Advanced Hydrology
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Lecture – 9

Good morning, and welcome to the next class of this video course on advanced hydrology. In the last class we look at the various parameters of the green ampt infiltration equation. Then we looked at a two layer model in which we can apply the green ampt infiltration equations, when we have more than one soil type in the ground. Then we look at the concept of pounding time. We found out how we can determine or estimate the pounding time using a certain procedure. Then we look at the method to determine or to calculate infiltration after the pounding time. What we are going to do today is look at a few examples on whatever we have done in this course till today or till the last class. So, I would like to get started with the the first example, today like to draw a attention here. So, what I will do is I just write the problem first, then look at what the data are for the data are given and then try to solve that problem.

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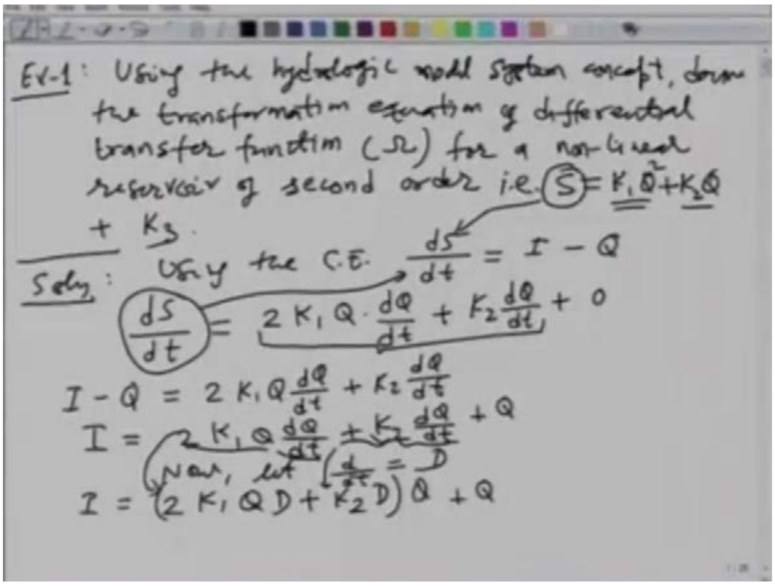

So, the first question or example problem we will take goes like this. Using the hydrologic model system concept, derive the transformation equation of differential transfer function which is your this omega we have defined for a non-linear. Remember earlier we had seen an example of a linear reservoir. So, today we will look at a non-linear reservoir of second order. That is your storage in the catchment can be represented using this equation K 1 Q square plus K 2 Q plus K 3. So, this is the problem in which we have to determine the expression for the transfer function operator for a catchment such that the catchment is simulated using a concept of non-linear reservoir in which the storage is a function of non-linear function of outflow Q and which is given by the second order polynomial like this.

How do we solve this problem? Well we will follow exactly the same procedure which we did earlier. So, what we will do is we will use the continuity equation and then replace the d S by d t term using the expression which is given to us. So, let us see how we are going to look at that. Using the continuity equation we have d S over d t is equal to I minus Q. This is the basic continuity equation we have derived. So, this first problem is basically coming from our first few lectures or the first chapter of your Ven Ti Chow's book. Now, we have the functions for the storage of the catchment, what we want to do is determine the first derivative of this storage function as a function of time. And then put that value into this equation. So, if we do that let us first find out what will be the d S over d t as you can see that K 1, K 2, K 3 are the constants and the only thing that various with the time is the either the storage or the outflow.

So, it will be twice of K 1 Q times d Q over d t. Q square as a function of time would be twice Q d Q over d t, because Q is a function of time plus this was for this part. And the second part what will be the first derivative of this, it will be simply K 2 outside and d Q by d t plus, what is the derivative of this term K 3? K 3 is constant. So, it will vanish so you will have 0. Now what we do is we replace this quantity or put the value of this in our continuity equation. So, let us do that. What I will have is I minus Q let say I put on the left hand side is equal to this whole thing which is twice of your K 1 Q d Q over d t plus K 2 d Q over d t.

