Environmental Air Pollution

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Lecture No. 31

Gaussian Model Useful Formulation

If you recall, we were trying to solve the advection diffusion equation as an analytical solution by making several assumptions and modifying our three-dimensional advection diffusion equation and finally getting to the concentration.

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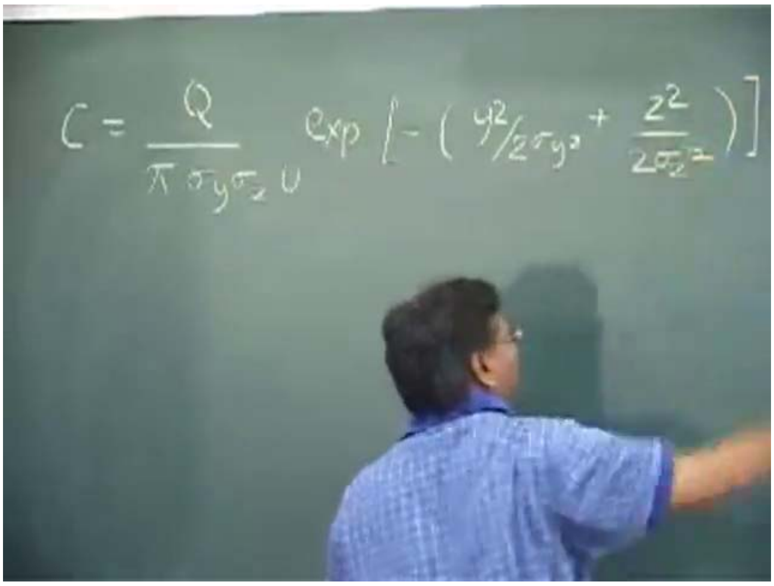

That concentration, correct me if there is any mistake, was Q by pi sigma_y sigma_z U into exponential minus (y square upon 2 sigma_y square) plus z square by 2 sigma_z square, this is closed and this is closed. This is the equation we derived yesterday. If you see in this equation, where was our source?

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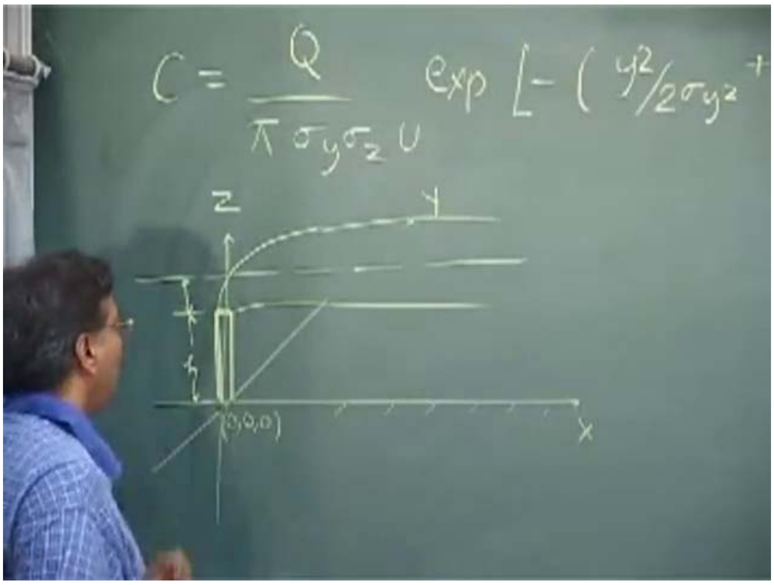

Again, if I draw the coordinate system when I am talking about z, this is my origin 0, 0, 0, X direction, Y direction and Z direction and all z I am measuring from here. In a way, my source or the emitting source is sitting at this point, but what you will see is that most of our chimneys are not sitting on the ground – most of the sources are what we call as elevated sources. If this is my ground level, my source – which is a chimney – is a little elevated and when the emission takes place if you look at the plume in general, that might look something like this. If I draw the central line of the plume and do this, most of the time, I will encounter sources that are not sitting here but are elevated sources. So we have to see that in this equation, we modify the equation for the source that is not here but somewhere here. In this case, to modify that, there are a few things we need to define. Let me call this as h and what is this (Refer Slide time: 04:08)? This portion will rise in the plume because of thermal buoyancy and other things that we will discuss.

