

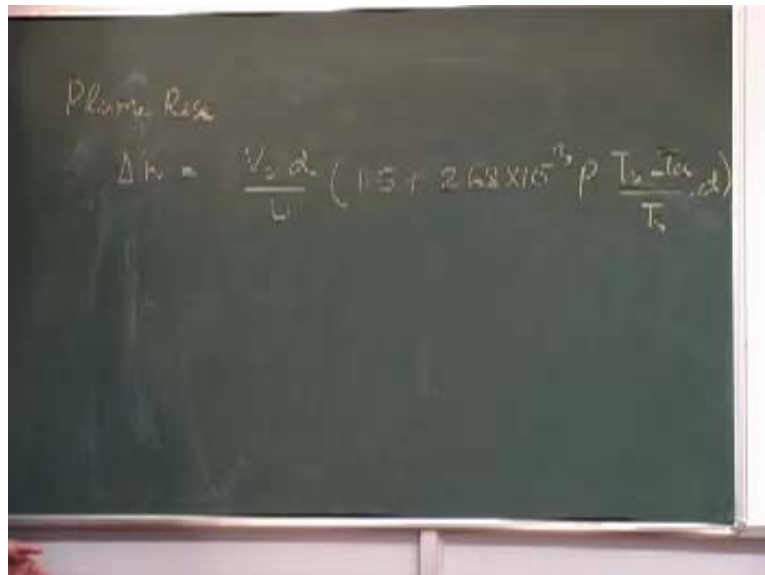
**Environmental Air Pollution**  
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**Lecture No. 32**  
**Plume Rise, Area and Line Source Model**

If you recall, we were discussing the plume rise and I gave you an expression for the plume rise. We said it has two terms or two components: one was more related to momentum rise and the other one more because of the thermal boils. There are many more formulas for the plume rise and much more accurate than the one I gave you, but for the time being we think this is all right. If you really look into literature, you find many people have developed more rigorous formulas and they have even validated the plume rise formula for various conditions.

Why is plume rise so important? If you see the expression of your Gaussian model, where does the plume rise appear in the formula? It is in the exponential term and therefore, anything in the exponential term is very sensitive – with a little change in the overall impact, you will see the concentration could be very high. People have essentially solved the momentum equation, the heat equations to get some formulas for the plume rise, but for this course we will be happy with the formula that we wrote yesterday. If you want more information on that, you can look for the Senfield book – that is one of the references that we gave. He deals the subject really in great detail, but for this course the formula that we had or described yesterday or in the last class is valid and we will use that.

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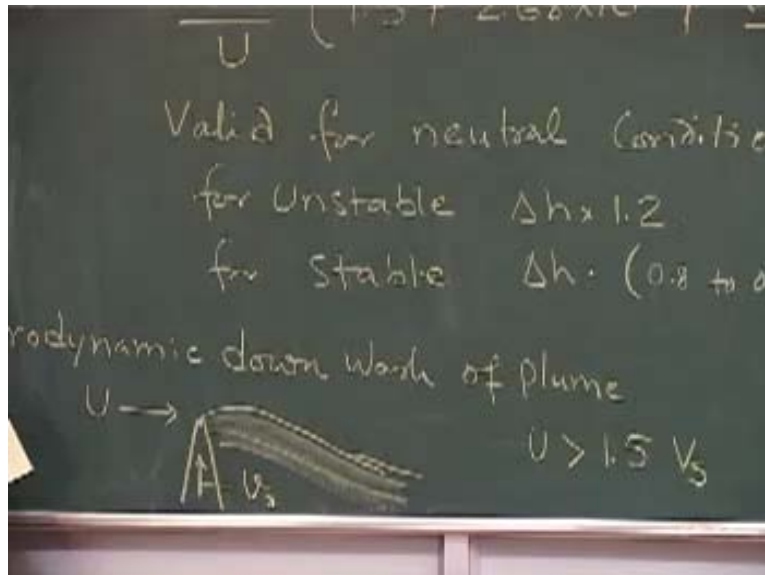


Plume Rise

$$\Delta h = \frac{V_s d}{U} \left( 1.5 + 2.68 \times 10^{-3} p \frac{T_s - T_a}{T_s} d \right)$$

So if I can write that formula **again...** let us write it again. delta h. We also defined the units for them because once we write some constants, then we have to define the units. I again want to remind **you....** What were the units as far as the pressure was concerned? Millibar. Millibar – always remember that, that is millibar. The other thing I ask you again and again is at what height is this U? You can see back or **at....** Stack tip. At the stack tip – do not forget that because this is the wind influencing my delta h; how much will be the rise will be defined by or decided by the horizontal wind at the tip of the stack. This formula for the plume rise is generally valid for neutral conditions.

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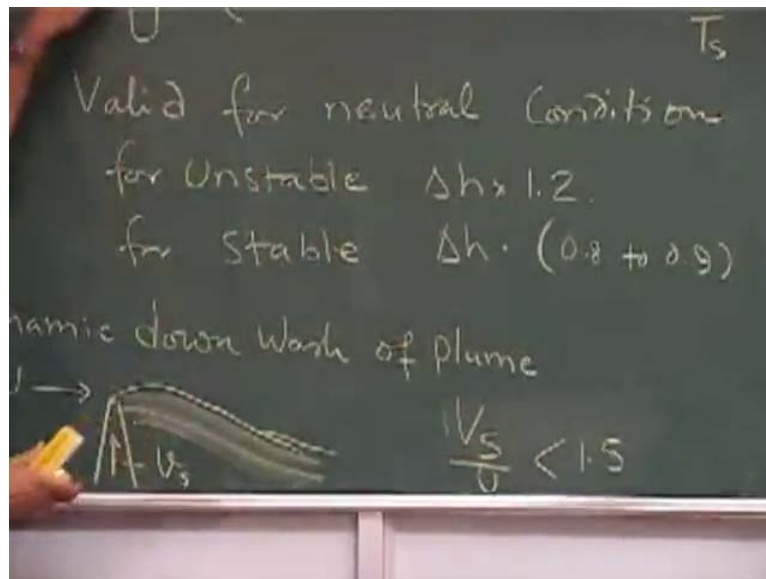