Now take the Q on the other side. So, that I is on the left hand side and outflow is on the right hand side so you will have twice K 1 Q d Q over d t plus K 2 d Q over d t plus Q. So, the Q has come on the right hand side. Now like we did earlier left d over d t the represented by some operator D. Once we use this concept we can write this equation as I is equal to twice K 1 Q D d over d t is the, and we are applying it to d outflow Q so that will be taken out outside. And then you have second term is K 2 D and it is being applied on Q. Is this clear? So, what we have is this twice K 1 Q d Q over d t this term corresponds to the first term that is twice K 1 Q d. And the second one this one corresponds to this term K 2 d operated on Q plus Q with this…

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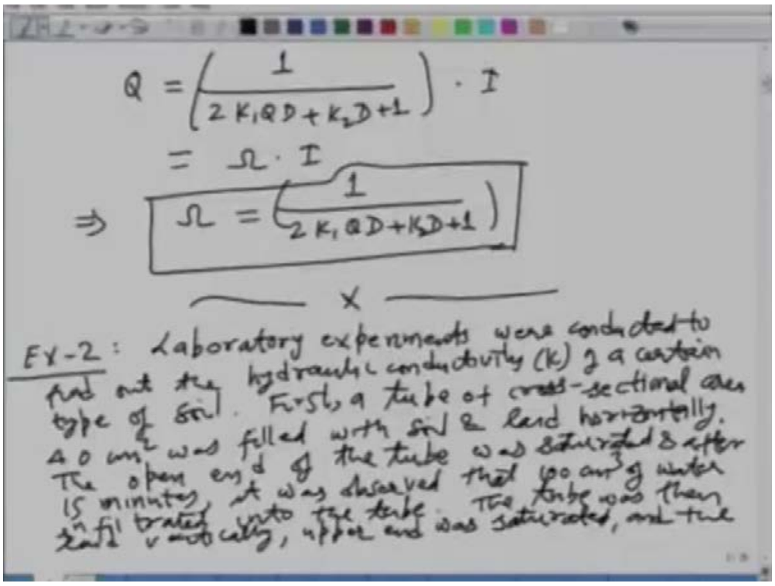

What you should be able to write then is the outflow from your system model as a function or operator applied on the input. And this whole operator is applied on I and this as we know we represent this as omega times I, that gives you the formation or the representation of omega S this expression twice K 1 Q d plus K 2 d plus 1. So, this is your transfer function operator for a catchment when we assume the catchment to be a non linear reservoir of second order. Now, you can see that you can extend this concept we can have any form of you know complex system we can try to simulate our catchment using in our more complex geometry. So, that we are able to model the complexity in the which are involve in the rainfall on a process or the recatchment process.

So, this was the first example. Now, what we are going to do is look at another example and this will involve the sub surface hydrology chapter use of the infiltration equation. So, looking at the first one, second example the first one from the sub surface chapter first I am going to write it down slowly, so that you can understand this. The lab experiments were conducted to find out what to determine the hydraulic conductivity K of a certain type of soil, how it was done? Well first a tube of cross sectional area 40 square centimeters was filled with the soil and laid horizontally. The open end of the tube was saturated or you know some water was applied. And after 15 minutes it was observed that, it was observed that 100 centimeter cube of water infiltrated into the tube. I am sure some of you may be getting the idea already how we are going to solve this problem, if not we will see in a minute. The tube was then laid vertically the tube was then laid vertically and the upper end was saturated so that the water will travel in the vertical direction now.