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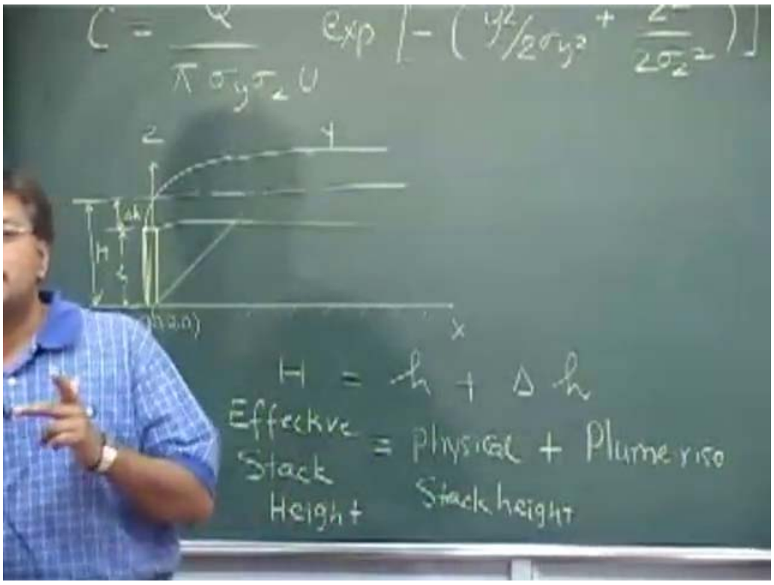

This I am calling as delta h and the overall height after which the dispersion is taking place we call as capital H. Whatever I have written here, let me write that in words – H is nothing but small h times.… This is what we call as the effective stack height and that is equal to small h (the physical stack height) plus delta h (what we call as plume rise). We will talk about plume rise a little later, but the plume rise will have two components: one is the momentum plume rise, like a jet action, and another portion will be the thermal buoyancy because the gases that come out are at a higher temperature – but we will discuss that later. What I want to do is modify this equation and say the height of the source is not 0 but this is at H. How do I modify this equation?

[Conversation between student and professor - Not Audible (06:07 min)]

Z is replaced by Z minus H. Does that everyone see that? Let us do that and I will write that equation again.

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C equals to Q by pi sigma$_y$ sigma$_z$ U into exponential minus (y square by 2 sigma$_y$ square) plus for Z I must write Z minus H whole square by 2 sigma$_z$ square and this is closed. But there is a little bit more change in this and that change is that you put 2 here in front (Refer Slide Time: 06:56) – this 2 has come from the way we estimated or calculate the constant k. Do you remember that we derived the value of constant k? If you derive the constant k under this situation…. This k was the constant which if you recall from the last time we defined as the large K. The new K for the situation when I am putting Z is equal to Z minus H comes out to be the K that we saw yesterday by 2, so you see here, the additional 2 has come, we are not discussing how that 2 has come but then we accept that the additional 2 will come. So, two changes from the equation that we derived yesterday: half or divide by 2 and Z becomes Z minus H – that part you can see very clearly. When the sources are elevated, some more things are to be observed, let us see.

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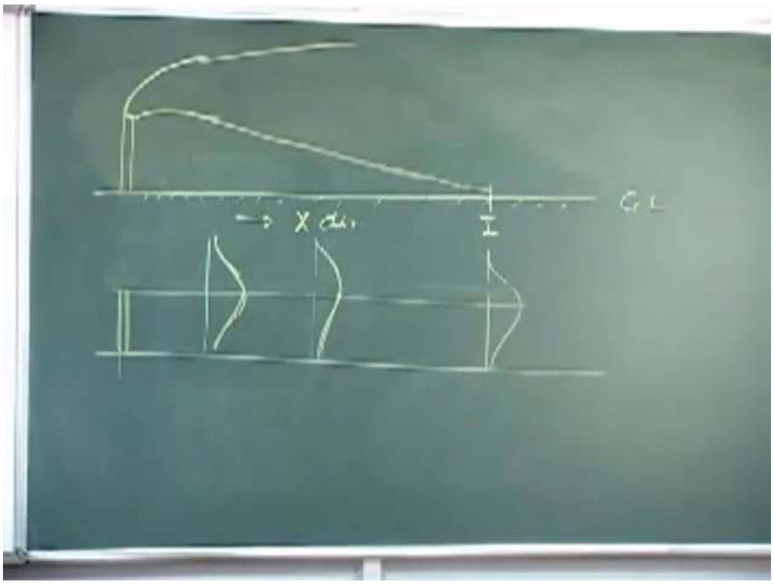

This is my ground level. At this point, at some point, the plume is likely to touch the ground and let me call this as point I. At some point, the plume is likely to touch the ground – let me call that point I and do not forget that this is my x direction. Suppose I am plotting the concentration at the… You see here, the concentration at various values of x and my stack height is somewhere here, so I am just taking that as the center line. If I am measuring the concentration in the z plane, that concentration will look something like this, then I can measure here, it will look… then I measure here, I think what we will do is make it a little lower here.