**Valid for...** for unstable,  $\Delta h$  times 1.2 and for stable, 0.8 to 0.9, whatever you take – that is the other thing you should know. We will do something but before we do that, I want you to recognize that if I want to increase the  $\Delta h$  because that is where I can minimize the impact, what are the things in my control? I cannot do anything about this one (Refer Slide Time: 04:42), I can increase perhaps the T stack, ambient temperature I have no control, the diameter of the stack I can increase perhaps, I can manipulate the velocity of the exit gases – put a larger blower to blow the things at larger speed, this again I have no control over the horizontal wind; I have no control but this (Refer Slide Time: 05:09) and this variation may not be so significant but we all know that  $U$  is the one that is highly variable. Sometimes, wind speed can be very large and sometimes wind speed can be low. So the trick here or the thing is my  $\Delta h$  for given conditions I can manipulate but this I cannot manipulate and then you see the  $\Delta h$  can or rather  $\Delta h$  will be inversely proportional to the wind speed; so more the wind speed, it means plume will not be able to rise too much, it will be forced down somehow.

This is one situation that we call as a downwash or rather aerodynamic downwash of the pollutants. We will try to define that one or we will quickly do it here because it is a very simple thing – downwash of plume. What happens is when you have a chimney or stack and all this you have noticed yourself that when wind is very high, instead of rising we will see sometimes the plume is going like this and you must have seen that from the brick kilns, for example, as you are

travelling on the road you see the brick kiln and the wind is high, then the plume just comes onto the ground or suppose you have gone to hilly areas, you know the people heat their houses and you see the plume is coming out from little chimneys and it just comes underneath the house. That is what typically we call the downwash or rather aerodynamic downwash of the plume. When it happens when the situation is,  $U$  is greater than  $1.5 V_s$ . The horizontal wind is too high compared to the exit velocity of the.... What is this? This is going this is  $U$  and this velocity inside the chimney is your  $v_s$ . See, one thing will be covered by the stability in that area, whatever that stability is.

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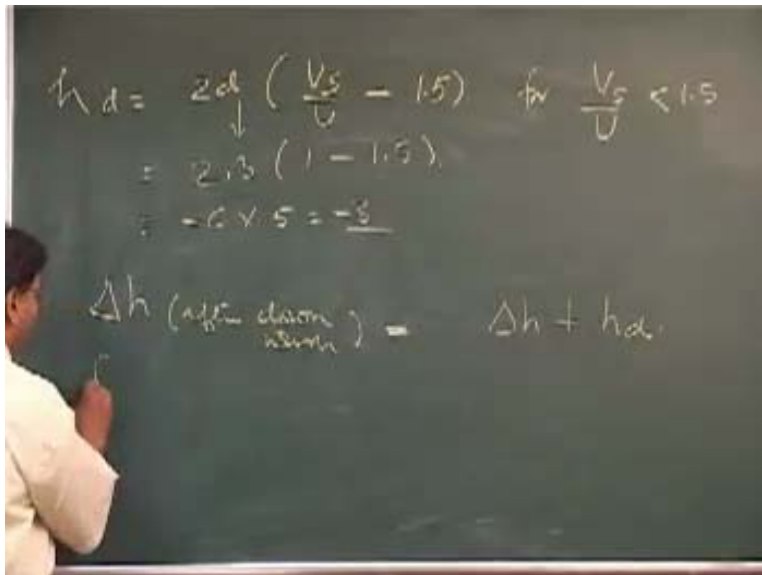
In the coastal area suppose the wind is high, what we should do is that we should modify this one and then this is what we take as the ratio of  $V_s$  by  $U$ . Just a second, this we should correct  $V_s$  by  $U$  is less than 1.5 because we have to take this ratio. Now let us answer your question. What you were saying is in the coastal area, what could happen to the plume rise? This plume rise will still be valid but only thing is that supposing the wind is high in the coastal area, that will be taken care by the wind and suppose the stability is different, that will be taken care by these factors. There are some more things that require very special operations into the coastal area; we will see if we have the chance to cover that thing or we may not have the chance to cover that thing in time.

Similarly, not only the coastal area but suppose you have a large water body near your chimney or within a distance of 15 kilometers, it can influence your plume to a large extent and that dispersion modeling is a little different – we have not talked about that but maybe we will talk provided we have the time. Essentially what happens is that at the interface of the sea and the land or the water at the land, the mixing height is very different onto the surface of the sea or of the water and at the land – it is entirely different and so that mixing height has the influence onto the land. When the wind is coming from the seaside, the mixing height is governed more by because of the mixing height at the water level and that has some changes. These things we are saying are generally valid for a source that is almost about 10 to 15 kilometers away from a water body, but as an approximate or as a tool to take some decision, this formula is still quite valid.

The coastal thing which may affect your this thing will affect early in the early hours – you may get little different results but once the whole day has come up, then this becomes more or less valid. But you are right; certainly the coastal area modeling is a little different at least in some early hours of the day. After that, things become the same what you are seeing but as I again say for routine purposes, even the coastal area you can take this and take broad decisions. Sometimes as engineers, you have to take broad decisions, not very minute decisions – whether the industry should come at the distance of 5.62 meters from the coastal area, you have to say it should come somewhere between 10 to 20 meters.

[10:59] Yes. Then you can just look into any of the books, you can say the water air quality modeling in the coastal areas or air quality modeling at the lakeshore and that can be... you will see the difference that will be shown. Maybe if we have the time, we will discuss that; if not, we will skip at least as far as this course is concerned – that is slightly in the higher level. The person who did lot of work on this coastal air quality modeling.... It is called lakeshore modeling because whether it is a coastal area or a large water body, it is the same thing. There was one person called Dr. Mishra from the Ministry of Environment, Ontario, Canada. He was the person who did a lot of work on this one and his work is the one that is always referred to when it comes to air quality modeling in the coastal area or near a large water body.

