

Surveying
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Module - 5
Lecture - 3
Theodolites and Total Stations

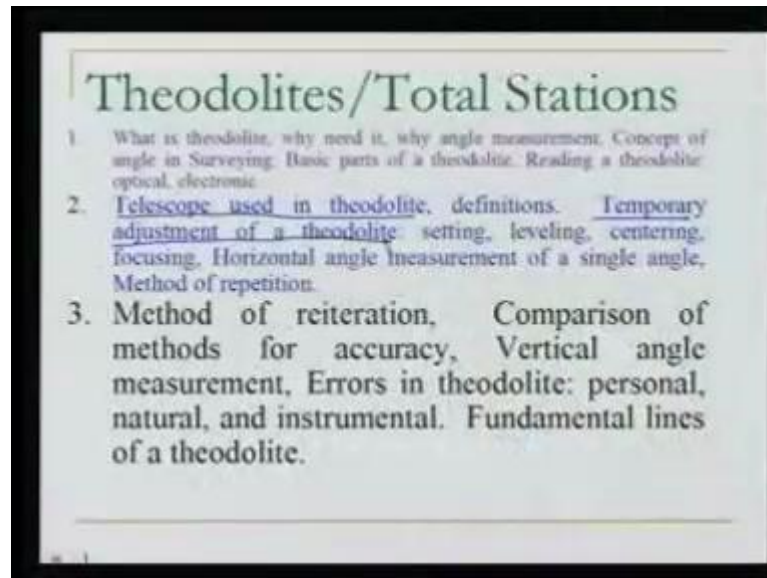
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Welcome to another lecture on basic surveying. This is module number five and this module is on theodolites and total stations. Today, we will do lecture number three of this module.

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Now, what we have done so far? As far as this module is concerned on theodolite and total stations, we have seen the instrument, the theodolite, why we need to use it and in particular, in our last lecture we discussed about the telescope, which is used in the theodolite. And then, from definitions like the line of sight, line of collimation and some more definitions, which are related to the theodolite. We also saw what is face left observation, what is face right observation. Then, if you are going to a field and we want to measure an angle, we saw the procedure last time.

So, there are some steps, which we need to do at every station. These steps are called temporary adjustments. What we do? We take the theodolite along with the tripod to that particular point or we can say, survey station. Then, we need to do a couple of things. Number one is centering. We put our tripod over that point, approximately center it; leveling, we level the tripod height approximately. Then, having done this, put the tripod or we put the theodolite on top of the tripod and after that we will start doing the centering of the theodolite, leveling of the theodolite and as well as we do the focusing.

So, having achieved proper centering and leveling, the focusing means, we sight to the object and we need to focus that. In order to focus the object we need to first focus the eye piece, so that we can see the cross wires very distinctly. Then, the image through the objective lens is focused on the plane of the cross wire.

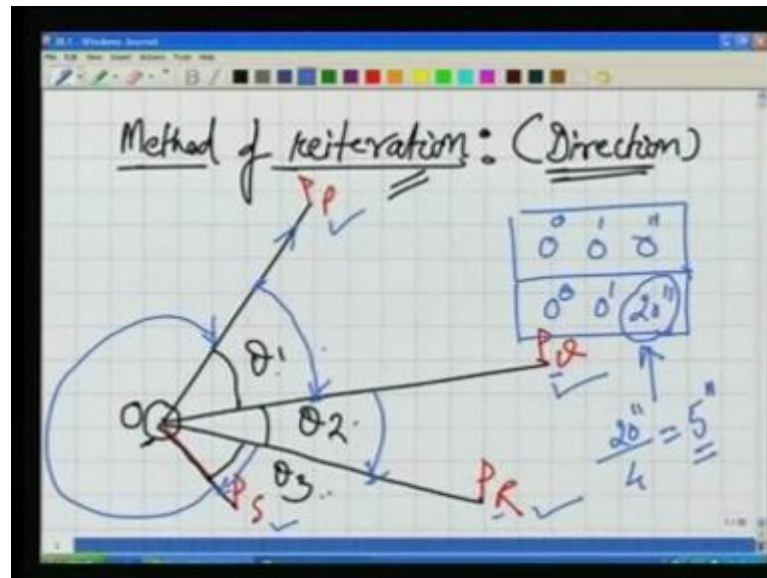
Now, here the term, which is called parallax. Parallax means, if this is the plane of the cross wire, the image, which is being formed is not at this plane, but rather here. Well, you may find this is clearly seen in the images, distinctly seen focus image as far your eyes are concerned. But is there any parallax between them? Yeah, there is the parallax between them because if you move your eyes here and there slightly, you will find, there is a relative displacement between the image and the cross wire.

If these two are exactly on the same plane. So, the image has been formed on the plane of the cross wire. Now, by moving your eyes here and there, there will not be any relative displacement between these two objects. Between these two objects means, the cross wire and the image. So, the meaning is, we have removed the parallax. So, we removed this parallax and now we are ready to take the observations. So, we have seen that thing.

Then, also we saw the method of repetition, how we can measure a single angle using the method of repetition. In the method of repetition, if you recall, we use the theodolite itself for mechanically summing up the angles. One single angle was measured time and again, and only the first reading and the last reading after n number of iterations was, they were recorded. And then, finally, we take the average of or not average, I will say, rather we divide the total reading divided by total number of the iterations, how many times we took the observation and this is how we can improve the precision of the instrument also.

What we are going to do today? We look into another method, which is the method of reiteration. Another method, useful in some other circumstances. Then, we will compare these two methods, the method of repetition and method of reiteration for the accuracy, which they can give. Then, we will also see the vertical angle measurement and possibly some errors of the theodolite, which are because of the person involved the natural errors and the instrumental errors. And finally, if time permits for this video lecture, we will also go into fundamental lines of the theodolite, what these are. So, we will start now with method of reiteration.

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In the method of reiteration what we do? If, for example, there is a point O where the observer is. We have some ranging rods, for example, one here at P, at Q, at R and at S. So, we are interested in measuring the angles between all these points. So, the angle, we can say, theta 1, theta 2 and theta 3. We are interested in measuring these angles. What we can do? We can go by the method of repetition and we can measure these individual angles separately.

But in the case of method of reiteration, which is also called method of direction, the procedure is slightly fast. Whenever we need to measure multiple angles at a single point, as in this case, we need to measure at O, three angles. So, we generally go for method of direction or method of reiteration.

What do we do in this case? The procedure is simple. Again, we will take our theodolite to the point O. When the theodolite is set at O, we will make the reading in the theodolite 0, 0, 0; 0 degree, 0 minutes and 0 seconds. You know how to do it now. Once you have done it, we will unclamp the lower screw and we will rotate the theodolite or the telescope. So, by unclamping the lower one and while the upper one is clamped, the reading is not changing, it still remains 0, 0, 0.

So, what we will do? Using that we will first bisect P. So, the P is bisected and the reading is 0, 0, 0. Next we bisect Q, next we bisect R, next we bisect S and we keep

taking all these observations, all these readings. The reading at P was 0, 0, 0; at Q; whatever is the value at R; whatever is the value similarly at S.

Then, next what we do? We close the horizon. The meaning of that is, after taking these readings at S, we close the horizon and we come back to P again. So, while we are coming back to the P again, the reading should be 0, 0, 0, as we had initially fixed for the point P.