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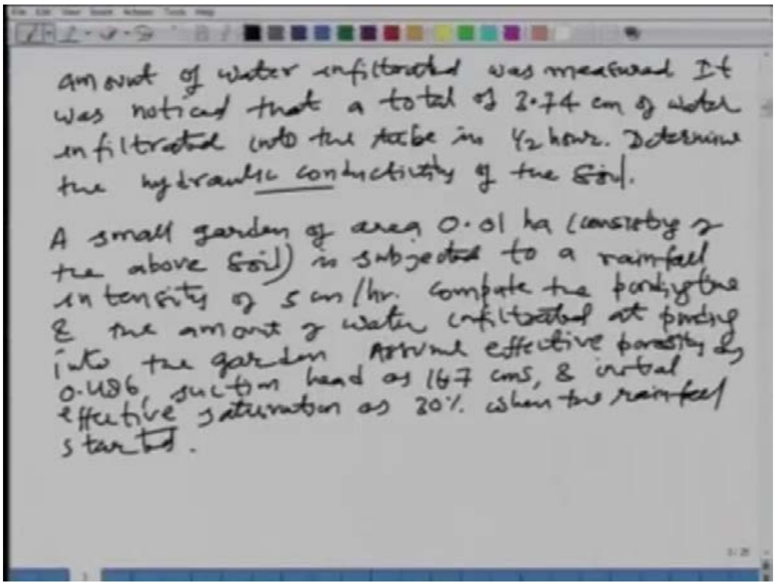

And the amount of water the amount of water infiltrated was measured. It was noticed that a total of 3.74 centimeter of water infiltrated into the tube, into the tube in half hour. So, using this data your first task is to determine the hydraulic conductivity of the soil. So, this is the first part of this problem it is a long statement I will quickly go over it again. The laboratory experiments, where conducted to find out the hydraulic conductivity of a certain type of soil first the tube was laid horizontally. And some water was applied on one side and the amount of water that infiltrated into the tube was measure and that information is given to us. Then what was done? The same sample was derived and then it was placed vertically in the vertical direction. And it was saturated from the top and the amount of water that get infiltrated in the vertical direction that was measured and all of that data is given to us.

Using these two information are these two data we have to find out the hydraulic conductivity. Once we have done that there is a another part of this problem a small garden. It may be in your hostel or in your home outside of area 0.01 hector which consist of the above soil consisting of the above soil means what means it has the same properties soil properties is subjected to a rainfall intensity of 5 centimeter per hour. Then what we have to do in this part is compute the pounding time compute the pounding time and the amount of water in filtrated at pounding that is just at the pounding time into the garden. Assume the effective porosity as 0.486 the suction had as 16.7 centimeters. And the initial effective saturation which as you knows is denoted as S e as 30 percent when the rainfall started.

So, this is the problem statement which is quite long but it is important to understand. So, you take a sample which is cylindrical in shape we laid first horizontally do certain measurement then we laid vertically measure the water that is infiltrated then we need to determine what is the hydraulic conductivity. And then we say that we consider a problem of rainfall event into a garden and then some of the data that are given to us we have to find what will be the pounding time of the garden the rate of rainfall is given to us and we have to find how much water will infiltrate into the garden at the time of pounding. So, it should be very easy to a see that which equation we are going to use here. Because the sample is laid horizontally as well as vertically we will use the Philips equation, because that is the equation we had seen which allows us to use or compute infiltration in the horizontal direction and also it is applicable for the vertical direction. So, let us look at the solution of this problem.