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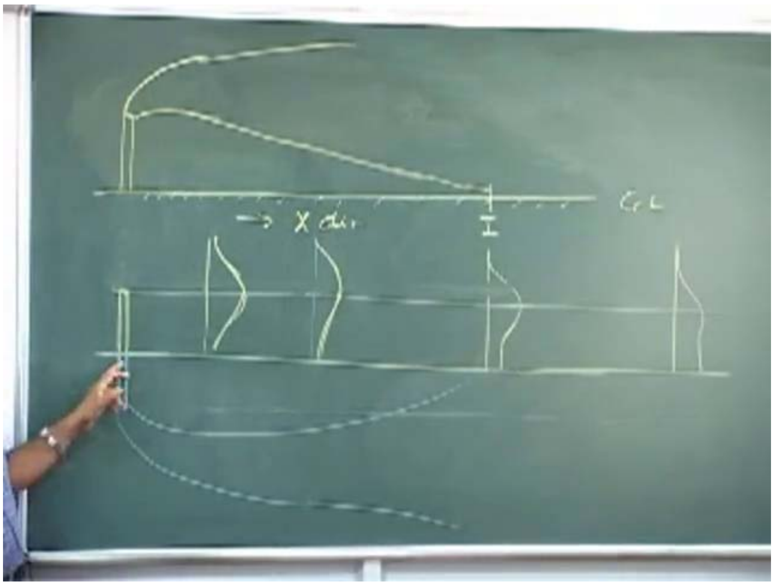

When the plume is reaching I and the plume hits the ground, there is a possibility that the plume will be reflected back – the plume is coming and then it could be reflected back and what has been found is that sometimes the concentration profile looks a little flattened here (Refer Slide Time: 11:03) – slightly at the ground level because once it is reflected, the concentration will increase and some people have observed that part. Suppose I again see it after some time very far away, you may find this is my center line, it should be like this, it may become more flattened because there is again… thing might be added up. Somehow, in that equation – this equation or the equation that I wrote earlier – we had not considered the ground reflection; we have to consider the ground reflection also – how much could be the change in the concentration because of the reflection from the ground. How to account for this? Let us see. The best way to do this is to think of… let us see if we can use another color here. It is another source here; we think there is an imaginary source underneath the ground and this is also causing the concentration. This source at the ground level, below the ground will have some center line and the plume from here will go like this – it is exactly the mirror image, exactly the mirror image. So here, this particular source is not contributing anything here at the ground level (Refer Slide Time: 13:23), it is not contributing anything here, but you will see that it is contributing something here.

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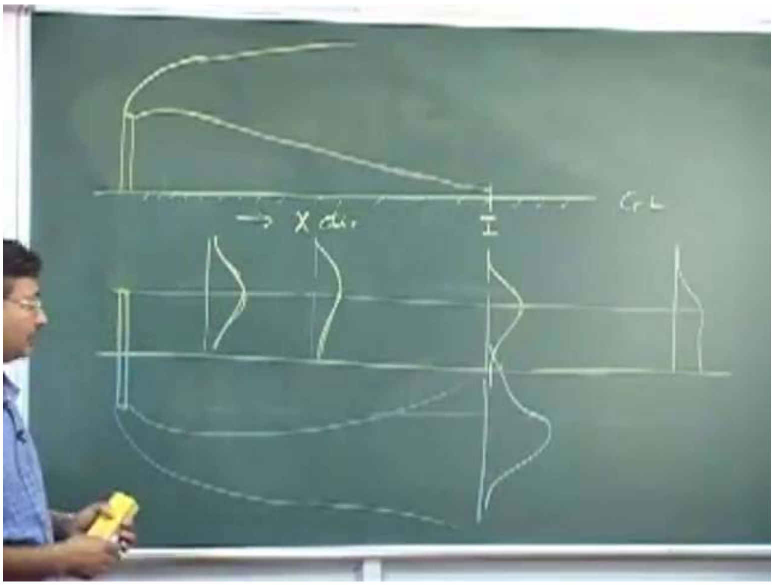

If I draw the profile considering only one source – of this particular source, my profile looked like this and if I draw the blue picture, the contribution from this source will also be… or this might slightly go like this and this source will contribute… – all hypothetical, but you see here, you have to add the concentration given by the yellow graph and the one that is given by the blue one.

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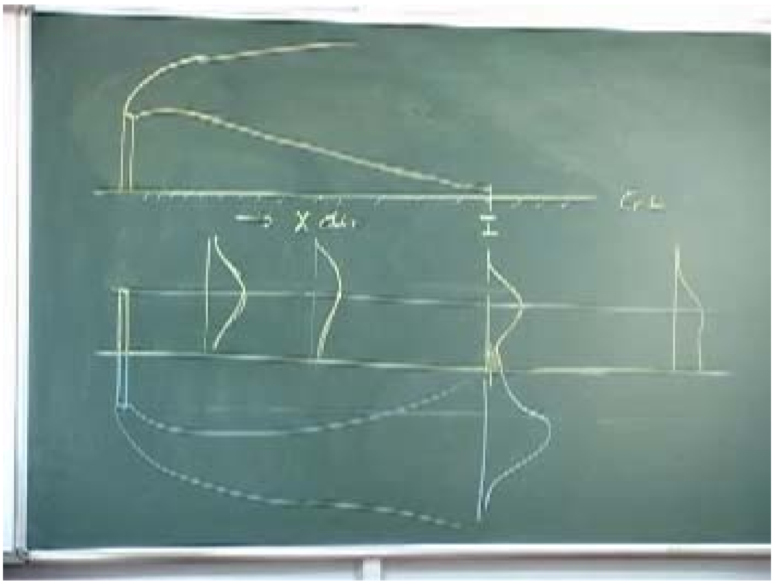