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$$h_d = 2d \left( \frac{V_s}{U} - 1.5 \right) \quad \text{for } \frac{V_s}{U} < 1.5$$
$$= 2 \cdot 3 (1 - 1.5)$$
$$= -6 \times 0.5 = -3$$
$$\Delta h (\text{after downwash}) = \Delta h + h_d$$

I want to define in the **same connection...** for this condition. This is the correction and  $d$  stands for downwash, this is the correction you apply to your plume rise if we are under the situation when your exit velocity, the ratio of exit velocity to horizontal wind speed at the tip of the stack is less than 1.5. [13:06] It should **be...** [13:08] **1 by....** [13:11] Correction becomes negative. Suppose your  $V_s$  by  $U$  is less than 1.5, let us do the example since you have asked. Suppose I take diameter as 3 meters fine and the ratio of the exit velocity by  $U$  is less than 1.5, let us say it is 1, 1 minus 1.5 that brings 6, that is equal to 3, is that right? So the plume rise that you calculated here – whatever that rise was – minus or this correction is that this or the final  $\Delta h$  after downwash equals to  $\Delta h$  that you have calculated above plus  $h_d$ . Does it make sense now?

So your effective plume rise will reduce because it has been subjected to aerodynamic downwash – that is why you see that factor coming out to be negative. All right? Clear? Just remember you do not apply this formula, this correction if the ratio of your exit velocity to horizontal wind speed is more than 1.5 – it means that horizontal plume will not have effect in bringing the pollutants down. This can be sometimes very important because you may have designed a lot of things – you might have designed a good temperature, you might have designed a good diameter – but if your exit velocity is not properly designed, you will unnecessarily cause serious problems to the people living in the area. Since discussions are going on, one time we

encountered this for a sulfuric acid plant. In a sulfuric acid plant, typically the way the chemical engineers design the plant is that they design some exit velocity let us say between 3 to 4 meters per second.

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$$h_d = 2d \left( \frac{V_s}{U} - 1.5 \right) \quad \text{for } \frac{V_s}{U} < 1.5$$

$$= 2.3 (1 - 1.5)$$

$$= -0.55 = -0.5$$

$$\Delta h \text{ (after down stream)} = \Delta h + h_d$$

for  $\text{H}_2\text{SO}_4$  3-4 m/sec

$$\text{if } \frac{3}{4} = 0.7$$

So if the  $V_s$  is let us say 3, we are drifting let us drift from what we normally want to do for a sulfuric acid plant. What we have measured is the exit velocity is anywhere from 3 to 4 meters per second. If on some day the horizontal wind speed is very high, let us say it is 7, then what happens? Or let us say **this is...** we are taking a hypothetical example: let us say this is **your...** what number you want me to take? Suppose the horizontal wind speed also was 3 meters per second or maybe even more – you can encounter 4 meters per second. So this comes out to be nearly something like 0.7 – much less than 1.5. You will see the sulfuric acid plant that is responsible for  $\text{SO}_2$  emission and acid mist if you recall – that thing will be brought down and the people in that area will suffer a lot because the plant was designed for  $\text{H}_2\text{SO}_4$  3–4 meter exit velocity or stack exit velocity. A chemical engineer may not know all these things but it all becomes interdisciplinary when you are checking or you are doing some calculations for a sulfuric acid plant – you ask him what your exit velocity is.

To give a little feel of the exit velocity in the power plant, for example if you go and measure in the chimney in Panki power station, this (Refer Slide Time: 17:22) might be anywhere from 15

to 25 meters per second so that the plume from the large power plants are not affected because of this section (Refer Slide Time: 17:30), whereas in the brick kilns (because I gave you the example of brick kilns), most of the time, it is the natural draft – there is no fan put inside this one. If there is no fan put inside, obviously the  $v_s$  will be always be determined by the temperature differences. So these smaller stacks will be forced or will face the aerodynamic downwash because the  $v_s$  is generally very low. In the village area in the brick kilns, the temperature of the brick kiln may be something like 250 degrees but the plume is just washed down and surrounded in that area and people suffer a lot okay – that is the situation.

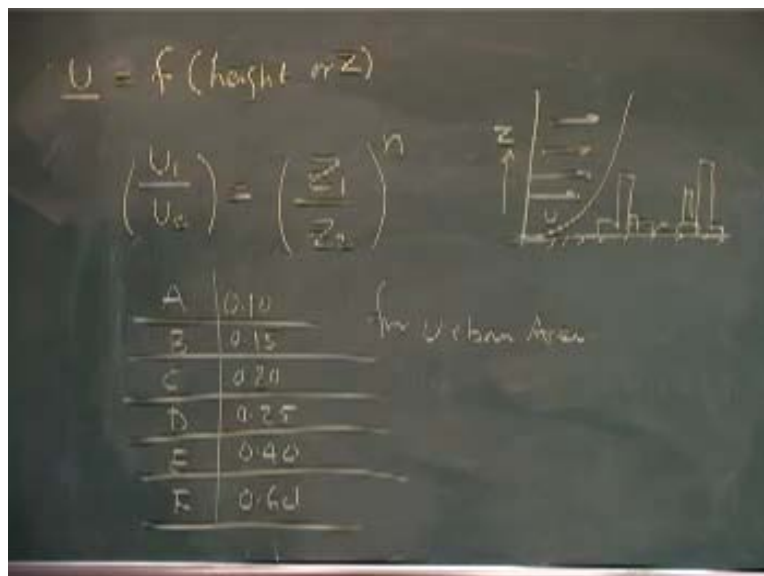
Question (by Student): So sir, plume rise depends upon the type of source from which the emission. Suppose we are taking thermal power plant particulate matter is high, so densities could be high. Suppose we are taking a ready mix of bituminous – there, emissions will be different and in oil refineries, they will be different. Does that affect this plume rise? [18:39]

This formula, again I repeat, is a simple formula and is based on the density of the hot air that is going out but these subtle things – the change in the density because there are particles or change in the density because it may not largely be air – are taken care by other complicated formulas and we are not discussing those in the class. The person who has done most of the work on plume rise is a gentleman called Briggs. Briggs is a person must be around 85 years old. Is he still surviving, I do not know. He says that he get 20, 30 emails everyday even today asking something about the plume rise. Although he tries to answer, he says most of the time “I am retired now, please ask some other people who are more actively involved in this.” Briggs is the person who has done this work and he has accounted for all the density part, what kind of plume it is, what kind of emissions are there from the refinery or from the power plant or from the particles – large particles and small particles.

