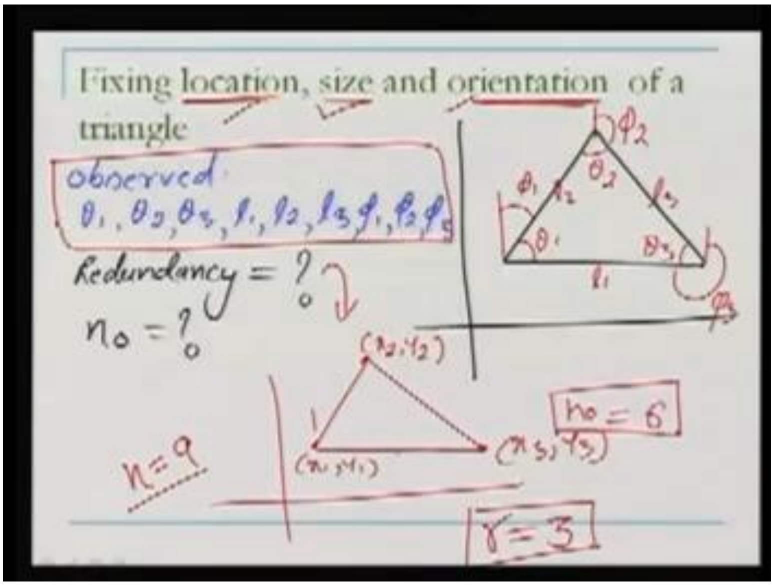

Well the idea is we want to fix location, size and orientation of a triangle. These are the 3 things which we need to fix location. Where it is located? It is size and its orientation. What all observations are required the minimum number of observations. Let us say someone who went to field has carried out theta 1 theta 2 theta 3 these 3 observations as well as he has also observed the bearings of the lines. For example, here it is phi 1 phi 2 and phi 3 these are the bearings as well as 1 has also observed the length l 1 l 2 and l 3. Do we really need all these in order to determine in order to fix the location, size and the orientation of the triangle? What is the minimum required? So, in this case these are the observations, what is the redundancy?

In order to know the redundancy, we should know what is the minimum number of observations which are required, in order to solve this problem in order to fix this problem? Well in this case the minimum required, because we need to fix the location size and orientation. If I somehow can observe the coordinates x 1 y 1 x 2 y 2 x 3 y 3. Let us say somehow we have observed these coordinates. If it is done the size of the triangle is fixed, the orientation of the triangle is fixed and the location of the triangle is also fixed all these are fixed. So, what we see only by 6 observations. So, we will to say our n naught is equal to 6.

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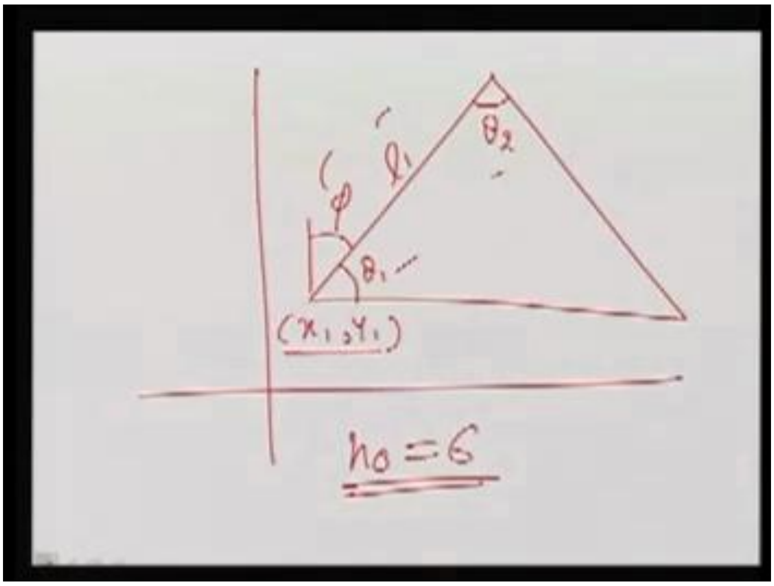

You can say this same problem in a different way also the same triangle. Well I observed 1 value of phi. So, orientation is fixed. I observe 2 angles theta 1 and theta 2. I observe the length l 1. What we have done so, far? So far we have fixed the size of the triangle and the orientation of the triangle. But still the location of triangle is not fixed. To fix the location we also observe x 1 and y 1. So, these are again 6 at some y 1 theta 1 theta 2 phi and l 1 again n naught is equal to 6. So, whichever way we go we need minimum number of 6 observations in order to find the solution of this problem. The problem was to fix the location size and orientation of the triangle.