It might not be so also depending the errors in the theodolite or may be, while we are working, you mishandled some of the screws a little bit, which we say slip off the screw. So, because of these reasons the reading may not be 0, 0, 0. It may be 0 degree, 0 minute and 20 seconds. So, what is the meaning of this? While we are measuring the angle θ_1 , θ_2 , θ_3 and this, for total angle we entered, we introduced an error equal to 20 seconds.

So, in method of direction, method of reiteration, what we do? If this error is small, as in this case, and of course, depending upon the kind of the survey we are doing, if it is small, we distribute it. 20 seconds can be distributed in θ_1 , θ_2 , θ_3 , as well as in angle SOP. So, 20 seconds divided by 4, 5 seconds. So, we will distribute this 5 seconds to each of these angles. So, we are correcting our observations for this misclosure.

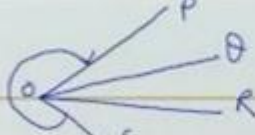
Now, this observation can be taken right now. For example, let us say we are doing it on face left. You know now what is the meaning of face left and face right observation. We can do the same thing on the face right also. For example, let us say, right now we were doing it on face left.

So, we started from P, went to Q, went to R, went to S and close this. Then, what we can do? We can change the face now and we can repeat the same procedure. So, we will have readings or angle values for face left and face right. Now, how to record these readings? How the tables look like? I am going to show you the tables now.

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Table for method of reiteration

Inst. at	Sight To	Face Left			Swing Right			Horizontal angle	Corrected Horizontal angle
		A	B	Mean	A	B	Mean		
O	P	0	0	0	0	0	0		
	Q	30	20	20	20	40	30	20	30
	R	✓	✓	✓	✓	✓	✓	✓	✓
	S	✓	✓	✓	✓	✓	✓	✓	✓
	P	0	0	20	0	20	0	0	20
		Average error of closing =			Correction to each angle =				



$20'' = 5''$

So, the table looks like this. And in this table, as you can see, once you start taking the observations from P, what above was the value at P? It was 0, 0, 0. This is the table for face left and we are swinging the instrument to right. Well, what we write? The first column is, instrument at, because the instrument is at O, then we are sighting to number one P, so the readings were 0, 0, 0. Now, this is for Vernier A and for Vernier B we will write only in minutes and seconds. So, 0 and 0, if it is so. And then, the mean of Vernier A and B, which is 0, 0 and 0.

Then, our next sighting is to Q. So, whatever is the value at Q, for example, 30 degrees, 20 minutes and 20 seconds and here it may be 20 minutes and 40 seconds in Vernier B. Well, we will take the average of these two. So, 30, 20 minutes and 30 seconds. Now, what it gives you?

We can similarly write the observations what we are recording for R and for S also. And finally, when we close the horizon as we go back to the P, we will again record the reading here. May be, this reading is 0, 0 and 20 seconds. Well, there is some error, 0 and 20 seconds. So, there is some error of 20 seconds, is the error of misclosure. So, this will be distributed.

Now, what we can do? Using these values here, using these values we can determine the value of the horizontal angle. I can write the horizontal angle. The reading to Q minus

reading to P will give me the angle here, which will be 30, 20 and 30. Similarly, we can get these values of the angles also. So, all these angles are written here.

Now, so far in these horizontal angle the correction has not been applied for the misclosure. What we can do? We can apply for the correction for the misclosure, as we found it should be 5 seconds to each of these angles and we can write these corrected angle values over here. So, this is one face, face left reading. What we can do?

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Table for method of reiteration

Inst. at	Sight To	Face Right			Swing Left	
		A	B	Mean	Horizontal angle	Corrected Horizontal angle
O	P					
	Q					
	R					
	S					
	P					
		Average error of closing =			Correction to each angle =	

We can have a similar table for face right. We are swinging to left now. And by doing the same procedure, nearly same, we are getting the angle values, again, for face right.

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The slide displays a table for the method of reiteration. The table has three columns and eight rows. The first row is labeled 'Average Horizontal angle'. The second row has three columns with '+' signs. The third row is labeled 'Face left'. The fourth row is labeled 'Face right'. The fifth row is labeled 'Average of face left plus face right by 2'. The sixth row is labeled 'Face left'. The seventh row is labeled 'Face right'. The eighth row is labeled 'Average of face left plus face right by 2'. Below the table is a diagram of a horizontal line with points and labels 'Average value as follows' and 'Average of face angle'.

So, we have the angle values of the horizontal angle for face left and face right and finally, we will take the average of these two, average of face left plus face right by 2 and this is what we will write in the final values of the angles. So, this gives us, finally, the angle values by the method of reiteration.

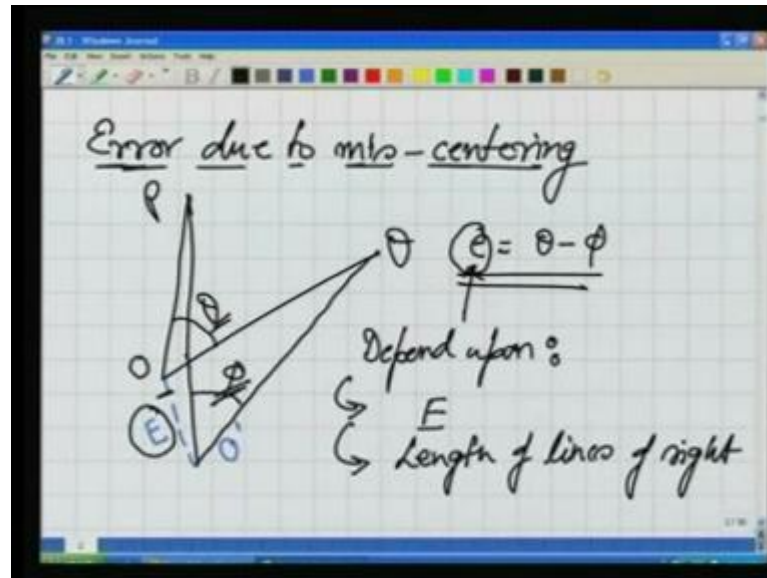
And whatever we have seen right now, this entire thing is one set. So, with this we say, the face left observation, face right observation, all these constitute to be one set of the observation. We can have multiple sets, you know, four sets, five sets, depending what kind of survey you are doing.

In order to measure the angle, as we have been seeing that what we need to do, we need to bring our tripod and the theodolite and we need to center it. ((Refer Time: 04:43)) as in this diagram, we wanted to keep our theodolite exactly over the point O. There, in the ground point O is a point and what we want to ensure, that the vertical axis of the theodolite, we have seen the vertical axis of the theodolite about which the theodolite rotates, should pass through the point there on the ground.

Now, this is slightly difficult exercise. Difficult, in the sense, there are lot of trial and errors are involved, and never we can reach the true value or the true orientation, true centering, perfect centering. So, what to do in that case?

We should know, if at all we are introducing some errors due to miscentering, what kinds of errors will be propagated into the angle that we measure. So, this is what we are going to see now. We are going to see error in angle measurement due to miscentering.