Finally when you add these things up, the picture will look like this – it is not reducing asymptotically but it is reducing like this; you might have a slight addition of the concentration in this range and I must account for this change – I must account for the reflection of my plume and I am accounting for that as if this is an inverted source. I can write the same equation for this particular source by putting z equal to… in this equation by putting z equal to.… Earlier, we put Z equal to Z minus H for this source. I can write the equation for this source by putting Z equals to Z plus H and I can write that equation. I can add those two equations and I can get the concentration here (Refer Slide Time: 15:29). You have one source contributing, you have another source contributing; by superimposing both of them and adding that thing, I can find out the net concentration by considering the reflection part – simple thing but we have to understand it. Does everyone understand that I can write the same equation that you wrote? Place Z equal to Z plus H in this (Refer Slide Time: 16:03), write 2 here and then contribution from this plus this and then whatever the final equation is, that is what I am going to write. If you agree with the equation I write here, that will be considering the reflection part – I will write that. Let me see if I can modify this.

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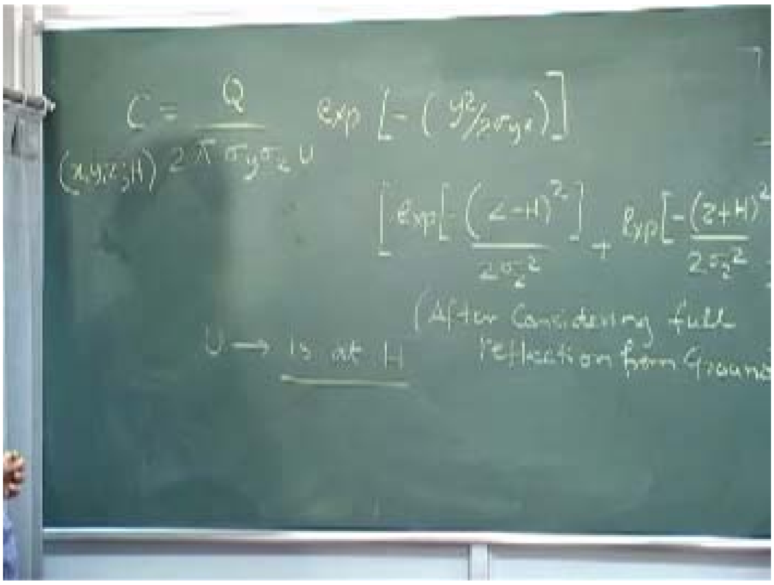

The 2 should certainly come and then a large bracket… exponential minus (Z minus H) whole square by 2 sigma$_z$ square plus exponential minus (Z plus H) whole square by 2 sigma$_z$ square, close this one and close this. You can work out and see if you are getting the same thing as I would get after considering full reflection from ground; you have been able to derive this and then you get the same concentration. This is my general formula in a broader sense and that is what I want to use depending on the situation I have. What is it that we want to do? Some more changes from here. Many times or rather most of the time, we are interested in the ground-level concentration because that is where people live.

If you agree, I can now write that C is the concentration at x, y, z, put semicolon, this is how it is written – the coordinate system x, y, z and from a source which is at H. A few things I should tell, rather you should tell me – we took this U as the wind speed at…? Ground level. Ground level, right? Now, at what height should this U be? Should it still be at the ground level or at some other height? At H because this is the wind speed that is affecting the plume. Earlier, we took this as the ground level, now you can make a note here: this U is at H. Earlier when the source was at the ground, U was at the ground level. This U is the mean wind speed affecting the plume. Where is my plume? My plume is at H, so my U should also be at H. Now, I very quickly want to do some variation of this.

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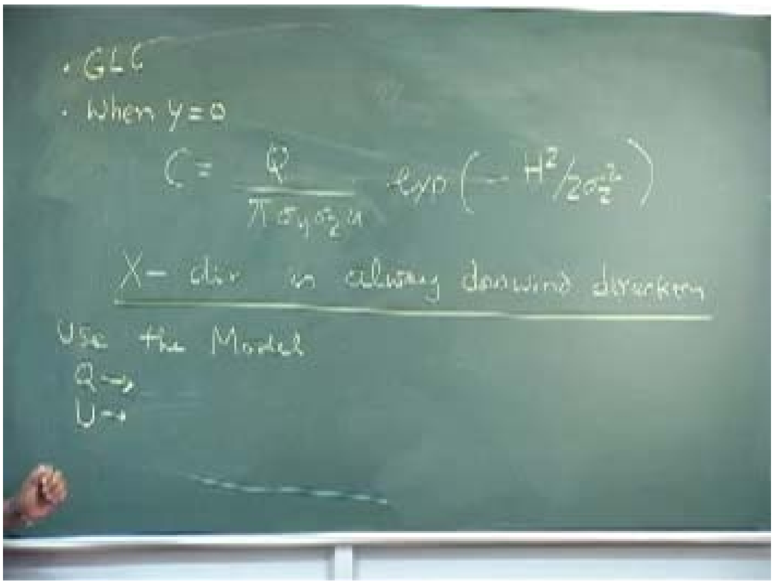