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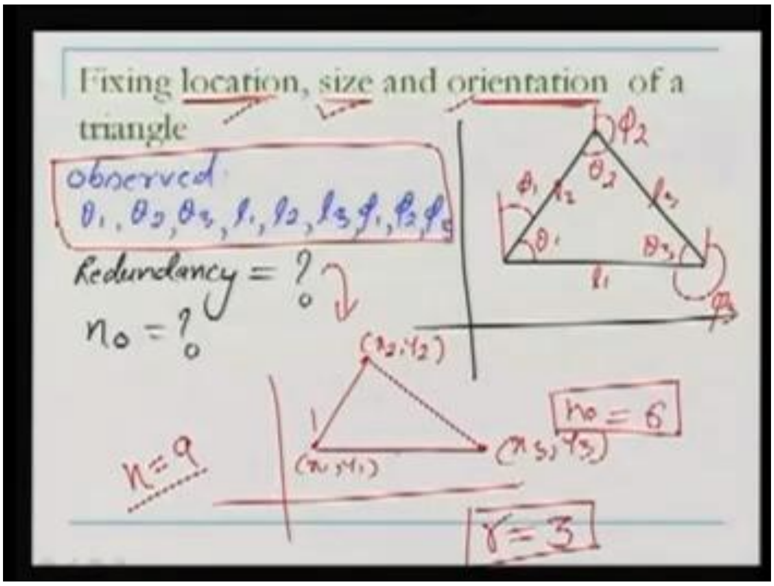

But the number of the observations that we had taken, we had taken the number of observations were nine our n was 9. So, our redundancy r will be 3. So, in any model we can work out the redundancy like that. And this is important because we make use of this later on1 final thing about this redundancy. As we saw that you know we define a model it should have proper redundancy ((refer time: 32:03)) we want our observations with the redundancy. But not always if there is redundancy we can solve for the model. Now, I will give you one example here.

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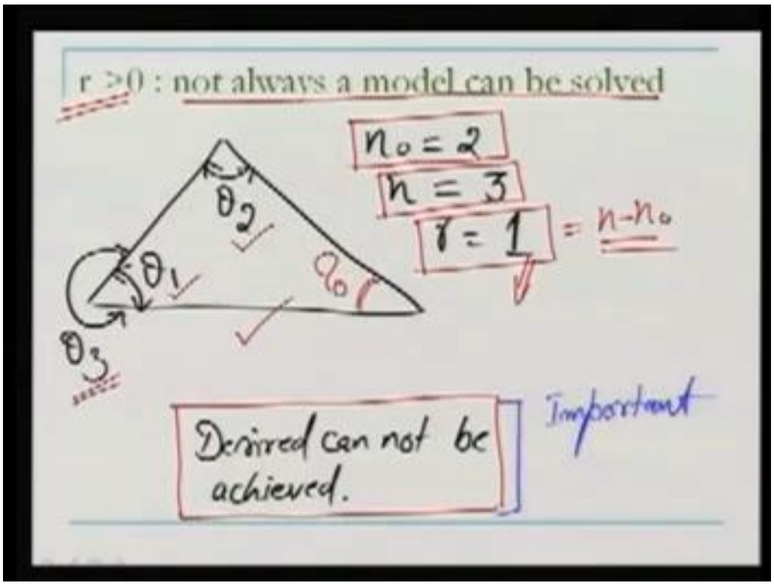

Let us say again the aim is to determine the shape of this triangle. If this is the aim well we can do, because we have now taken the observations theta 1 and theta 2. And we need only these 2 observations in order to solve for this problem, but the solution will be unique something which is not desired. Unique means we do not have any redundancy in the observations. In this case if only number of observations that we have taken h 2 that is is not desired. What we want to do? We want to have redundancy well what we do we also observed now theta 3. So, now what we have done? We have increased the number of the observations n is equal to 3 and now the redundancy is 1 that is n minus n naught.

So, we have a redundancy in our sets of observations, but can we solve for our model. Can we find the estimate of our desired quantity? No, still we cannot find the best estimate for our desired desired quantity, because still for the shape of the triangle. For the shape of the triangle this angle still needs to be determined from theta 1 and theta 2, because we do not have any observation here. So, what do I mean? I mean in this case the desired cannot be achieved cannot be computed. So, not always a model can be solved even if the redundancy is more than zero. So, this is important. Introducing redundancy in the observation is not the only thing rather introducing the redundancy in such a way that the desired can be estimated is required.