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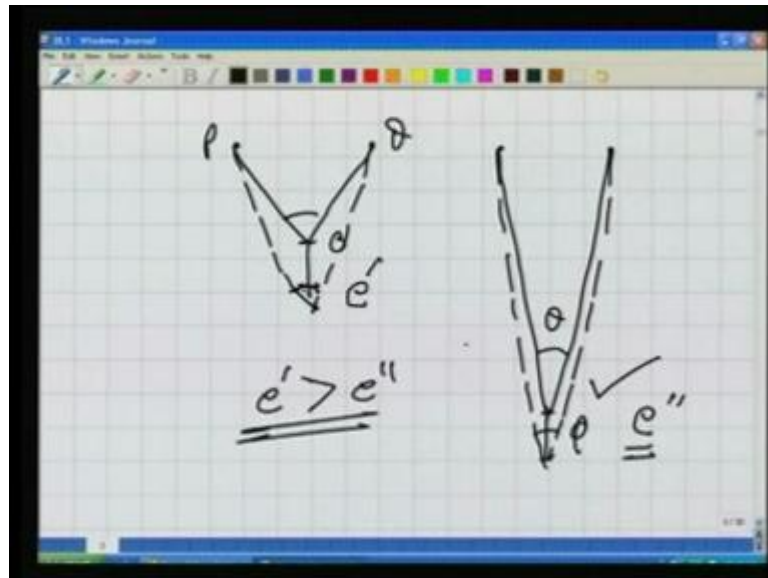
So, basically, if I draw the diagram here, what it is like? Let us say, there are two points, P and Q and O has to be here. This is the observer and we want to measure angle theta. The meaning of miscentering is, somehow we are not able to center our instrument at O, rather we are centering at O dash. Now, because of this what will happen? Instead of taking OP and OQ as the line of sight, we will be taking O dash P and O dash Q as the line of sights.

So, what we are doing? Instead of measuring theta, we are measuring phi. So, this error e in angle due to theta minus phi or we can say, due to this miscentering, is the error introduced in our angle. So, we can see by this simple diagram, that whenever we are measuring the angles and if there is miscentering, there is some error, which is going to be introduced in our angle measurement. Because there in the field what angle we need to measure? In the field we need to measure theta because the angle in the field is theta. But what we are measuring? We are measuring phi and because of this there is an error of e here.

Now, what this e will depend upon? What it will depend upon, let us see. Number one, it will depend upon the amount of e, how far is the miscentering. You know is the

miscentering is small or large? If it is smaller, this O dash point is somewhere here, then the value of e will be small. It will also depend upon the length of lines of sight. What is the meaning of this?

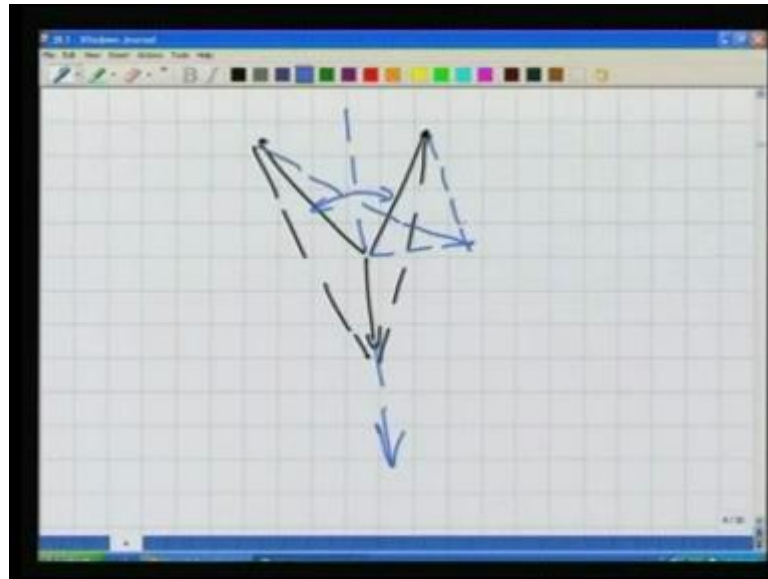
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The meaning of this is, if I draw a diagram here, let us say, my original P and Q are there, this is O and in one case, and now the miscentering is some amount here. And what we are doing? We are taking these at the line of sights and there will be some error due to miscentering. Well, for the same P and Q if the line of sights are, lines of sights are very, very long and the same amount of miscentering is there.

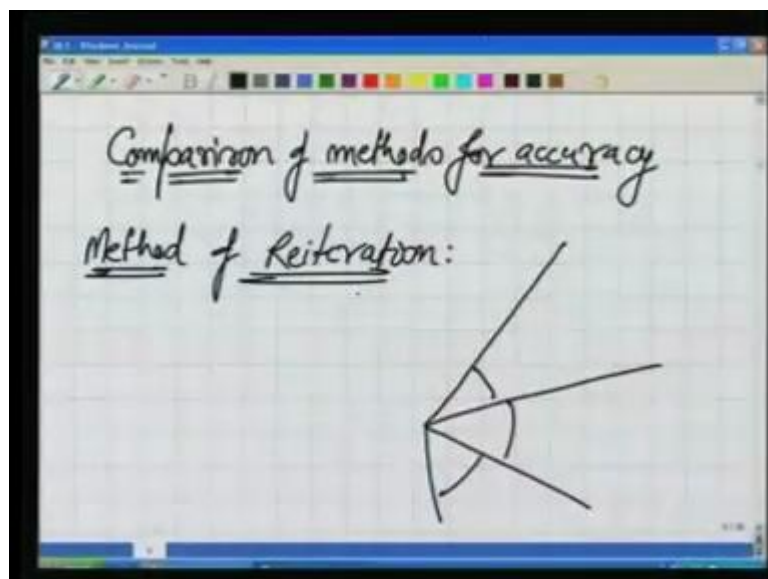
We take now the lines of sights from O dash and then, there will be e . So, θ and this is ϕ . So, here in this case, you will observe, that the value of e error due to miscentering will be less than the value here. If I write e dash e double dash, so e dash will be more than e double dash. So, just by this graphical representation we can see if the lines of sights are longer, the error in our angle due to miscentering is less.

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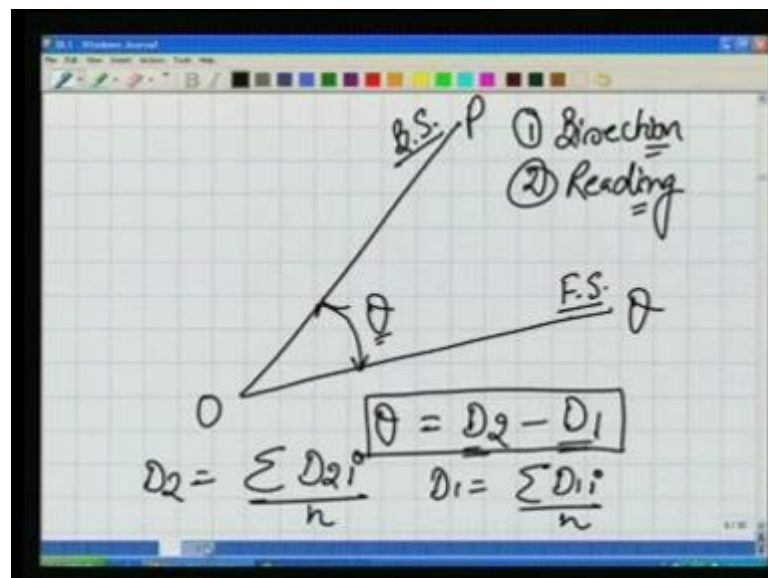
Anything else in which this error due to miscentering will depend upon? It also depends upon the direction. The direction means, the miscentering could be in this direction or it could be by same amount in this direction. Now, it can be proved very quickly, that the error in the angle due to miscentering will be more if the miscentering is in the direction of bifurcation of this angle. For example, in this direction, this, the miscentering error will be maximum in this case than in any other case. It can be proved very quickly. We have seen now two methods method of repetition method of reiteration to measure any angle.