Sometimes, I am interested in only ground-level concentration; I am also interested in at the ground level, what value of y you expect concentration to be more. We want to see the maximum impact, so we say maximum impact at the ground level where people live, so what do you think? Again, qualitatively. The plume is going, so what level of y do you expect or what value of y do you expect the ground level will be maximum at?

[Conversation between student and professor - Not Audible (21:27 min)]

y, I am asking y; z is 0 because the moment I am saying ground level, it means z = 0 – that goes automatically. Just see here: as I increase y – does not matter plus or minus – what will happen? My concentration will decrease, maximum concentration will come when y = 0, that means you are standing right underneath the plume, the ys are 0, so the plume is just passing over you and then you think it might affect the ground level. Many times, we are interested in the ground-level concentration; I will not call it maximum but when y = 0 – that is our interest. Suppose the plume was passing over IIT Kanpur from the Panki power station, then I want to see what is the concentration to the people sitting at IIT Kanpur, taking y = 0; that is a simple thing you do here – you put y =0 and z = 0, you come up with another expression and that expression is most commonly used because that is where you want to find out when ground-level concentration that is when z = 0 and y = 0 and you can very quickly see that this term will disappear (Refer Slide Time: 22:58), this will disappear, this will become H square, this is also H square, so it will become 2 times exponential H square by 2 sigma$_z$ square, that

2 will cancel out with this 2. If you agree, I will write that C becomes Q by pi $sigma_y$ $sigma_z$ U into exponential y should disappear minus H square by 2 $sigma_z$ square – that is all, that is all.

Let us see what is it that I want to cover in some sequence. One thing I want to remind you is very important and if you keep that, you will not make much mistake: always remember and underline that your x direction of the coordinate system is always.… Underline this: in the system, my x direction or the x coordinate is always the downwind direction because the x has been the assumption all through; the moment the wind direction changes, your coordinate system changes and x changes, so you should be careful – that is a very important thing. Now, let us talk about how we can use the equation – use the model, if you like. What you need are certain things: Q. Where will the Q come from? Emission inventory, right? Do you remember we worked out the emission inventory? This is the source, this is the power plant burning so much of the coal with sulfur content of this one, the coal consumption per day is so much tons per day, so what will be the sulfur dioxide emission? You did that in one of the examples and you found out that the emission of sulfur dioxide will be 19 times the S – that is the sulfur content times your tons per day of the coal used. That Q can be estimated or Q will be most of the time given to you or you can estimate it given the data.

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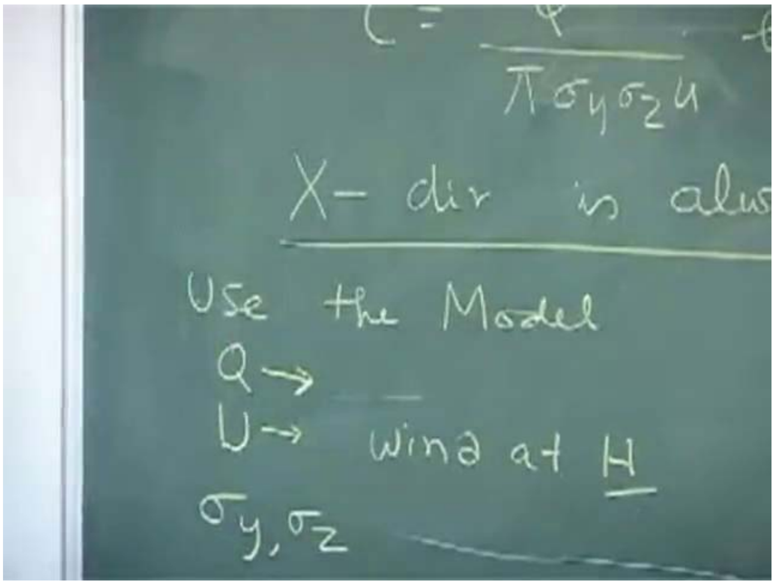

The other thing you need to know is U. Where will you get it and how can you get this U? You cannot get this U so easily, but the U you get is from the ground-level measurements; if not, you can at least go to the airport and find out what is the U in that area, but this is the U they will give you – they will give you the surface wind; no one will be able to give you very easily what will be the wind at H, so this one we need to answer, but we will answer it a little later; the other thing you need is $sigma_y$ and $sigma_z$. We will have a little bit discussion as to how and where the $sigma_y$ and $sigma_z$ come from.