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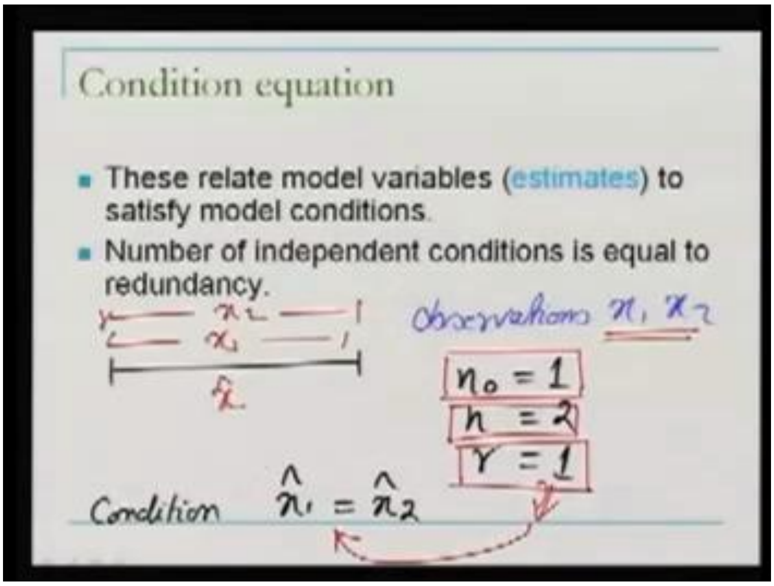

Now, we will go for our approaches for adjustment. We have 2 methods for carrying out this we say the condition equation method and observation equation method. We will see them one by one. What these are? So, first we will try to understand these, what is the concept of condition equation and what are the concept of observation equation? This is what we will try to understand now. Now, in the case of the condition equation, the condition equation method it tries to frame conditions on the observations that we have carried out. It does not take the parameters of the model into account parameters means the desired things.

For example, if we are observing a length of line as you can see here. A length is there and we want to observe its length. So, our parameter is x hat. That is the desired thing that we want to determine. So, we do not take this into account rather we frame our condition equations using the observations. How we can do? Now, here in this case let us say we have taken 2 observations x 1 and x 2 this was measured once x 1 1 measurement and then another measurement of the same line x 2. So, our n that is number of observations is equal to 2.

How many observations we need in order to find that unique value of that length of the line only 1 observation of course, in that case there will not be any redundancy, but that is the minimum 1 which is required to find the solution. So, we say n naught is equal to 1 only 1 observation. So, the redundancy is 1. Now, here it is important. What is the redundancy in the model? We can write the equal number of condition equations using our observations. Now, here r is equal to 1. So, we need to write 1 condition equation and that condition equation will take into account only the observations. How do we write this? We write it this way well let us get into it. What is the meaning of this?

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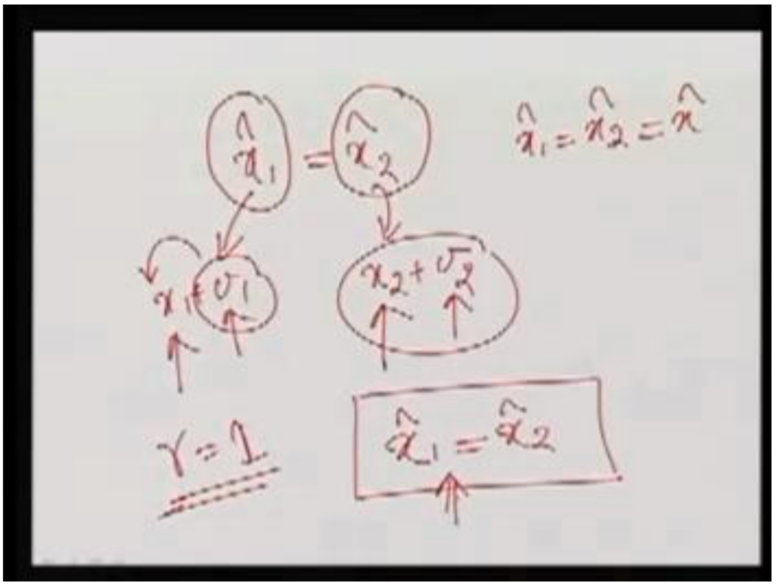

I am writing x 1 hat is x 2 hat. Obviously x 1 hat is x 1 plus v 1. X 2 is x 2 plus v 2. X 1 is an observation x 2 is also an observation while, v 1 and v 2 are the residuals. So, once we apply this residual on x 1 we get x 1 hat and similarly here we get x 2 hat. So, in any observation if we are applying the residual, we are getting the estimate of that observation. And all these estimates should be same, because finally, it should give me the line the actual line. So, all these x 1 hat should be equal to x 2 hat means that is equal to x hat.