Again, in summary, we want to stick to what we have for the time being, but you just write on the Internet ‘Briggs plume rise’ and you will find pages after pages on that one or you say Senfield book, look at the Briggs and then it will give you everything. So this point is clear – small thing but very important thing, because this will cause a very serious problem to the people and the concentrations here can be very large in spite of all efforts that you might have made and that is why the aerodynamic downwash of the pollutant is so important to know about that.



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We have talked about a lot that about the U. In many lectures, I have told again and again: take U at this step or take U at this, U is this, U is this or maybe your U is... suppose your plant is let us say at the tip of a hill, then maybe you have to take the U at much higher height. I have not defined so far as to how you can find out the U at different heights. It is very simple, the formulation is like this. Where has this come from? Vertical profile of the wind we all know about – we study in pipes and open channel flow.

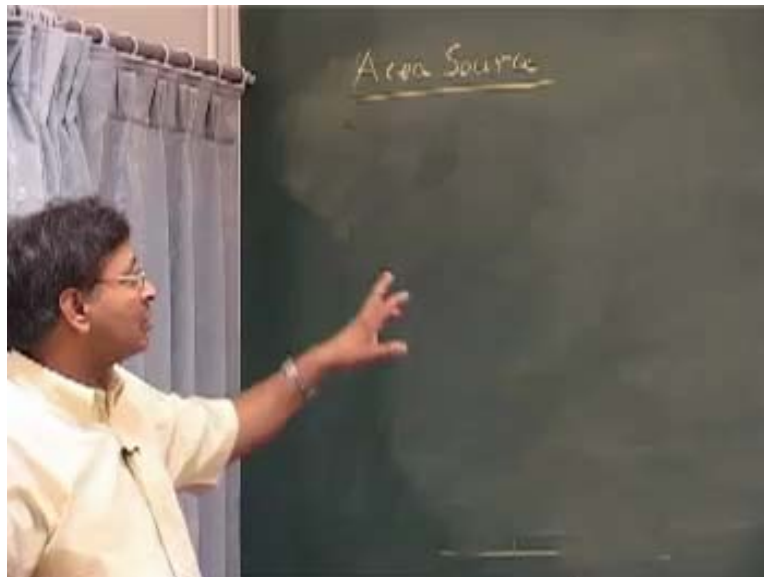
This is what is your Z and these are your U. There is also this parameter n; it depends on two things: stability because that stability can disturb your wind profile and also the conditions – whether these are there in urban conditions or these are in rural conditions. So wind profile if you go to Connaught Place area of Delhi, it will become very different – the building, the roughness; if you go to an area like the Gangetic plain – flat terrain – things will be different.

Suppose there are large buildings here, this could be affected, so this n will be different for urban areas and n will be different for rural areas, which is kind of flat terrain. It also depends on surface roughness. How it depends – that we will probably not see, but what I will do is to try to give you the value of n. Suppose you have stability A, B, C, D, E, F.... I have given you a methodology or technique: if you know your wind speed at particular height, you can translate that wind speed depending on the stability that you are talking about and the area being urban or

rural area and you can find out the new wind speed – that is what you require in your calculations. I am not giving you for the rural area.

By the way, you do not have to remember these things – no one can really remember – but you should have the idea that this is how it works and then you can always refer where the  $n$  is but what should you should really know is where this has come from – the physics part you should know, not the numbers; no one can remember the numbers but the physics is what we should not only know but we should understand.

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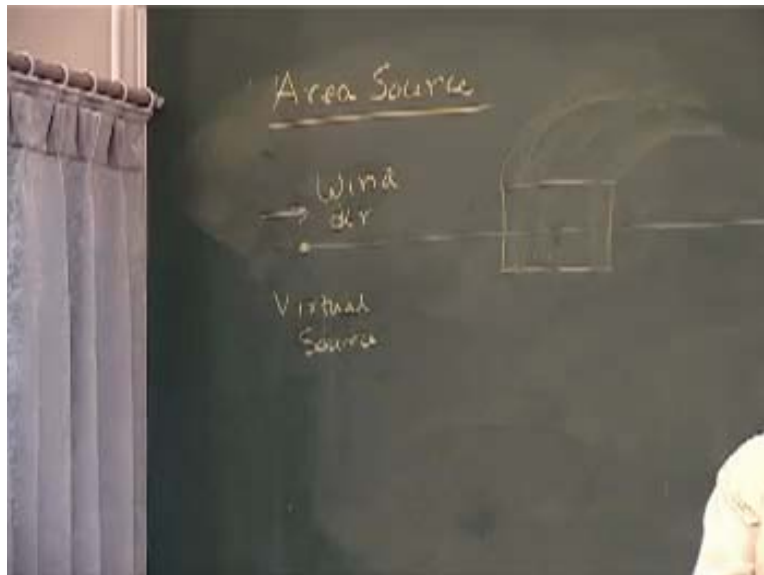


We want to do something more – a new thing, slightly new thing: area source. We had partly talked about the area source if you recall using the mass balance approach and we did the example **about...** What was the example? In the city of Kanpur, the sulfur dioxide levels etc., and then whatever you came up with the concentration it should be so much, but there we did not utilize the property of turbulence – we simply assumed that everything was thoroughly mixed from the ground level to the top of mixing height. But then since we have developed some more theory, we should improve our concept of the area source model and account for the turbulence.

How this is done I will show you, but before that, what we **do is....** What is the importance of considering the area source? The formula that we developed was generally for the chimney or stack – that is what we call as the point source; one single chimney is there and that is your point

source. But for area source, it is almost impossible to go to every house and see what is the chimney you have or go to every little tiny industry and see what chimney you have. In order to facilitate a modeling, we take that as the area source and then you see what is happening in that particular area. Suppose you go in the [26:24], I cannot count every vehicle, so I will take that as the area source and try to model that as the area source. The significance of the area source is to simplify the things. Otherwise, I have to do calculations. Suppose there are three lakh vehicles, then I have to model three lakh vehicles – that is almost impossible for me. This is how the concept of the area source comes in and that was the little significance that I wanted to talk to you about, but let us consider the area source.