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What we can do? The angles, which are there in the field, as we have seen for example, here each of these angles we can measure individually by method of repetition and also, each of these angles can be measured by method of reiteration. Now, what we want to see? We want to see in terms of accuracy, which of these method is better. So, for that we need to do a little analysis and what we will do in this analysis we will revisit some of the things which we had discussed in our earlier video lectures when we were talking about the error propagation. We will see, we will do one analysis here how the error propagates from a simple thing to a complex thing ((Refer Time: 19:19)) final measurement. This is what we will see. So, we will start with method of reiteration.

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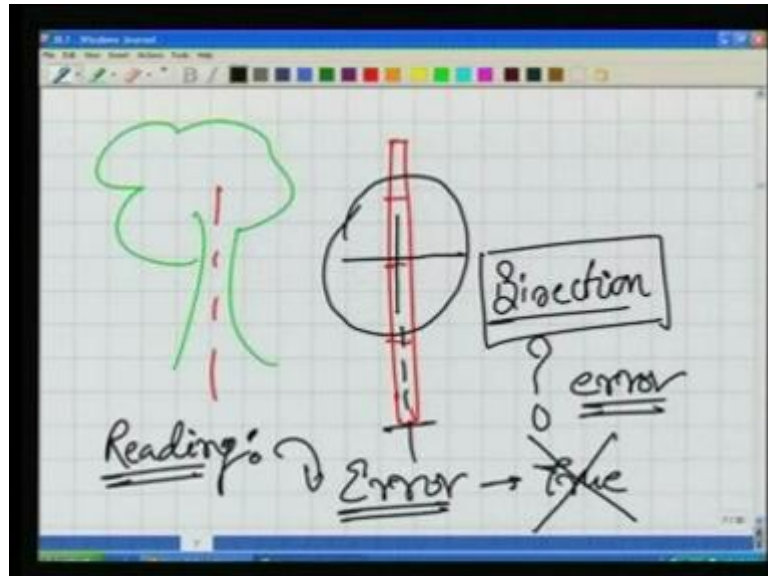


Now, in the method of reiteration, what we do, this is our P and Q, observer is here and we measure this angle theta. How do we measure the angle theta? Theta is, basically, given by D 2 minus D 1. What is D 2 and D 1? We are taking a back sight at P, we are taking the foresight at Q and D 2 and D 1 are the averages. So, I can write the D 2 as sigma D 2 i, I will explain this terms in a moment by n, and as well as D 1 as sigma D 1 i by n.

Now, why I am writing this in this way? Because here, as you see, theta is D 2 minus D 1. The average of all the foresights minus the average of all the back sights, this is what we do in reiteration, gives me the angle theta. Now, what is the single back sight or fore sight. A single back sight or fore sight is when you are bisecting from O to P. What we

are doing? We are doing two things. Number one, we are doing the bisection and number two, we are taking the reading, we are taking the observations.

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Now, how this bisection and taking the reading will introduce error? Because when I am bisecting, what is the meaning of bisection? For example, let us say, if there is a tree or may be, for let us say, be there is a ranging rod here. Let us be more precise, because in case of tree where to bisect will be something, you know, more of confusion.

Let us take a case of the ranging rod. Now, we want to bisect this ranging rod. So, what we do? We put our field of view, this be something, which you are seeing through the telescope on the ranging rod, and we align our cross here with the center of the ranging rod. So, this aligning of the cross hair with the center of the ranging rod is the bisection.

Now, the question is, can we really align it? Will there be some error? Yes, we can never align it truly to the point or to the center of the ranging rod. So, there will be some error because of the bisection. Second thing, once we have aligned it wherever, what we are doing? We are taking the reading; we are taking the observation. So, while we take the reading through our graduated circle what we do? We look through the Vernier and try to see the reading. Again, this is also one more source of error. What about we read? We cannot read the true value. No, we cannot read the true value.

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Bisection: σ_p^2
Reading: σ_R^2

D_{2i}
 $i = 1..n$

$\sigma_{D_{2i}}^2 = \sigma_R^2 + \sigma_p^2$

So, they are error sources because of bisection and reading and these two errors can be estimated. Let us say, the standard deviation for bisection error is sigma P square and the standard deviation for reading error is sigma R square.

Now, as we have seen, we are, in the method of reiteration our D 2 is average of all the foresights. Now, what is one foresight? If you look at one foresight D 2 i, this i varies from 1 to n. The meaning of this is, there are n number of observations of the angle. This angle we are observing n number of times. So, there are n number of foresights and n number of back sights and this is how we are determining this angle. So, this i varies from 1 to n.

Now, in any single foresight, let us say, in any single foresights the error of D 2 i, the standard deviation of this is depended upon and can be written as sigma R square plus sigma P square because these two are the sources of error in taking the foresight. So, in my foresight these two are the error sources. So, the standard deviation of the error for this fore sight will be given by this formula.

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$$D_2 = \frac{\sum D_{2i}}{n}$$

$$Y = f(x_i) = f(x_1, x_2, x_3, \dots, x_n)$$

$$\sigma_Y^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2 + \dots$$

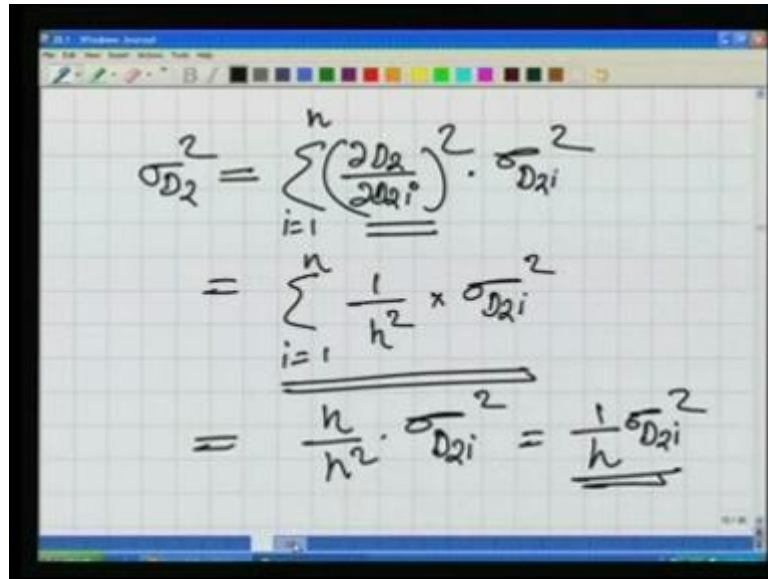
Now, if it is so, what we are interested in? We are interested in finding D_2 where D_2 is the mean D_{2i} by n .