So, our condition equation in this case is x 1 hat is equal to x 2 hat. So, what we have done in this condition equation. We have framed a one condition equation, because our r was equal to 1. So, we can frame one again this important one independent condition equation. In our last problem it may be possible that we may frame some condition equations which are dependent on the previous ones. So, it should not be. So, all the condition equations should be independent. So, that is the condition equation that we have framed.

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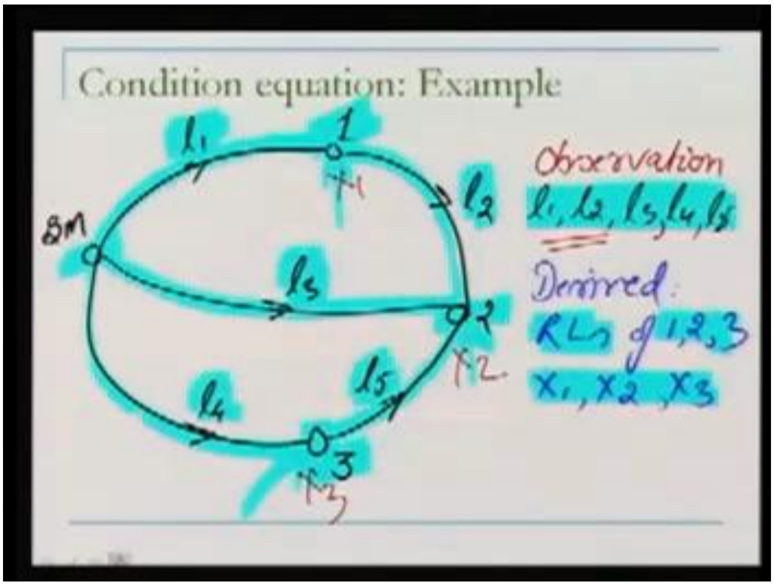

So, similarly we can frame the condition equations for more complex problem. I am taking one example here. Again the example is frame starting from the benchmark we are carrying out the levelling in order to determine the RLs of various points. Let us say the points for which we need to determine the RLs are point number 1, point number 2 and point number 3. We want to determine the RLs of these points. So, what we do? starting from the benchmark which is known I take an observation along this route and the value of the difference in RLs is l 1. So, by that I can determine RL of point 1. So, this is a unique solution. We do not want it that way.

Well further because we had to determine RLs for 3 points. I can go further 2 point to by observing this l 2. Then 1 redundant observation l 3, because now, this l 2 can be computed l 2 can be computed through this route as well as this route. So, we have introduced the redundancy in the observation. Further in order to determine the RL a point 3 have been known the RL of point 2. I am finding the difference in elevation between these 2 points that is l 5. So, I can determine the RL of point 3 also.

But I am also taking 1 more observation here that is l 4. So, again there is redundancy. So, what all observations that we have carried out? We have carried out l 1 l 2 l 3 l 4 and l 5 these are differences in elevations. While the desired are relative valuations RLs of 0.123 let us write them as x 1 x 2 and x 3. So, the RL of these points are x 1 x 2 and x 3 these this is the desired thing. While we have carried out l 1 l 2 l 3 l 4 and l 5 observations now, what we will do we will try to analyse this problem now.

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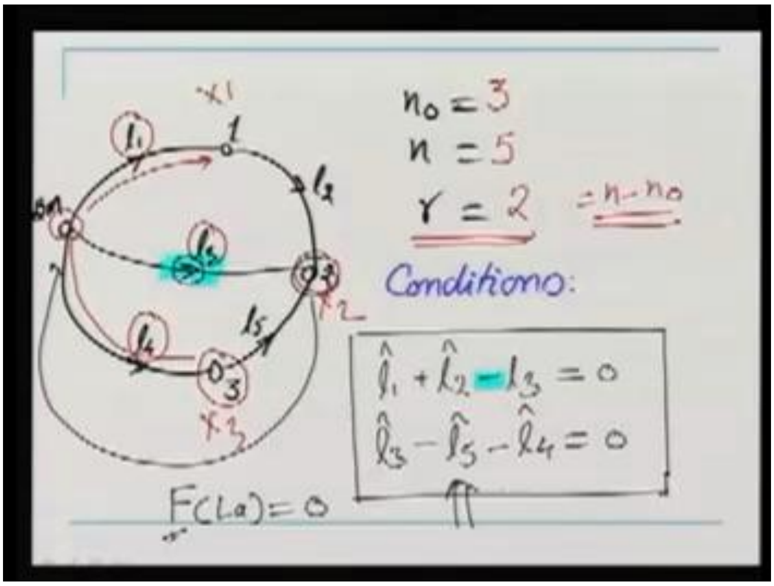