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Let us say this is a square. The emission is occurring from this area and the plume is not from the chimney but it is going like this. This is the plan by the way. Now, how do we model this one is very simple. We try to define some point source because we can model the point source, so we will try to find out a virtual point source and how this is done is I try to locate this source somewhere back, but as a little chimney; let us say if I put the equivalent of this one at some this thing and treat this as the point source –this is what we call as virtual source. Just imagine, suppose I had a chimney here and it had travelled up to here, what would be the difference from having this travel from here to here? The  $q$  is the same, the mass is the same, but it has got spread – that is the only difference that has come. So what we do is that we say this was a virtual

source somewhere, but right now what picture we are seeing is the source with initial spread and if... [28:53] This is the wind direction that I am assuming, this is my wind direction; you can take the wind direction the other way also and then you will say the wind is blowing like this. So if you have the wind like this, you can take that as the virtual source as the wind.

[Conversation between student and professor - Not Audible (29:19 min)] But then, is it real approximation? Then you are saying the chimney is located here and at the tip of the chimney, your  $\sigma_y$  and  $\sigma_z$  are nearly 0 – the spread in the plume is nearly 0. Is that correct here? This is not correct. In the source, already there is some  $\sigma_y$  and  $\sigma_z$ , so there is a spread already – it means, it is tantamount to saying that there is some source that was a point source and by the process of wind transportation, it has already spread to something.

[Conversation between student and professor - Not Audible (30:07 min)]

How this can be...?

[Conversation between student and professor - Not Audible (30:11 min)]

How accurate is this one? All right. See, when you make some assumptions, one of the things is you do the measurements on the field. People had done the measurements and they found this is all right and then more by experience and empirical things and empiricism in the modeling that we always employ – it is based on that one, so it is reasonably accurate, because sometimes we just want to... as an engineer, you want to make an estimate, not the accuracy of the thing, we want to make an estimate and the modeling is nothing but the estimates. These are not the measurements – the measurements are always much accurate; estimates are the ones that the model provides. This has been reasonably estimated and that is how it is modeled.

[Conversation between student and professor - Not Audible (31:01 min)]

You can do that it – it just requires how much efforts you want to put in. If I have limited time, then I would rather do this one and even if I make this as a grid and suppose the improvement in my predictions is not so large compared to what is the resolution I want for large source area, large cities like Kanpur and Agra or Delhi, you cannot really do so much of tiny calculations. Of

course, you have the powerful computer – you can do it, people do it, you can also do it but this is the simple way where you can do it quickly.

[Conversation between student and professor - Not Audible (31:42 min)]

Yes. And we can also improve temperature, density. So our city can be divided into regular grids of 1 kilometer. In such cases, can it be done? It can certainly be done.

[Conversation between student and professor - Not Audible (32:01 min)]

Correct.

[Conversation between student and professor - Not Audible (32:04 min)]

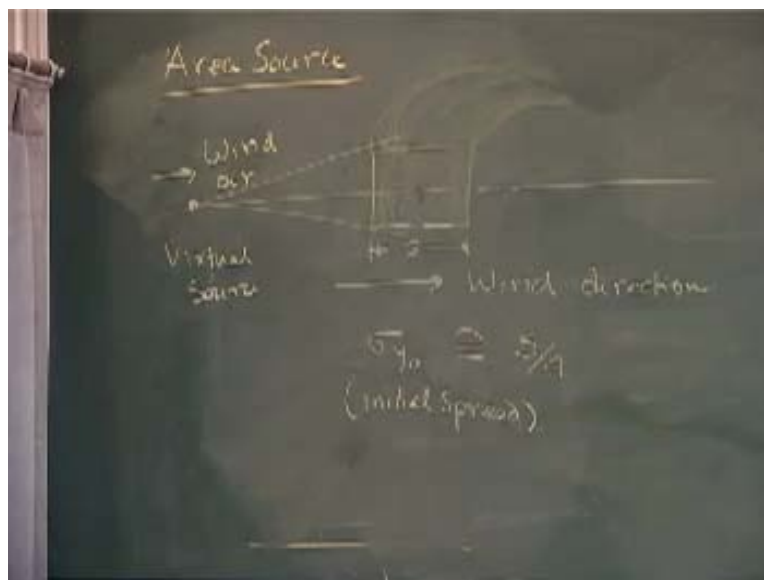
That is correct. The thing is what is your requirement, how accurate you want to be and in order to be accurate.... You may have fantastic mathematics, you may have a fantastic grid, but if your input variables are not very accurate, your wind speed is not accurate, you are doing the measurements here but the wind speed is very less, let us say even if I take the example of Kanpur, you are trying to do something let us say in Sharada Nagar and your wind data has come from the Chakeri airport, you might have a fantastic thing here but the wind is not such a good thing, so you might still not get reasonable results. So in order to get an estimate, you want to apply such a thing where you can quickly see as to what is happening but you can certainly do, you can divide it a grid of point or 500 meters or half kilometer. You can not only divide the grid in a horizontal way, you can also have a grid that is a function of x, y and z, you can solve the equation in every little grid and find out the concentration and you can also not only model the concentration but you can also model the wind vector.

Suppose you say go there and you strike to your building, the physical factor has gone into your model or GIS base, the wind cannot go this way, then it turns this way but that accuracy boils down to how much accurate you want things to be. If you have to take a quick decision, if you want to take a broad decision, is that all right? That is why we do it in the broader sense but if you want to do it more accurately, more efforts are required, more energy is required and if the situation so warrants, we should do that and then what we have to apply is a numerical modeling.

Maybe time permitting, we will have one or two lectures on the numerical modeling of air pollutants – we will see that and we will make grids there.