Now, we have seen one formula. We have seen, that formula is, if Y is $f(x_i)$. We are computing this Y from several observations, which are for example, x_1, x_2, x_3 , so on, up to x_n . So, in, by, in computing this particular quantity we are making use of all these observations and in all these observations there are errors, $\sigma_{x_1}^2$, $\sigma_{x_2}^2$, $\sigma_{x_3}^2$ and so on.

For example, we know these. So, we have seen in the error propagation, that how these individual errors will propagate into Y and this is given by, if you recall, σ_Y^2 is $\sum_{i=1}^n$ because there are n number of variables and partial differentiation of this function f against each of these i , square of this and multiplied by $\sigma_{x_i}^2$.

Now, this we have seen. Now, what we will try to do? We will try to use this for this formula. Now, if it is so, what we are interested in? We are interested in square. Now, how do we find it? So, the meaning is, if you take the note from here, we are going to differentiate now this D_2 partially with each individual D_{2i} according to this formula. So, what it becomes?

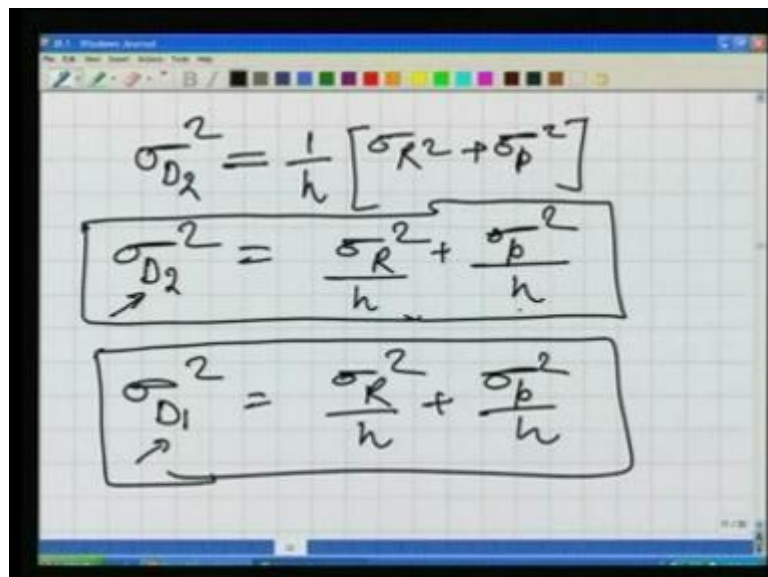
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The image shows a handwritten derivation on a grid background. The first line is $\sigma_{D_2}^2 = \sum_{i=1}^n \left(\frac{\partial D_2}{\partial Q_i} \right)^2 \cdot \sigma_{D_{2i}}^2$. The second line is $= \sum_{i=1}^n \frac{1}{h^2} \times \sigma_{D_{2i}}^2$. The third line is $= \frac{n}{h^2} \cdot \sigma_{D_{2i}}^2 = \frac{1}{h} \sigma_{D_{2i}}^2$. The final result is underlined.

It becomes sigma D 2 D 2 i square of this into sigma D 2 i square and this i varies from 1 to n. Well, if you solve it. By solving this you will find the value of this is, i is 1 to n and this total terms are to be 1 by n square into sigma D 2 i square. And if you find, further, solve it further because the sigma is there, i is 1 to n, so that will become n by n square into sigma D 2 i square. Well, we can write it as 1 by n into sigma D 2 i square.

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The image shows a handwritten derivation on a grid background. The first line is $\sigma_{D_2}^2 = \frac{1}{h} [\sigma_R^2 + \sigma_p^2]$. The second line is $\sigma_{D_2}^2 = \frac{\sigma_R^2}{h} + \frac{\sigma_p^2}{h}$. The third line is $\sigma_{D_1}^2 = \frac{\sigma_R^2}{h} + \frac{\sigma_p^2}{h}$. The final result is underlined.

If you expand this D 2 i square, we know the value of D 2 i square already that is the value of the D 2 I square here. If you write that value there, what we can write now?

$\sigma_{D_2}^2$ is equal to $\frac{1}{n} \sigma_R^2 + \sigma_P^2$. So, that becomes σ_R^2 plus divided by n plus σ_P^2 divided by n and that is my D_2 square. Let us look at this, what it is so for.

If you are looking at this, then you will notice the $\sigma_{D_2}^2$ is the error, which is propagated because of the reading error and because of the bisection error in the average of the foresight. Well, similarly with the, same we can write also $\sigma_{D_1}^2$ as σ_R^2 divided by n plus σ_P^2 divided by n . Now, what is the meaning of this because when we are taking the back sight that was D_1 , when we are taking the foresight, D_2 . The things are same, there is no change. So, there is no change. The error, which is propagated will be the same.

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$$\theta = \frac{D_2 - D_1}{\dots}$$

$y = f(x_i)$

$$\sigma_{\theta}^2 = 1 \cdot \sigma_{D_2}^2 + 1 \cdot \sigma_{D_1}^2$$

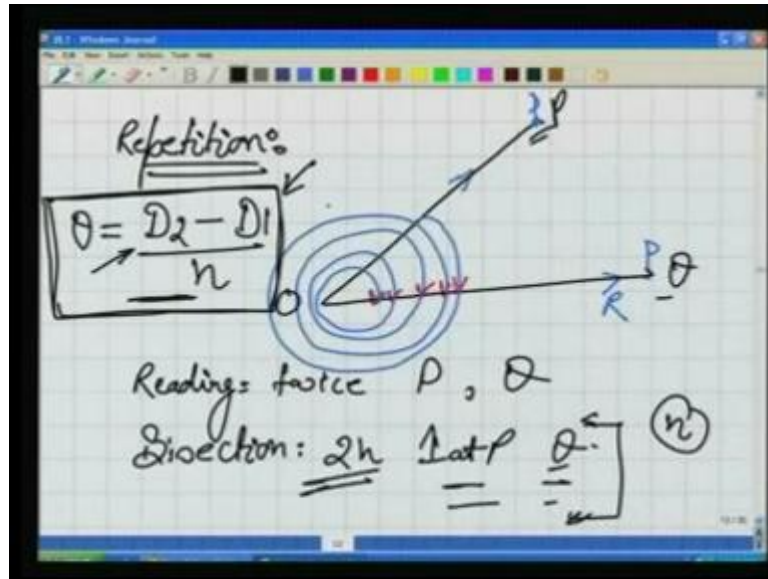
$$= 2 \left[\frac{\sigma_R^2}{n} + \frac{\sigma_P^2}{n} \right]$$

This is still not clear because we want to determine our theta and theta is D_2 minus D_1 . Now, the error, which is there in D_2 , which is $\sigma_{D_2}^2$ and the error, which is in D_1 , $\sigma_{D_1}^2$. We are writing these errors in terms of the standard deviations here, how they propagate finally into theta.

So, in order do it again we will use what we had done for $f(x_i)$, that particular formula we will use. In order to use that we can write σ_{θ}^2 as if we, part, partially differentiate now this with D_2 and D_1 individually. We will find that it will be 1 into $\sigma_{D_2}^2$ plus 1 into $\sigma_{D_1}^2$.