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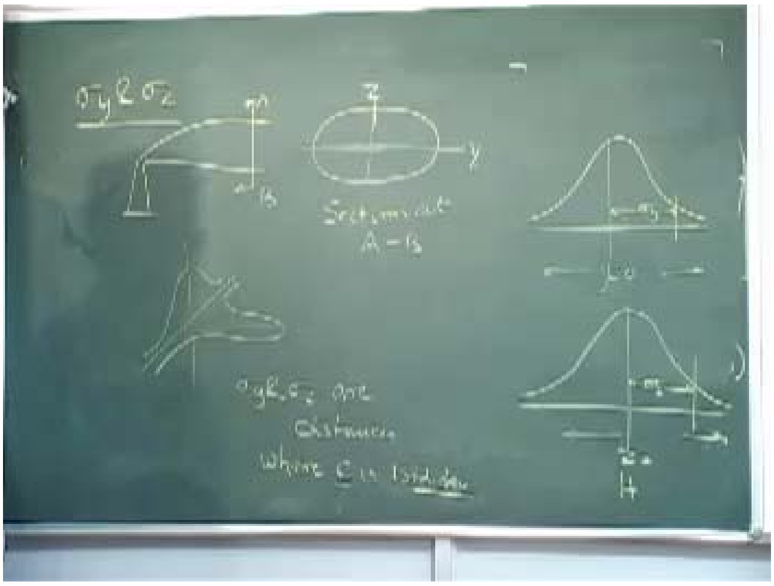

Let us see what these are and how we can measure them. Again, let me draw a picture of the chimney. I will take the section at AB and if you see the section, in what direction is this? Y. Y, good and this is my Z then – Y, this is Z. Let me see if I can draw how the concentration will be as seen in the Y and Z direction. Suppose I keep on measuring, suppose I fly in the aircraft and start measuring the concentration at different places of Y and at different places of Z. Here whatever the value of z is, it is not changing at this axis and I went on to do the measurement here, here, here, here, here, here, here, went up there and inside the plume and then I will get a concentration profile, y goes to 0, people have found that this is the Gaussian kind of concentration. I can also within the plane or rather not the aircraft but with a helicopter, if you like, I am not changing my Y coordinate but I fly my helicopter vertically up and down and do the measurements at different heights. I will then get a profile that will look like this and what is the z here at this line? What is the z here? What is the value of z here (Refer Slide Time: 29:40)?

[Conversation between student and professor - Not Audible (29:41 min)]

It is capital H – obviously, this is my capital H, so you see that is my capital H, so do not put z = 0. I can find out the y and so this is what is the concentration at a fixed z and by varying the ys; here my y is constant and I am varying the z; this is my Z, which is equal to H. I will try if I can draw this one – not so easy to draw, but let us see. If this is my y plane, I am measuring.… This is my y plane and my z plane will be something like this and if I have to plot in the z plane, that will look like this. If I have done this concentration $sigma_y$ and $sigma_z$, that is the same thing I have drawn here.

Wherever is the distance where you get this as 1 $sigma_y$, 1 standard deviation and here, there will be some point where you will get…. This distance where concentration is equal to 1 $sigma_y$ is called $sigma_y$ and the distance when concentration is equal to 1 $sigma_z$ is called $sigma_z$ – remember that $sigma_y$ and $sigma_z$ are not the concentration but they are distances. We again write that $sigma_y$ and $sigma_z$ are distances, where concentration is 1 standard deviation. Having said that, we have only explained this but we have not said how these are measured.

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The first attempt to measure $sigma_y$ and $sigma_z$ was, I hope the spelling is correct, at TVA; there was a power plant in Tennessee, USA and the power plant was located in a plain terrain. They did measure the $sigma_y$, they did measure the concentration at different this thing.

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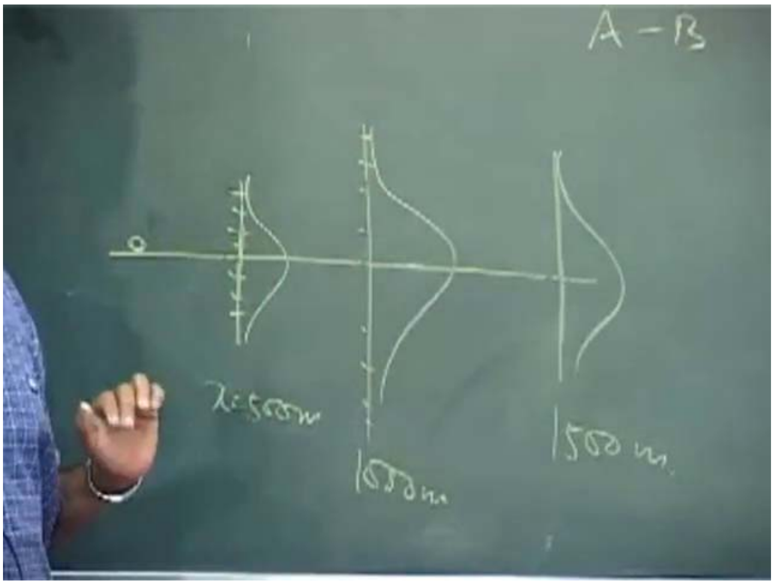