The other thing is if you want to make the grid, especially when you want to have a good time resolution, I am interested in not what is happening minute to minute, I am interested in what is 24-hour average when you take the assembled or assembly, the little things that you might get with your final modeling and final grids will be kind of somehow suppressed in there and that may be your objective; if your objective is to see the 1-hour change in the concentration of carbon monoxide, then you better do it this way, but if I want to find out the 24-hour average sulfur dioxide concentration, I will be happy with doing this one. In fact when you take the larger average, many things cancel out and then you might be just happy. So it all depends on your requirements – what is the situation and what you really need this one for.

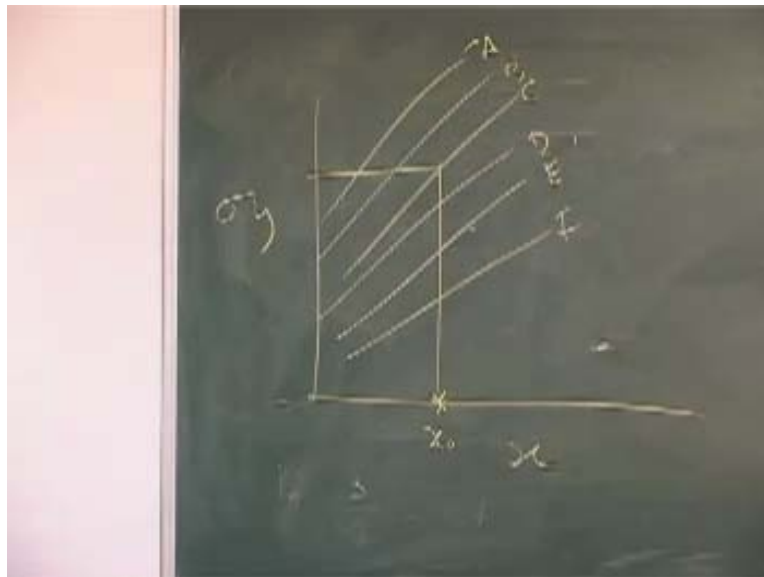
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We can think that is a virtual source that is starting from here and dispersing like this. I can think of a point source at the up wind and do not forget this is my wind direction – my wind direction is like this. With experience and with many other studies, we found out suppose this distance is  $s$ ,  $\sigma_{y_0}$ ,  $\sigma_{y_0}$  that you are observing here is approximately  $s$  by 4, I will confirm this  $s$  by 4 in a moment; the exact number is 4.2. All right. So whatever is your area of your source and one of the sides of this one is  $s$ , so you get the  $\sigma_{y_0}$ ; for this corresponding  $\sigma_{y_0}$ , this

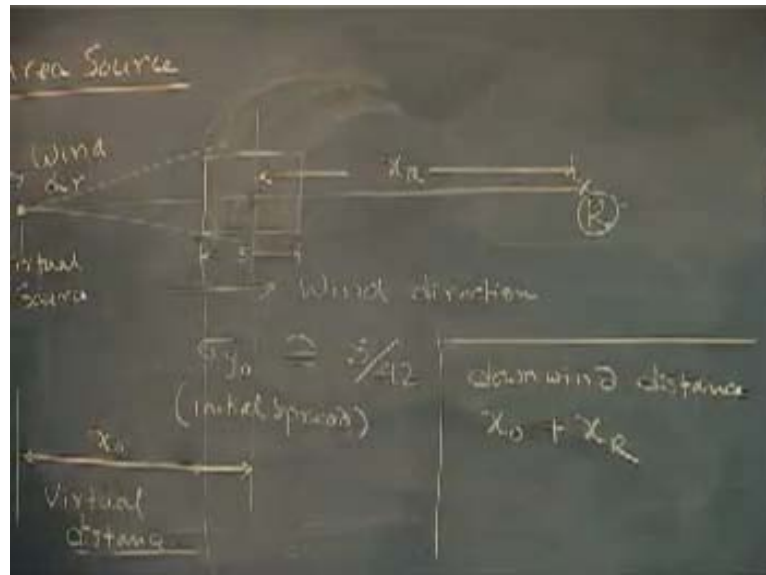
distance is what I can call  $x_0$  and call this as the virtual distance. Now, the  $s$  is known to me – that area that I have defined. Somebody has to tell me how I can find out  $x_0$ .  $\sigma_{y_0}$  is the horizontal dispersion coefficient that we talked about, but since I am taking that the initial spread is so much, please tell me how I can find out  $x_0$ . We have a graph that we talked about – that was the Turner that he developed in Tennessee Valley Authority and what he gave was the relationship between  $x$  and  $\sigma_y$ .

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If you recall, what kind of graph did we have? Normally, what have we been doing so far? We know the  $x$  and we find out the  $\sigma_y$ , right? Here, you know the  $\sigma_{y_0}$  (Refer Slide Time: 38:17) and take this stability, let us say my stability was  $C$ , find out the distance and this will be my virtual distance for obvious reasons – this will be my  $x_0$ . You can find out the  $x_0$ . No problem there? If you find the  $x_0$ , then it means you have physically relocated the source from here to here.

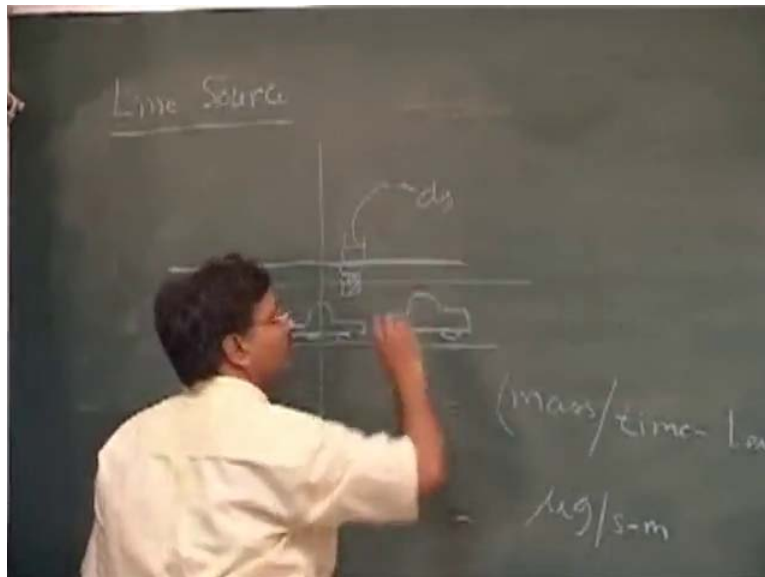
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Then whatever is your distance, which you are of course measuring from here, the distance that **you will consider...** suppose I want to predict the concentration at R, the distance for the R is actually let us say  $x_R$ , but for all my model calculation, I will take the distance equals to  $x_0$  plus  $x_R$ . Clear? For all my calculations of sigma y and sigma z further down from  $x_0$ . So the distance that I will use in the rest of the formulation will be the distance or the downwind distance that I will take for my this thing will be virtual distance plus actual distance. It is what we talked as a concept. We will do some examples – do not worry about that – but you understand what I have done. You can also find the sigma z similarly. I want to talk about a new thing and we will finish it. Keep some patience if we go overboard with time.