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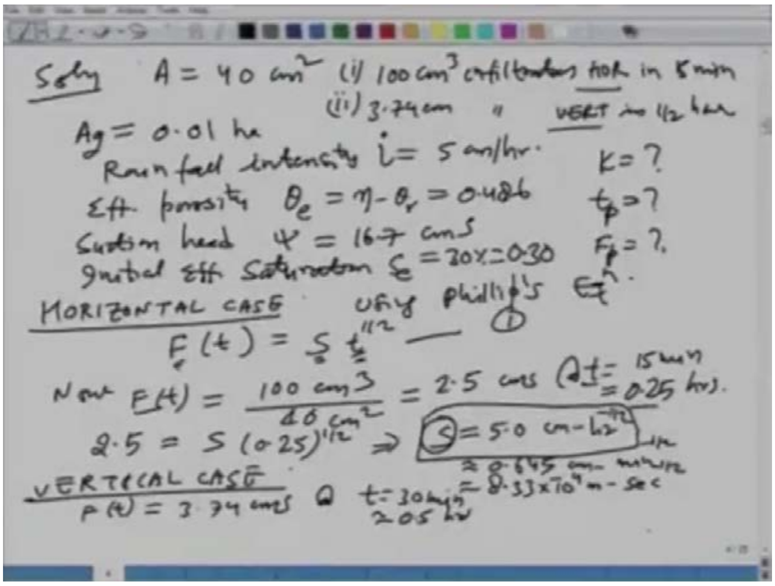

So, what we are going to do is first thing is a just list out all the data that are given to us. So, first I will write down or compiled all the data the cross sectional area of your soil sample is 40 square centimeters. In the first case, two data are given first one is the 100centimeter cube of water infiltrates for the horizontal case in 15 minutes. For the second case 3.74 centimeters of water infiltrates for the vertical case. And that happens in half an hour. Also given to us is the area of the garden I will use the subscript as g here to distinguish it from the other area this is given to us as 0.01 hectors. The rainfall intensity is given this is for the second case I as 5 centimeters per hour. We have to find what is K? What is time to pounding and what is the amount of water that is in filtrated at pounding that is F b? The other data that are given to us is the effective porosity. What is the effective porosity? This theta e which is eta minus theta r is given to as us 0.486 the suction had psi is 16.7 centimeters.

And the initial effective saturation of the soil which is denoted by S e is given to us a 30 percent or as a fraction we say that it is 0.30. As I said we will use the Philips equation with all this data that are given to us. So, first we will look at the horizontal case using the Philips equation. For the horizontal case what we have is F is equal to nothing but S t half. Now, in this equation what are the known and unknowns? You can see that in this equation we know the time we also know F how much water gets infiltrated. So, basically the first data point can be used to determine the value of S or the sorptivity. So, if we did that using the data for the horizontal case we know what is F t? It is nothing but 100 centimeter cube of water in filtrates and we divided by this cross sectional area of the sample. So, that we have the infiltrated water in the length units. So, it will be 2.5 centimeters at t is equal to 15 minutes and let us convert everything into hours so 15 minutes will be 0.25 hours.

So, we put these values and let me say that this is a equation number 1. So, I put the value of F and t in equation 1. If you do that you will have 2.5 is equal to S times time is 0.25 raise to the power half. So, that will give you should be easy to see that you can calculate the value of S as 5.0, and its units will be centimeter hour minus half as per this equation. Some of you may find this into some other units. So, I will just give you the answer it will be 0.645 centimeters minute minus half or 8.33 times 10 minus 4 meter second minus half. So, we can represent the answer in any units we want. So, I am giving you all this numbers so that you can verify this.

So, for the horizontal case, we have determined the value of S. Now, what we do is we apply the same Philips equation for the vertical case in which the gravity forces will be predominant or will be important and the K term will appear. And that is where we can find out the value of K or the hydraulic conductivity. If we do that we would have or let us first write the data vertical case, the data that are given to us is that F of t is equal to 3.74 centimeters it is already given in length units. So, we do not need to convert it and this is at t is equal to 30 minutes which is 0.5 hours.