So, if you write it further down because we know these values of σ_{D_2} and σ_{D_1} , so if you write it further we can write it as 2 times now from here σ_i^2 by n plus σ_p^2 by n . This σ_θ^2 is the error in angle measurement for this θ in the case of the method of reiteration and it is being given by this particular value here.

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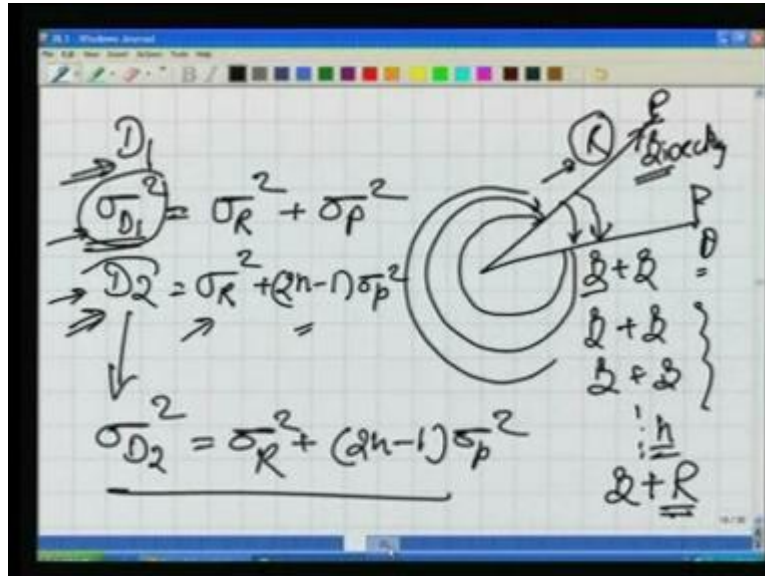


Now, what happens in case of method of repetition? If you look at the basis of the method of repetition, in the case of the method of repetition, our observer is here, point P and point Q, what we do? We first of all bisect P, the ranging rod there and we bisect and we take the observation. Then, after doing several rounds of the instruments, several rounds of the instrument because we are summing up the values mechanically. And then, we finally take the reading here, take the reading at Q while in between when we are doing these rounds, we always, at every time bisect Q.

So, what we are doing? We are doing the reading, we are taking the reading only twice, only twice. Twice means, once at P and at last at Q while we are doing the bisection. How many times we are doing the bisection? If for example, there are n number of repetitions we are doing it $2n$ times. One at P, because initially, first of all we bisected P and then, we are going to the Q, again bisecting it and again we are bisecting, again we are bisecting and we keep bisecting that way. So, all these bisections are involved. So, $2n$ number of bisections; the total number of bisections are $2n$.

Now, how it is going to affect our angle theta? The angle theta, we write here as our final reading, D 2 minus initial reading divided by number of the reiterations that we have done.

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Now, how the error will propagate again in this case? Now, in the case of the D 1, what the D 1 was? D 1 is, we are taking a reading, we are taking one reading only, one reading here and we are bisecting it and we are bisecting it, first reading and then we are bisecting this one.

So, in the case of the D 1 we can write it as the error in D 1 will be sigma R square plus sigma P square because D 1 means, the very first reading. Very first reading means, we are bisecting this P, we are bisecting it once and then we are taking the reading and that is the value of D 1; that gives us the value of D 1. While in the case of the D 2, what we are doing in the case of the D 2? For each D 2, P and Q. So, for this Q what we are doing?

We come over here, we bisect it, we again close the horizon and again bisect it. So, one more bisection now. Again, we, over here one more bisection, we close the horizon, one more bisection, again we come over here. So, bisection plus bisection and finally, after doing this for n number of times, we do our final bisection and the reading.

So, we are taking the reading only at the last, but while we are doing this bisection, we are bisection, bisecting P and then Q and then P, then Q, this is the procedure of repetition. So, if it is so in the case of the D 2, the error will be $\sigma_R^2 + \frac{2n-1}{n} \sigma_p^2$ because we have, in order to take this D 2 we have done only one reading while we have done the bisections $2n-1$ number of times. Well, what we know in the case of the repetitions? The angle is given this way, $\theta = \frac{D_2 - D_1}{n}$. We know now the error in D 2 and error in D 1, how they will propagate in theta. So, this is what we are going to see because we know the error in D 1 and error in D 2.

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top left, the angle θ is defined as $\theta = \left[\frac{D_2 - D_1}{n} \right]$. To the right, a box contains the function $y = f(x_i)$ and the variance formula $\sigma_y^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \cdot \sigma_{x_i}^2$. Below this, the variance of θ is derived as $\sigma_\theta^2 = \frac{1}{n^2} \sigma_{D_2}^2 + \frac{1}{n^2} \sigma_{D_1}^2$. This is then expanded into $\sigma_\theta^2 = \frac{1}{n^2} \left[\sigma_R^2 + (2n-1) \cdot \sigma_p^2 + \sigma_R^2 + \sigma_p^2 \right]$.

So, error propagation in theta, because the theta we are writing as $\frac{D_2 - D_1}{n}$. So, using again and σ_y^2 we have done it, i is 1 to n , $\left(\frac{\partial f}{\partial x_i} \right)^2$ into $\sigma_{x_i}^2$. Using this same philosophy we can compute now for σ_θ^2 and if you not differentiate partially, this particular function against D_2 and D_1 , what we will get? We will get $\frac{1}{n^2} \sigma_{D_2}^2 + \frac{1}{n^2} \sigma_{D_1}^2$.

Now, we put the value of D_2 and D_1 from here, $\sigma_{D_1}^2$ and $\sigma_{D_2}^2$ square. Sorry, I should write it as $\sigma_{D_2}^2 = \sigma_R^2 + \frac{2n-1}{n} \sigma_p^2$ and $\sigma_{D_1}^2 = \sigma_R^2 + \sigma_p^2$.

Now, if I take these values here, so what kind, what I can do? I can write it as now 1 by n square, I take it out, sigma R square plus 2 n minus 1 into sigma P square plus for sigma D 1. Sigma D 1 square is sigma R square plus sigma P square; sigma R square plus sigma P square.

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$$\sigma_{\theta}^2 = \frac{1}{n^2} [2\sigma_R^2 + 2n\sigma_P^2]$$

$$\sigma_{\theta}^2 = \frac{2\sigma_R^2}{n^2} + \frac{2\sigma_P^2}{n}$$

$$\sigma_{\theta}^2 = \frac{2\sigma_R^2}{n} + \frac{2\sigma_P^2}{n}$$

We solve it further now. In solving this further what we will find? We will find, that sigma R square and sigma R square, they will add. So, we can write it further as sigma theta square as 1 by n square sigma R square, I have got twice of that, plus 2 n sigma P square. Now, this can further be simplified as 2 sigma R square divided by n square plus 2 sigma P square divided by n, and this is what we get here. Now, we compare this with the one, which we got for method of reiteration. In that case, we got R square by n plus 2 sigma P square by n. This is what we got in the cases of method of reiteration, as you can see here.