But what they had was a ground-level source, if I recall correctly; then at various places of y, they measured the concentration and at different xs; at x = 500 meters, they measured the concentration, 1000 meters, 1500 and so on and so forth; they plotted the concentration here, they plotted the concentration here and then found out the $sigma_y$ as a function of x (Refer Slide Time: 34:39), $sigma_y$ as function of x, $sigma_y$ as function of x. Similarly, they did the measurement for $sigma_z$ at various heights. Of course, we do not have to get into the details but you see these were actually measured. What else will $sigma_y$ and $sigma_z$ depend on, other than the x? That we are seeing with the x.

Tell me, will $sigma_y$ increase or decrease with x? Increase. Increase, so that is clear – because the plume is expanding, so $sigma_y$ is increasing. What did I ask you? $sigma_y$ will increase. What else will the $sigma_y$ depend on? Turbulence. Turbulence is indicated by the atmospheric stability. What they did was they ran this experiment by releasing some tracer because suppose you are doing it with sulfur dioxide, sulfur dioxide will always be in this area by some other source and the results will be a mixture of sulfur dioxide from various sources. They released some tracer so that that tracer is not there in the atmosphere and so they can accurately measure. They went into the fields with some small handy gas chromatograph – whatever that instrument is I do not know – but they measured at different places of y and they have to measure at the same time. Suppose they measured at ten places, maybe ten people can stand with their monitors, with gas chromatographs and then they release some tracer; I do not know what tracer they release, but the normal tracer that we use in air pollution study is SF_6. The person involved was the same person who talked about stability – that was Turner and his team and then finally, once they came up with this data, they plotted the sigma_y and sigma_z.

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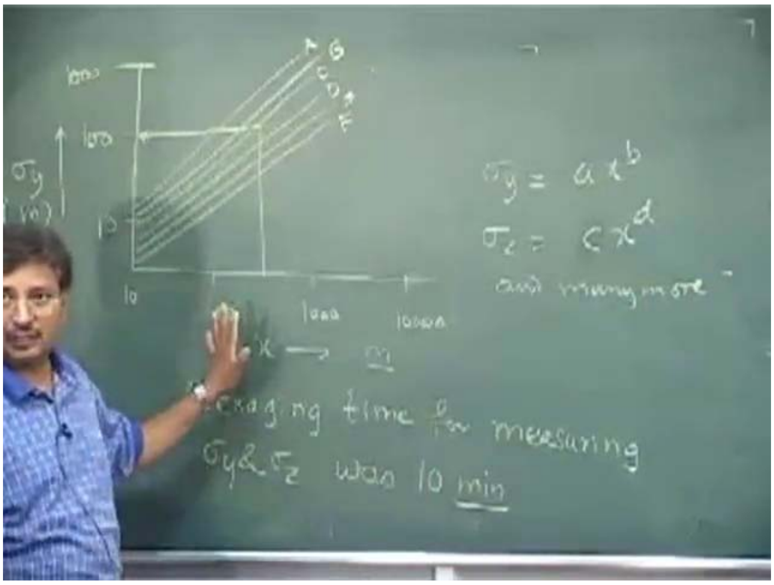

This side is the downwind distance x, this is your logarithmic scale – 10, 100, 1,000 and 10,000, this is in meters and this is sigma_y and since I am drawing, they are approximate but you can get the exact numbers in the book. They reported this to be something like this, they reported this for stability A, B, C, D, E, F. Whatever your distance is, take that distance, suppose you want to find out 1,000 meters from the stack, you know the stability – let us say stability was D, the corresponding value of sigma_y can be found out from the graphs developed by TVA. Let us say you want to find out at 500 meters and your stability was C, the sigma_y value could be 100 meters – this is not accurate but this is how you use this.

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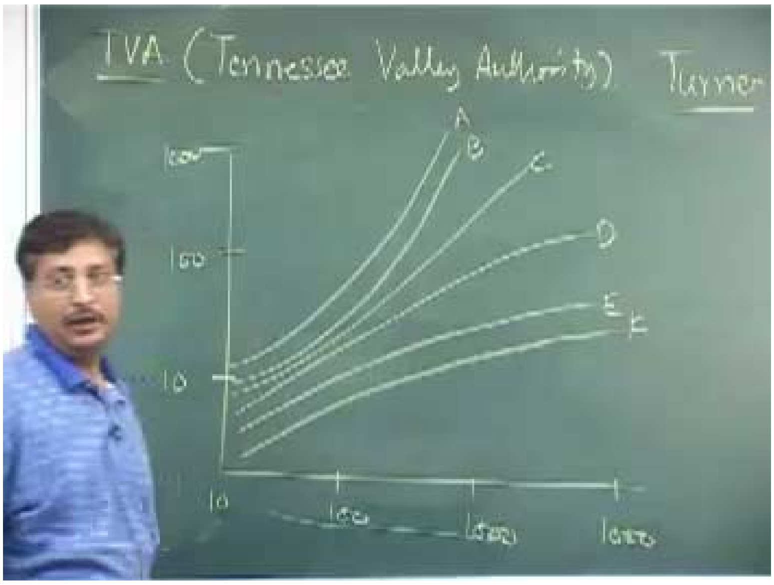