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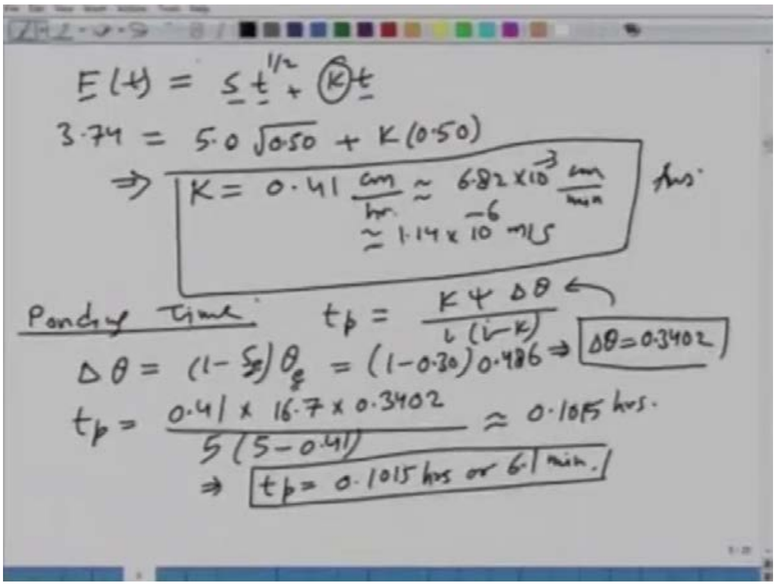

So, what you do is the Philips equation for the vertical case is has this form in which both suction forces and the gravity forces are important. Now, we put the values of S we know the value of t; we know the value of t and we also know F, only thing that is unknown is K which we can find out. So, if we put the values what you will have is 3.74 S is 5 the time for the vertical case is half an hour so on the root of 0.5 plus K times 0.50. You can solve this I am not going to go through the steps just give you the value of K that will be 0.41 centimeters per hour or it will be 6.82 times 10 minus 3 centimeters per minute or it will be 1.14 times 10 to the power minus 6 meters per seconds any of this 3 answers is fine.

So, this is your answer to the first part in which we found out the value of the hydraulic conductivity using the lab experiments or the observations which we made using the horizontal and vertical cases. Now, we go to the second part of this problem in which we need to find out what it will be the pounding time and the humilities infiltration at pounding. That should be fell easy to see you have derived the expressions for the pounding time using green ampt infiltration equation. And it was given by this K psi delta theta over pie times i minus K.

So, on the right hand side you see the K we have just found out psi is given to us the rainfall intensity is known K we have just found out. And we just need to evaluate what is delta theta, what is delta theta? Is it is a measure of initial conditions or initially how much is the saturation or how much is the moisture contact? If you go back to your notes you can see that this delta theta is given by this expression. We have derived this while looking at the effective saturation for the green ampt infiltration equation different parameters. The effective in initial effective saturation is given the effective porosity is also given. So, we just put these values so that it will be 1 minus as e is 30 percent. So, it is 0.3 times effective porosity is 0.486. So, that with that will tell you that delta theta is 0 0.3402. Put this all these values into your expression for t p and we should be able to find out what it will be the time to point?

So, if you did that you will have t p K is 0.41 times psi is 16.7 times 0 0.30402 is delta theta and everything divided by i is 5 times 5 minus 0.41. One thing I like to question all of you is that what ever expression is use we should be consistent with the use of our units rainfall infiltration are the parameters 5 etcetera. We should make sure that we are using in the same units it is not that one thing you use in centimeters per hour other thing you use in meters and insight. So, everything should be consistent so that the units would cancel out and the final answer will be hours and seconds or whatever you’re using. So, you can notice that this aspect has been taken care of. So, this answer would be in what units hours 1015 hours then you can convert it into minutes if that is desired. So, that you t p would be equal to 0.1015 hours or it will be about 6.1 minutes. So, what does that tell us that it will take about only 6 minutes for water to start pounding in the garden once the rainfall starts at that particular rate? So, that is the first part of the second component of this problem then we just need to find out.