For this problem number 1 what is the minimum number of observations which are required in order to find x 1 x 2 and x 3? Well you can guess very easily while we can do starting from the benchmark I can take this route I take the observation l 1. So, I know the RL of 0.1. Similarly, starting from benchmark I can take this route and determine l 3. I can find the RL of 0.3.2. And finally, I can take this route and determine the RL of 0.3 by taking these observations. So, this is the only you know 3 observations. We can carry out only 3 observations in order to solve this problem, but in a unique way. That is important we need only 3 observations yes in order to solve this problem, but our solution is a unique solution. We do not know about errors it is not. It is a unique solution.

You know we have been talking about that the significance of the redundancy there is no redundancy in the observations in our model. Well the actual number of observations that we have taken is 5. So, the redundancy here in this case is 2 which is n minus n naught that is the redundancy. So, what we will have we should have 2 number of condition equations which are independent. So, we need to write those condition equations now. How do we write it? Well the condition equations can be written now, as if you look first in the loop here starts from benchmark go to 0.1 go to 0.2 and again go back to benchmark.

So, here is the loop. Here is a loop means, we are starting from the benchmark going to 0.1 going to 0.2 and again coming back to the benchmark. And what we are observing? We are observing the difference in elevation l 1 is difference in elevation between benchmark and the 0.1 l 2 is difference in elevation between 0.1 and 0.2 and l 3 is difference in elevation between 0.2 and the benchmark is a closed loop. So, what we can do? using this closed loop we can write one condition equation and that condition equation will look like l 1 hat plus l 2 hat minus l 3 is equal to 0.

Now, please mind it, I have put an arrow here. That arrow shows that this is the direction or this is the positive observation. This is the positive observation this is the positive observation. So, the sign this is why, I have used minus here. Further we have another loop and in this loop if we start from the benchmark go to 0.2 then come back through 0.3 to benchmark. We can write for this loop as l 3 hat minus l 5 hat minus l 4 hat this is equal to 0. So, what we have done? These are 2 independent condition equations. So, the condition equations basically have a form of, because these are the condition equation.

This is a matrix notation here it is a function of the adjusted observations the estimates that is why you are using l hat here. And the relationship between adjusted observations whatever all the possible relations which are there which are independent we write them in this form. So, these are the condition equations. So, later on we will make use of these condition equations in order to find the solution by least square.

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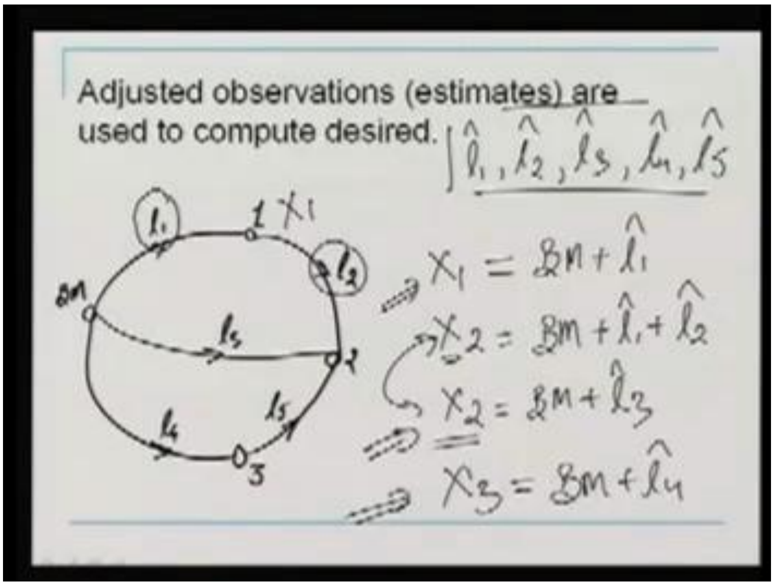