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Line source: many times, we encounter the situation when the vehicles are going on the highway. This is your highway or GT Road, it is kind of infinitely long and you have vehicles, vehicles are travelling and I will assume that the wind is like this, but we will change it at some point but for simplicity, let us take the wind blowing like this. Now, somebody please tell me what is my x direction and y direction because I want you to be clear as to what is my x direction and what is my y direction. This is my x direction – always the wind direction is my x direction. As the wind changes, my x direction changes obviously. All right, this is my y direction. Now if you recall the formula we have developed for the point source, we will try to modify that based on the situation we have.

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$$C = \frac{Q}{\pi \sigma_y \sigma_z U} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left(\exp\left(-\frac{H^2}{2\sigma_z^2}\right)\right)$$

$$dC = \frac{q \cdot dy}{\pi \sigma_y \sigma_z U} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left(\exp\left(-\frac{H^2}{2\sigma_z^2}\right)\right)$$

$$C = \frac{q}{\pi \sigma_y \sigma_z U} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) dy$$

$$s = \frac{1}{\sqrt{2}} \frac{y}{\sigma_y} ; ds = \frac{1}{\sqrt{2}} \frac{1}{\sigma_y} dy$$

$$\boxed{dy = \sqrt{2} \cdot \sigma_y ds}$$

If you recall the C, what is this concentration? At what is it – some height or at the ground level? This is at the ground level because my  $z = 0$ , so let us say so:  $x, y, 0$ , coming from a height of  $H$ . When you have an infinitely long source, you want to define your emission rate  $q$  in terms of mass per unit time per unit length because there you had one point source, you could say okay, you want to say here what is your emission from all these things in terms of the mass per unit time per meter. So normally, we want to define in this situation which is more acceptable to me is that I will define this  $q$ , which I am writing as a small  $q$  with respect to the line source – that is in terms of mass, time, length if you like or let us say microgram per second per meter. if I am saying that, then the best thing for me is to rewrite this **equation as...** Suppose I take a small strip here in my  $y$  direction (Refer Slide Time: 44:46).

I can say that this amounts to the same as the large  $Q$  dimension-wise – I am just considering the small strip. So we are doing nothing great under this situation, under this situation, I want to find out the concentration the way I have defined  $q$  in terms of  $dq$  and the small distance in the  $y$  direction as  $dy$ . If I want to find out the concentration, I can integrate both the sides, so  $C$  will **become....** Let us take the constants out –  $q$  is constant,  $\pi$ ,  $\sigma_y$  and  $\sigma_z$  do not depend on  $y$  but they depend on  $x$ , so I can take that out,  $U$  is also not function of  $y$ , what about  $H$  and  $\sigma_z$  square? They are also not function of  $y$ . Then, this (Refer Slide Time: 46:41) is function of  $y$  obviously. Is there anything else with respect to  $y$ ? Nothing else. So I can integrate this from

$y - y$  can expand from minus infinity to plus infinity, minus infinity to plus infinity, so we have (exponential minus  $y$  square by  $2 \sigma_y$  square) times  $dy$ . If I make substitution, suppose I say  $S$  equals  $\dots$ . Suppose I make a substitution, so  $ds$  will be  $\dots$   $\sigma_y$  is constant and  $dy$ . All right? No problem there? Therefore, my  $dy$  will be equal to root 2 times  $\sigma_y$  times  $ds$ . Suppose I call the whole thing for  $\dots$  to save some time because we do not want  $\dots$

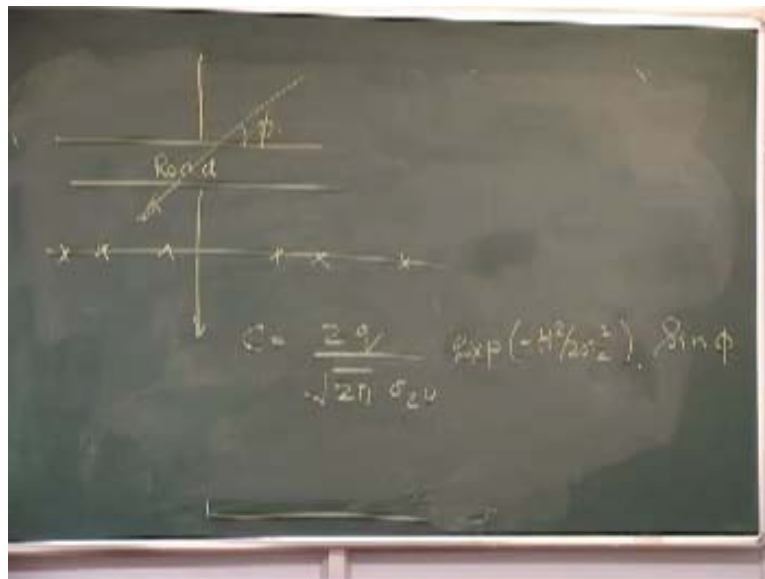
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$$\begin{aligned}
 \text{Line Source} \\
 C &= K \int_{-\infty}^{\infty} \exp(-s^2) \sqrt{2} \sigma_y ds \\
 &= K \sqrt{2} \sigma_y \sqrt{\pi} \quad \left( \int_{-\infty}^{\infty} e^{-s^2} ds = \sqrt{\pi} \right) \\
 &= \frac{q}{\pi \sigma_y \sigma_z u} \sqrt{2} \sigma_y \sqrt{\pi} \exp(-H^2 / (2 \sigma_z^2)) \\
 \boxed{C} &= \frac{2 q}{\sqrt{2 \pi} \sigma_z u} \exp(-H^2 / (2 \sigma_z^2))
 \end{aligned}$$