Similarly, they did for $sigma_z$, which looks a little bit more… By the way, do not forget that this side is also log scale, so 10, 100, 1,000 and 10,000 and 10, 100, 1,000. Here the lines are straight for $sigma_y$, but here they got curved lines; this is your A, B, C, D, E and F – that is your stability A, B, C, D, E and F. The same thing: at what distance you are, you can find out what the $sigma_z$ will be. I have the graphs but I did not bring the laptop, so this particular graph some of you in the front can see (Refer Slide Time: 40:58). This axis is downwind distance x, this side is $sigma_y$ and these are the values of $sigma_y$ for stability A, B, C, D, E and F – that is something I have plotted here.

Then, the same thing for $sigma_z$ and $sigma_z$ you see here is a more curved graph – same downwind distance on the log scale, the value of $sigma_z$ is on the y axis but again log scale and you see here A, B, C, D, E and F. You can find out what will be the value of $sigma_y$ and $sigma_z$ depending on what x you are interested in and what is the value of turbulence or what is the value of the stability class. I will just quickly pass on this to you. All this is there in the book – there is nothing I am teaching that is not there in the book. You can all have a look at this – it will show you how you can find out the $sigma_y$ and $sigma_z$ given the value of downwind distance. What is x? Again, remember that x is the downwind distance and you can find out the $sigma_y$ and $sigma_z$.

Another thing you should note and I will write that so that there is no confusion… was 10 minutes (Refer Slide Time: 43:01). The person stood there for 10 minutes for taking the sample, so that was the 10 minutes, so essentially by using these sigma$_y$ and sigma$_z$ graphs, you will be finding out the 10-minute average concentration. Any questions so far? If you are given the Q, if you are given the U at the height H, you are given the H, you can find out the concentration at different locations.

Let us see one more thing. What happens once you have a graph, it is a normal thing to fit an equation because a graph cannot be fed into computers, so people have fitted the equations to this one and there are so many formulas but this was the basic work. Now, people have fitted this kind of equation that is generally in the form of sigma$_y$ equal to… if you do not want to use the graph. What will AB and CD be function of? They have fitted the curve. It will be the function of stability. For the same x if the stability is different, then sigma$_y$ is different; for the same x, stability is different, then sigma$_z$ is different; I will give you those numbers or maybe I will pass on through mail to you rather than giving and spending some time. There are many more other formulas – I will probably will put a slide in the next time to show you how the sigma$_y$ and sigma$_z$ can be calculated without using these graphs – graphs were fundamental.

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Before you are set to use that equation, you need to know the plume rise and that is…. We will not derive delta h but I will give you again some formula and some ways – that is enough for us for the time being. There are many things for delta h to find out. One of the ways is using the…. Let me write that. U is… you should write the units here, U is the wind speed at…? H. Small h or big H? Big H. It is at small h. What we are saying is that this particular wind is affecting the plume rise. When wind is there, something wants to rise and there is a horizontal force – that will affect the wind, so there will be no wind rise. Again, I am asking you, where is this U? Not here but this U is at h (Refer Slide Time: 49:34) – at h or stack tip. These are small things but can make a difference.

This U is at the stack tip and that u which was in our formula was at h, because that is what is the mean wind that is causing the advection in the plume. This U is causing the problem or disturbance in the plume rise because we are discussing plume rise. Since they are constants, you have to use the same units – you cannot change the units than what I written have there. The most important thing is that people make a mistake sometimes in the pressure, which is always in millibars because we have done some changes and got some constants; P is the atmospheric pressure in millibar. Normally, how much is the millibar pressure?

[Conversation between student and professor - Not Audible (50:47 min)]

Around 1,000. 1,100, 1,200, 1,000 – that is the normal millibar pressure and there will be some more changes here. We apply some factor here (Refer Slide Time: 51:06) and we will discuss that factor in the next class – that factor takes care of the stability because here we have not talked about the stability. One more thing maybe we can talk later but this component – 1.5 times V_s times d by u, this component or this term will give you momentum rise because of the jet action; this component V_s times d by u times this term gives you the thermal buoyancy and maybe I will write it in the next class. There are two components of plume rise: momentum rise and thermal buoyancy. Some plumes are dominant in terms of the momentum rise, some plumes are dominant in terms of the thermal buoyancy – we will see what it comes out to be. I think we will stop here and try to do some… I still need to define how you can find out the u at different heights – we will talk later about that and maybe we will start some example to get a feel as to how things are done. We will stop it here.