Now, here in this case, once using our solution of these equations will have a method in order to solve for this equation of for these 2 equations. So, that we can find l 1 hat l 2 hat l 3 hat l 3 hat and l 4 hat and l 5 hat. So, once these are known means l 1 hat l 2 hat l 3 hat l 4 hat and l 5 hat these are known; that means, we have now, the adjusted values of these. If you write for example, l 1 plus l 2 minus l 3 that will not be equal to zero we know it because these are the observations, but for l 1 hat plus l 2 hat minus l 3 hat that will be equal to 0, because these are the estimates. Once we know the estimates what we can do? we can find the value of desired quantity. So, what is the desired quantity? For example the x 1 X 1 is the RL of this point this is benchmark plus l 1 hat.

Similarly, the x 2 could be benchmark plus l 1 hat plus l 2 hat and x 2 can be also written as benchmark RL of benchmark plus l 3 hat. Now, in this case this 2 x 2 will be same, because in our least square solution this functional model has been taken into account. So, whatever the way we find the value of x 2 what the values should be seen if our solution is correct. So, similarly we can also determine the x 3 h benchmark plus l 4 hat. So, our desired quantities can be now computed if we know these estimates. So, this is the method of condition equation. Now, we will see the method of observation equation.

Now, we are going for the observation equation method. In the case of the condition equation method we saw that, we frame the condition equations using estimates of the observations. For example, l 1 hat l 2 hat l 3 hat that we have seen only estimates or observations were taken into account and we try to write r number of r means the redundancy independent condition equations. And then we solve for them in order to determine the value of estimates of the observations. And then we find the desired quantity. Now, here in the observation equation method, it is different. Here in this case we include the estimates of the observations. Yes, but in addition, we also include the model parameters means the desired quantity. What is the desired quantity? We also include that in the equation here. Now, how we do it let us see it?

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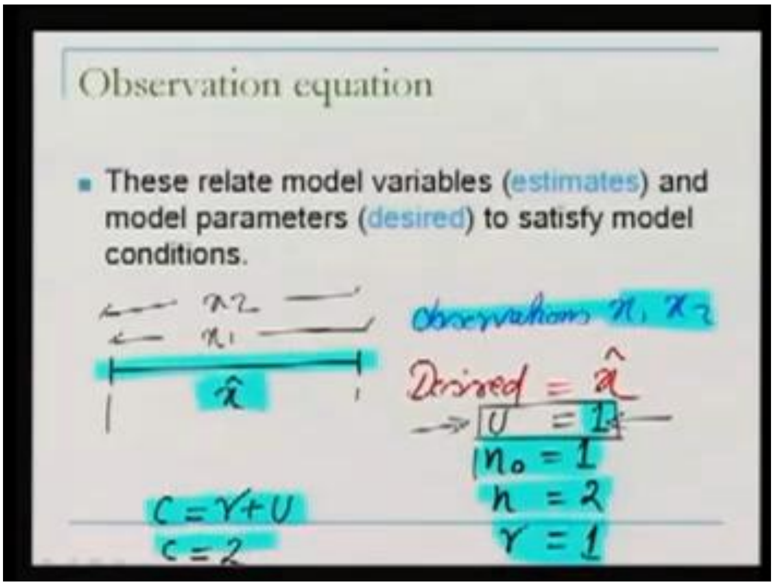

We have a line over here and we want to observe this line. So, the desired quantity is x hat. What we do we carry out 2 observations x 1 and x 2? Someone measured this line x 1 and then again x 2 2 observations have been taken. Now, we know now, u here comes for the number of unknowns in terms of the desired quantity. So, the desired quantity here is the length of the line. So, here is 1 unknown. So, u is equal to 1. n naught as in the previous case the minimum number of observations. Which can determine this unknown? n the actual number of observations which are actually taken. R is the redundancy. Now, here in this case the total number of observation equations that we can write is equal to C. C means r plus u. Over here in this example C C is equal to 2. Because we are including the model parameters also into the computation processes in our least square solution or in writing this observation equation. So, we will need to write C number of observation equations. Now, how do we do it particularly now, in this example?