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top, the angle θ is defined as $\theta = \frac{D_2 - D_1}{r}$, with $r = f \sin(i)$ written to the right. Below this, the error propagation is shown: $\sigma_\theta^2 = \frac{1 \times \sigma_{D_2}^2}{r^2} + \frac{1 \times \sigma_{D_1}^2}{r^2}$. This is then simplified to $\sigma_\theta^2 = 2 \left[\frac{\sigma_R^2}{h} + \frac{\sigma_p^2}{h} \right]$. The final result is boxed and underlined.

Now, what is the meaning of this? Let us look into the meaning of this. In the case of the method of repetition and reiteration the error, which is propagated into our final angle, that we are writing, by sigma D square, the standard deviation of the error. In both the cases, it is dependent upon the error due to the reading and the error due to bisection; the error due to reading and the error due to bisection, similarly here.

However, as you will notice, for the reading the term here in the case of the repetition is n square while in the case of reiteration it is n . What is the meaning of this? The error propagated in theta is less in the case of the repetition because of the reading because the term is square here and this term is only one.

So, what does it indicate? This indicates, because in the case of the repetition method we are taking very few or rather only two number of readings, the first reading and the last reading, so the error propagated in our final angle value because of the reading is less. So, that means, the angles, which we compute or which we observe by repetition are better than the angles, which we determine using the reiteration.

Now, this is very important and this analysis is very important and the steps, which are involved in this analysis, you should do it yourself, then only you will understand all this clearly.

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So, far we have seen the methods for measuring angles in horizontal. Now, we will see the method for measuring vertical angle. Now, what is the meaning of vertical angle? The meaning of vertical angle is, for any object we are measuring the angle of this, from the point of observer with the reference as the horizontal line. So, that is the horizontal plane. So, from this horizontal plane we are measuring this angle to an object there. So, that is my vertical angle. We have seen this definition.

How do we measure this using the theodolite? So, this is what we are going to see now with the help of theodolite. Now, here in the theodolite, the procedure will be same, some of the procedures as in the case of the horizontal angle. First of all, we take this theodolite to a point where we need to measure the angle because obviously, whenever we are measuring the angles, we are measuring the angles at the point of the observer from a station. So, we will take this tripod there, we will center it, we will level it.

Now, once we have centered it, leveled it, we will fix this instrument, the theodolite, on the tripod and again we will level using these three foot screws. So, the idea is to make this horizontal plate horizontal.

Now, this is the horizontal. What it does, while we are leveling this instrument or the theodolite using this foot screws and making this plate bubble in the center, it also ensures, that my vertical circle is also vertical, which is important.

Henceforth, I am going to measure the angles in vertical plane, I do not want to measure the angles in inclined plane. I want to measure them in a vertical plane, so my vertical circle has to be vertical.

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Well, one more thing, because in the case of the vertical angle measurement, that is, the vertical circle and this is the Vernier or the index plane, what happens if I rotate, as we do here? If I rotate my telescope, what is happening there in the case of the vertical circle? It starts rotating with the rotation of the telescope. The vertical circle rotates, but the index or the Vernier, they stay where they are. So, these index or these Vernier, they form a line, which is the horizontal line or horizontal reference. So, I need to ensure, that this is also horizontal.

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How do we ensure it? We will make use of the altitude bubble. We have seen on the index frame, there is an altitude bubble. This altitude bubble is more sensitive, more sensitive than the plate bubble. So, after centering this plate bubble, again using the foot screws we centered this altitude bubble.

Now, this is very interesting. I am saying the altitude bubble is more sensitive than the plate bubble. Plate bubble is in center. Now, it is center wherever I rotate my instrument. Now, I try to bring this in center. While I try to bring it in center, my plate bubble will not change, will not shift its position because it is more sluggish than this one.

Well, once this altitude bubble is also in center, what it ensure? This ensure, that my index is horizontal. Now, at that time when my index is horizontal, if I rotate my telescope in such a way, that the reading over here in Vernier, generally for the vertical circle we say Vernier C and Vernier D, is 0, 0, 0. If it is so, my line of sight should be horizontal.

I repeat, when, because my index is horizontal, now I make this, I rotate this telescope. In order to ensure what is happening while I am rotating the telescope, the vertical circle is rotating against the Vernier. I make the reading 0, 0, 0. We can make use of the tangent screws and the clamp screw in order to do these things. So, once that is ensured, that the reading is 0, 0, 0 here, 0, 0, 0 here, sorry, 180, 0, 0, 0, then in that case, the line of sight, which we can see here should be horizontal.

Now, in order to measure the vertical angle what next we do? We sight to the object. Of course, we will need to focus the eye piece and focus the objective lens, so that particular object is bisected. Once that is bisected, whatever is the reading over here now in Vernier C and Vernier D, we note down those readings and these give us the value of the vertical angle. The vertical angle as defined here, the line going there, the horizontal line and the angle here.

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This we are doing right now for me in face right. So, we repeat this entire procedure on the face left also.

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Table for vertical angle measurement

Inst. at	Sight To	Face Left				Face Right				Average vertical angle
		C	D	Mean	Vertical angle	C	D	Mean	Vertical angle	
0	P	✓	✓	✓	✓	✓	✓	✓	✓	✓
	Q	✓	✓	✓	✓	✓	✓	✓	✓	✓

How do we record these readings? We will come to the slide here. Now, this is the table for vertical angle measurement. The instrument, as in the case, that is the point where observer is. So, instrument is at O, our two points are P and Q to which we have to measure the angles. So, P and Q, that is the sight to right.

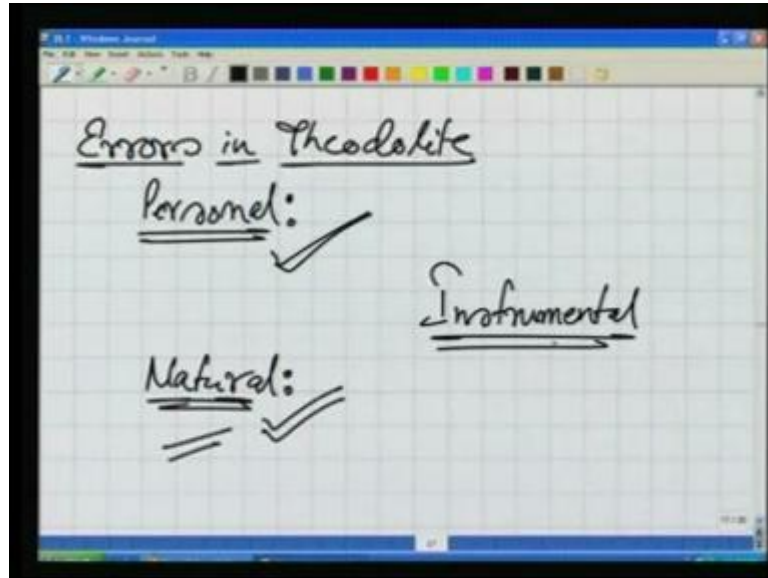
Now, we are taking the observation, for example, in face left. Now, we are writing the readings here in degrees, seconds and sorry, minutes and seconds. So, whatever was the value of the reading we will write it here and this is for another Vernier, the Vernier D, this is Vernier C and we take the mean of the each, mean of Vernier C and D. We write the mean here and that gives us the vertical angle. Similarly, we do it for Q and it will give us the vertical angle for Q also. So, vertical angle for Q, vertical angle for P.