So what you can say here is that  $C = K$ , a constant which is equal to this term minus infinity – limits will not change as you can see – minus infinity to infinity  $e$  to the power exponential what else did we get? Root two please see if is there any mistake. No mistake, so this will be equal to  $K$ . These are all constants so that comes out to be exponential square negative integration from minus infinity to infinity is equal to  $\pi$  and a root  $\pi$ . What I am trying to say here is  $e$  to the power minus  $s$   $\dots$ . Have you done the things all right? Let us now write the value of  $K$ .  $K$  is your  $q$  by  $\pi$   $\sigma_y$   $\sigma_z$   $u$  and exponential we will write a little later, so I can write  $\dots$  and then  $H$  square, correct me if I make some mistake. What can I write here?  $2 q$  by  $2 \pi$  – that is correct –  $\sigma_z$   $u$  exponential minus  $H$  square by  $2 \sigma_z$  square. Did you get something like this?

Why we are writing in this form is that that is how people sometimes prefer it because this is a Gaussian kind of thing – square root or root 2 pi, otherwise you can write in whatever way you like to write, but if you see many of the books, they have not derived this – they will simply give you the understanding like this. This is your C – make a note. What is q in? Mass per unit time per unit length. Where has sigma y disappeared? Obviously, sigma y is cancelled.

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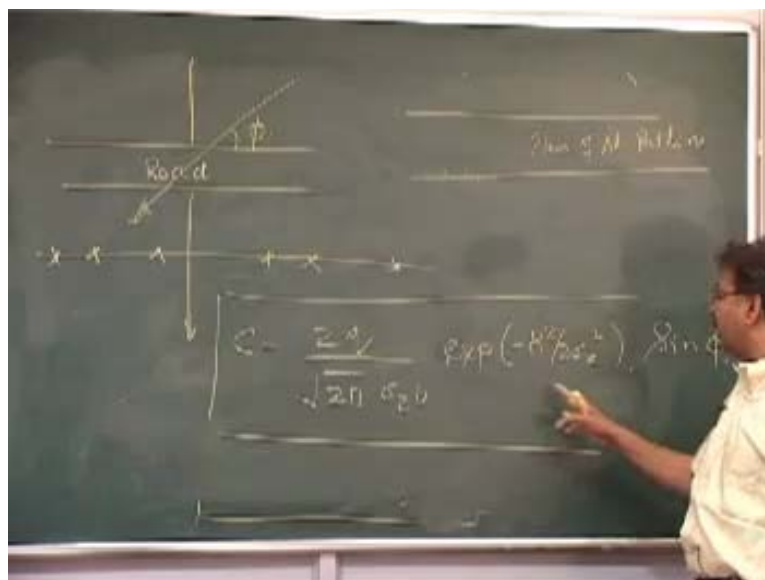


What does it mean to you when you do not have any sigma y? It got integrated out. The wind is blowing like this. Your y term has also disappeared and one thing what we have assumed is we have integrated from minus infinity to plus infinity, so you have taken this line to be infinitely long, so what happens in this case is that when I change the y and your emission is the same all throughout. Is your concentration anymore a function of y? It is not a function of y and that is why your y has completely disappeared from there.

If I take anywhere in the y as long as my x is the same, I measure the concentration at any place or model – measurement is of course a little tricky – but that will be the same, so that is why that got integrated out. The situation is the little variation that we need to give **here is....** Suppose if wind direction is like this making some angle phi with respect to the road. In that case, simply what you do is C equal to 2 q by root 2 pi sigma z u exponential minus x square by 2 sigma z square – take the component in the x direction of that thing, so you simply do the sine phi.

Obviously, if the  $\sin \phi$  is 0, concentration will be... and  $\cos \sin \phi$  is the  $\phi$  if the  $\sin \phi$  is 0, then it becomes 0. What is  $\sin \phi$ ? It is 90, then it is 1. But we apply this particular thing only for  $\phi$  equals 20 or 30 degrees or more because otherwise you will get the concentration as 0. This is how we can get toward through the line source. Little variation, I could have still maintained  $z$  here (Refer Slide Time: 55:07) – there was no problem right? I put  $z = 0$ . If I want to maintain the  $z$ , I can still keep this  $z$  intact;  $z$  is not a function of  $y$ , so I can still get the expression containing the  $z$  there. You can further simplify this one. If you think your cars and scooters and trucks are emitting at effective height zero because this is more [55:35], you can put this equal to 0 this becomes 1 – so that variation you can do. But why we are writing in terms of  $H$ ? Sometimes, we model many of the industries as a line source.

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One example I will give because you have the background is the aluminium plant. You have many pots and this line may run into 700 meters, 800 meters and you know that there are emissions at the top in the aluminium smelter, there will be openings on the top, so there will be emissions. This is a plan of aluminium pot line, it is very long. If you remember, I had shown you the picture and that it looked very long. So in that thing, this may be about 30 meters high from the ground, so then you will maintain the  $H$  here but for some sources that are more or less ground-level sources, you can do this one. Sometimes, people set the wall of their field on fire – then you can apply these conditions; sometimes, people have also tried to model the drift of the

snow because the wind and then there is a snow drift – some people have applied these kind of... not exactly the same but something similar, but then you treat this line as infinitely long having the same some emission at each point. So now, you can apply this model at the situation where it wants. Time and again, I say this:  $q$  is mass per unit time per meter because many people make the mistake and this is how you can do it. In summary, what we have done is you can model a point source, we can model an area source, we can model a line source. Maybe in the next class, we will do one example.