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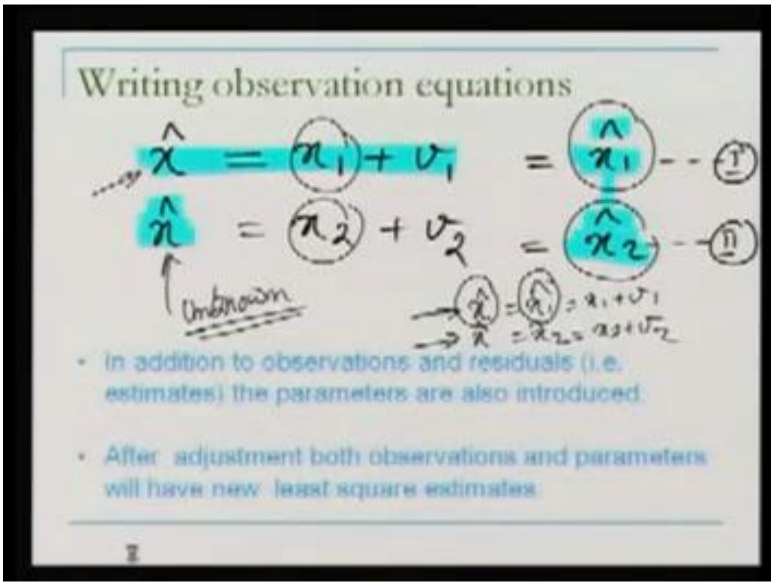

We know x hat is x 1 plus u this is equal to x 1 hat also you know we can write. Because we know for this problem x 1 hat and x 2 hat these 2 are same; obviously, which are equal to x hat. So, x hat is our unknown. This is the only unknown. What we have done? We have taken these as observations. So, what we are doing? We are relating unknown with the estimate of the observation. So, how we can relate it? We can write 1 equation as we have written over here you know. X hat is equal to x 1 hat. X hat is equal to x 2 hat this all we can write. And x 1 hat is x 1 plus v 1. X 2 hat x 2 plus v 2. So, these are the 2 observation equations. Which we are able to form they are independent observation equations. They are relating important thing is that they are relating the unknown with estimate of the observation. So, generally, this will be the form of our observation equations.

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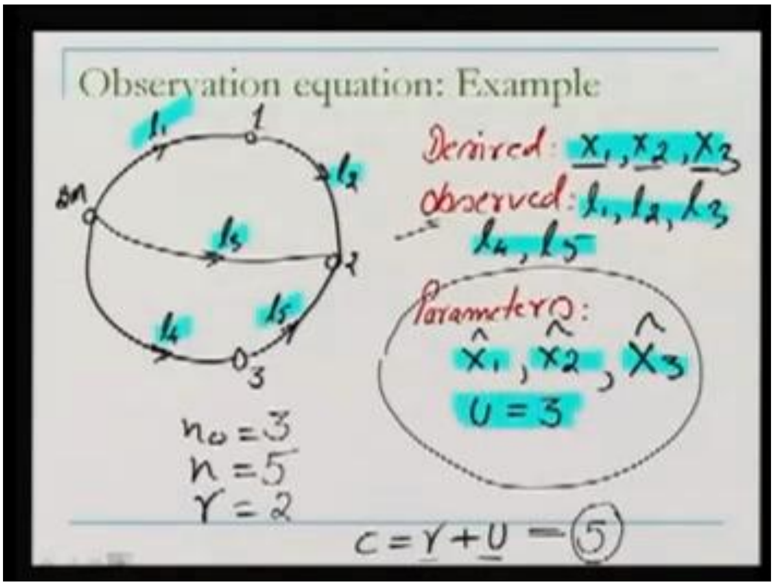

Now, we look in the same example which we have done earlier. Now, in this example we know the desired quantities are x 1 x 2 and x 3 desired quantities. And we have carried out the observations l 1 l 2 l 3 and l 4 which we know these are the observations which have been carried out. Now, what are the model parameters the unknowns? The unknowns are the final estimates of x 1 x 2 and x 3. So, the total numbers of unknowns in this problem are 3. N naught means 3 we have seen it before and in this case is 5. So, r is 2. So, the C will be r plus u. So, that is equal to 5 what is the meaning of this? The meaning is, if we are including these parameters also into the equations along with the estimates of observations. We will need to write 5 observation equations in this case. What this observation equations will be?