We can do the same procedure on face right again reading the Vernier C and Vernier D, taking the mean and finding the angle and then, finally the angle, which is coming from the face left and the angle value, which is coming from the face right will average these two and we will write the average of face left and face right. Similarly, for Q also average of face left and face right.

So, what we have done? This is what is recorded here is for any object P only one set of observation. We can repeat this multiple times depending, you know, how accurate we want to be in our observations. So, this is how we observe the vertical angle.

Now, we have seen all these angle measurements, we are going to another aspect then. What are the errors? Are there any errors in measuring these angles? What are the errors, which may be introduced into the theodolite? So, we will look into those errors now.

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Now, in the errors in the theodolite, as usual, we will talk first about the personal errors. How the personal errors will be introduced when we are doing the theodolite measurement?

If you look at here, the observer, the surveyor has not done the leveling proper. If the leveling is not proper, my horizontal circle is not horizontal. If my horizontal circle is not horizontal, what is happening? Now, my graduated circle is inclined. So, all the observations, all the angles, which are, which I am taking now horizontal angle, I am not in horizontal plane, rather in inclined plane. Now, this could be one miscenter, one error, which can be introduced by the observer. Miscentering, we have seen the effect of the miscentering, what will happen. Then, the bisection is not proper. We need to bisect the object properly. So, the bisection is not proper, again there may be error.

Error in the parallax, if the parallax is not removed, we have seen how the parallax can be removed. If the parallax is not removed properly, again that will lead to the error in the theodolite because of the surveyor. Finally, while he is taking the readings, he is not taking the readings properly, again this is also a source of the error. So, these all are the personal sources of the error.

Similarly, the natural errors. There may, there may be many, some of them, while we are working in the field with the theodolite. It might happen if the ground is soft, one of the legs goes down. So, what will happen? The level will, leveling will change and now our observations will be wrong, or if it is too windy in the ground and there are vibrations in the instrument, again the readings, which are taken will be wrong or may be, the refractive index of the medium.

Many times in the summers you will see when you are looking through the theodolite or the telescope, you will see there is shimmering effect. Your ranging rod will appear to be shimmering. So, it becomes very difficult because of the natural condition, bisection of the ranging rod becomes difficult. So, that also leads to the error.

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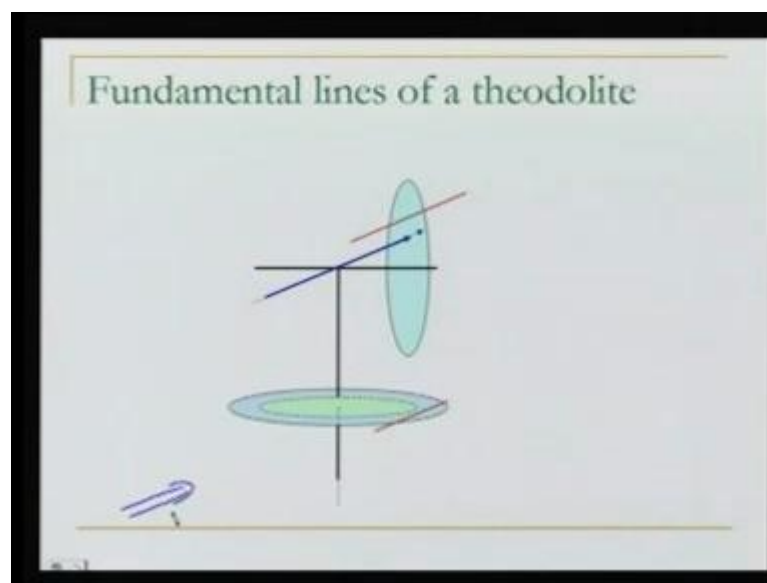
One more error, for example, let the sun is on this side here. So, only part of the instrument is being heated. Now, if only part of the instrument is being heated, what will be the reason? What will, what might happen? The graduated circle may expand in one part while it is not so in the other place. So, again that might lead to some error.

Also, that is my bubble tube here and the sun is shining from here, the sun light is falling on the instrument. So, only part of the bubble tube is being heated. Because of that the bubble will not remain in center, it will go out of the center. While we might constitute, that the leveling has changed and we will try to level it so. In fact, what is happening? We are changing the horizontal circle. We are not, it is not any more horizontal, we are

changing it to be inclined. So, because of these natural conditions also there might be several errors, which might be added in the instrument.

Finally, the errors due to instrument, which we say, instrumental errors. Now, before we get into the instrumental error we need to understand couple of fundamental lines of the instrument. These are very, very important. We have understood the construction of the instrument, how it is made and all these things, but we need to look into some of the fundamental lines in this instrument.

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When we are talking about the fundamental lines of the theodolite, you have to follow the instructions of what I am showing you here very clearly where that will make you understand what these lines are. We start with the vertical axis. If I rotate this theodolite, it is rotating about an axis, which is the vertical axis of the theodolite.

So, we can show this as a line here, then this is the horizontal plate and the axis of the bubble tube. If, if i showed in this diagram, this is the horizontal plate and that red line is the axis of the bubble tube. There is some relationship. The relationship is, the horizontal plate should be parallel to the axis of the bubble tube, they should be parallel to each other. Then, as well as these two should be perpendicular to the line of, to the vertical axis. As we can see here, these are perpendicular to the vertical axis.

Then, further we have now two axis system in this part of the instrument. One is for the upper circle or the Vernier frame and the other one is for the bottom plate. Now, these two axis again should coincide. As you can see here, by the dotted lines, that is what the inner one and that is for the outer one. So, these two should coincide.

If you go further up the lines, one here is the horizontal axis, what is this. Now, in this theodolite if I rotate my telescope, it is rotating about a line here, this line, we say, the horizontal axis or trunnion axis. So, this is the horizontal axis or the trunnion axis. Is there any relationship between vertical axis and this horizontal axis? Yes, they should be perpendicular to each other.

As we can see here when we go further up, this blue line is the line of sight. So, the line, which is passing through like this, if I rotate it, the line of sight is rotating, as well as there is one more line, which is shown here by the dotted one. This is the line of collimation. Ideally, the line of collimation and the line of sight should be coinciding as well as these should be perpendicular to the horizontal axis and the vertical axis.

For example, here if I rotate it anywhere, the angle formed by line of sight to the horizontal axis is 90 degree, as well as the angle formed by the horizontal axis with the vertical axis is also 90 degree or we can say this way. So, these relationships should be maintained in the instrument. If you go further up we have the vertical circle, that is, the vertical circle. Now, the relationship of the vertical circle is this. Vertical circle should be perpendicular to the horizontal axis or trunnion axis.

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And we have the altitude bubble. So, this altitude bubble is in such a way, that once this altitude bubble is at the center, the line formed by the index of Vernier should be horizontal. So, this is what I am trying to show you here by this diagram. So, these all are the various lines of a theodolite, which is a fundamental lines of the theodolite. So, what we have seen? We have seen the inter relationships of these lines. When we discuss further about the theodolite we will look into these lines again.

So, what we did today? We looked into the method of reiteration. We compared the method of reiteration with method of repetition for their accuracies. We saw how to measure the vertical angle, and then finally we have seen now the fundamental lines of a theodolite, which we will again repeat in our coming lectures.

Thank you.