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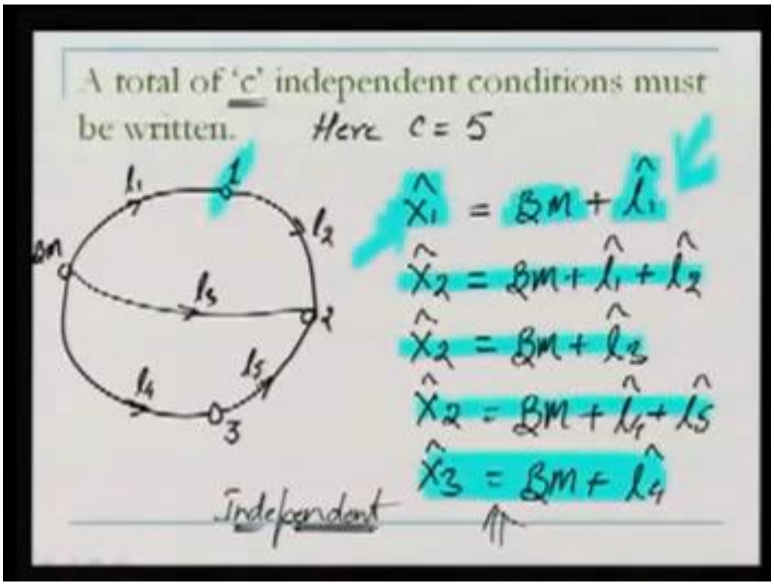

The observation equations I have written over here. First observation equation x 1 hat I am writing ((refer time: 53:45)) here, because that is the estimate of this is benchmark plus l 1 hat, because it is relating the parameter with the estimate of the observation. Similarly, we can write one more also. And finally, in all these observation equations are it is important these are independent. This is important. Whenever we have a problem, we are going to solve a problem. We must ensure that the observation equations which we are writing should be equal to first the C and all these should be independent. Once we have written these observation equations. We will apply the least square solution to these. And we have a method by the least square when we can find the solution of these equations. And through that solution as is obvious we will find the values of the model parameters as well as we will find the values of the estimates of observations.

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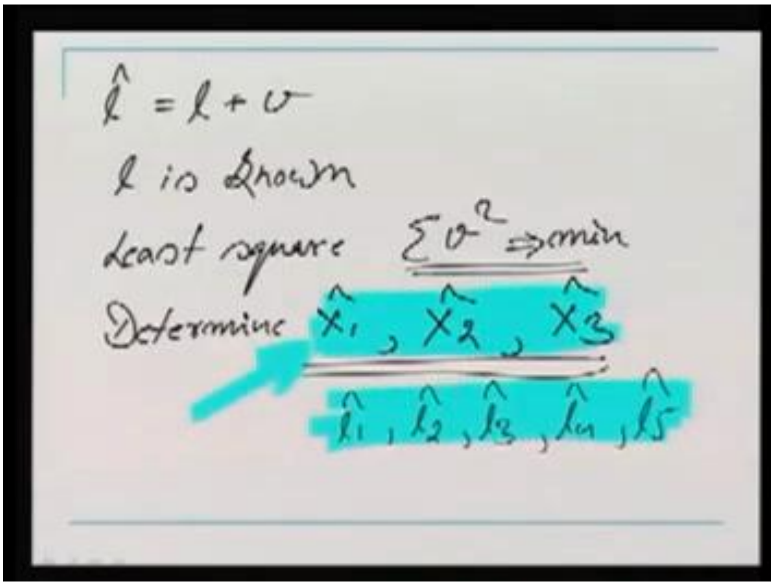

So, what we will determine by applying the least square we will find the desired quantities as well as l 1 hat l 2 hat l 3 hat l 4 hat l 5 hat. Of course, these are not really the desired thing. We do not need them, but well these are computed in between our desired quantities are the one is which are actually being computed here. So, our effort will be to determine these desired quantities value. So, what we saw today? We saw again the concept of the least square, then the criterion how to we you know go about the least square, then the redundancy of a model why it is important, why it should be there?

We also saw one important thing even, if there is redundancy it is sometimes so, that the model cannot be solved. So, whenever you are planning the survey we should ensure that the model or the observations are defined in such a way that we can have the redundancy in the observations as well as we can solve for the desired quantities. Then we saw 2 method or 2 approaches of condition equation and observation equation. We can form the condition equations using the estimates of observations and the numbers of the condition equations are equal to the redundancy.

Then in the observation equation not only the estimates of observations we also include here parameters of the model the unknowns the desired quantities. And the number of the equations in this case was redundancy r plus u number of the unknowns. So, we need to write that many number of unknown equations. So, we just have an idea of condition equation and observation equation now. Next what we will you do we will frame for the

((refer time: 57:03)) for some example and we will try to solve it by the least square. So, we will see the least square solution in these 2 approaches.

Thank you.