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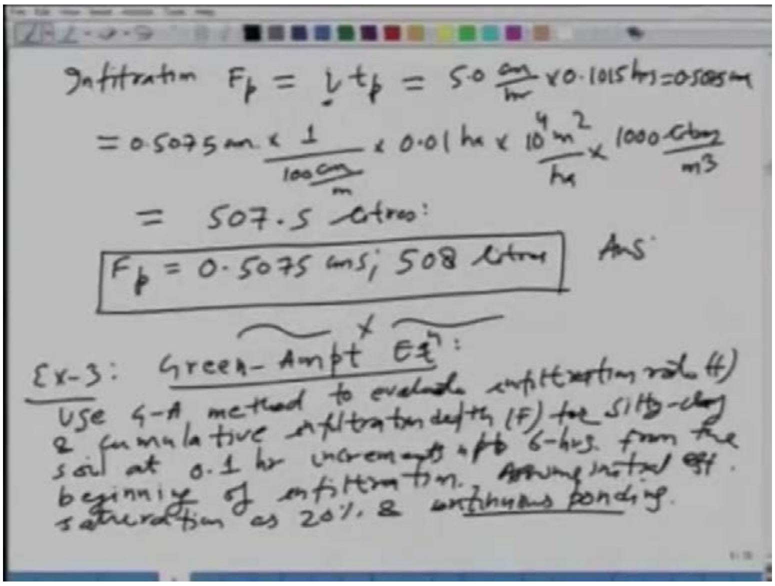

What will be the infiltration amount of water that it filtrates humiliating filtration just at pounding as you know is nothing but 5 times t p, because before the pounding this not enough supply of water available. So, whatever water falls on the ground or on the garden everything will get filtrated. So, i is the infiltration times t time to point it you can find out it will be 5 centimeters per hour times, 0.1015 hours is just what we found out, so that it will be 0.5075 centimeters. You can convert it into other suitable units so it will be 5075 centimeters multiplied by 1 over 100 centimeters per meter times area of the garden is 0.1 hectares can convert the hectares into square meters. So, that is 10 to the power of 4 square meters per hectare. Then if you want you can convert it this is in meter cube all of this you can convert it into liters so in a 1000 liters in 1 meter cube.

So, you can see all the things will cancel out, what you will have is the answer in liters 507.5 liters of water will infiltrate that t is equal to 6 point minutes So, your final answer is F t is equal to 0.5075 centimeters or approximately 508 liters. So, that is your final answer as far as the cumulative infiltration is concerned. So, you see that you can solve some practical problems using some of the equations or some of the matters we are using for in filters. So, this was the example of Philips equation. And the next example which I would like to take would be on the use of green amplification remember we had said that the green amplification equation is a little complicated as far as the solution is concerned, why because it is non-linear in nature and also it is implicit.

So, when we have any implicit equation, we do not have the expression to calculate out desired variable exclusively. So, we need to do some trail in error or some manual trail. So, I would like to demonstrate how we do that by taking one example on the green abbreviation? So, if we see that what we have is this example. So, this is my third example today that is all we will do it is on the green amplification or the use of the green amplification. First I will state the problem like I did for the previous 2 examples use the green ampt method to evaluate infiltration rate which is half and cumulative infiltration depth, accumulative infiltration depth which is capital F for silty clay type of soil at 0.1 hour in 3 minutes that is at every 0.1 hour up to 6 hours from the beginning of from the beginning of infiltration or your rainfall people. Assume the initial A factor saturation which is S e as you know as 20 percent and the conditions of continuous ponding is important. Because we had said that all these infiltrations give us the potential infiltration or they assume the continuous ponding.

So, if we have sufficient amount of water available that is ponding depth is there whatever, we calculate from green ampt equation will be equal to the actual a filtration. Under this assumption, what we need to do in this problem is calculate the small f and capital F at 0.1 hour interval into 6 hours. Obviously, we will not have time to do that, but what I would like to demonstrate is to carry out the alterations for one time tab that is 0.1 hour. So, let us see what will be involved in this. And once you understand that you can write a small computer program or you can use excel to compute this or to complete this problem.

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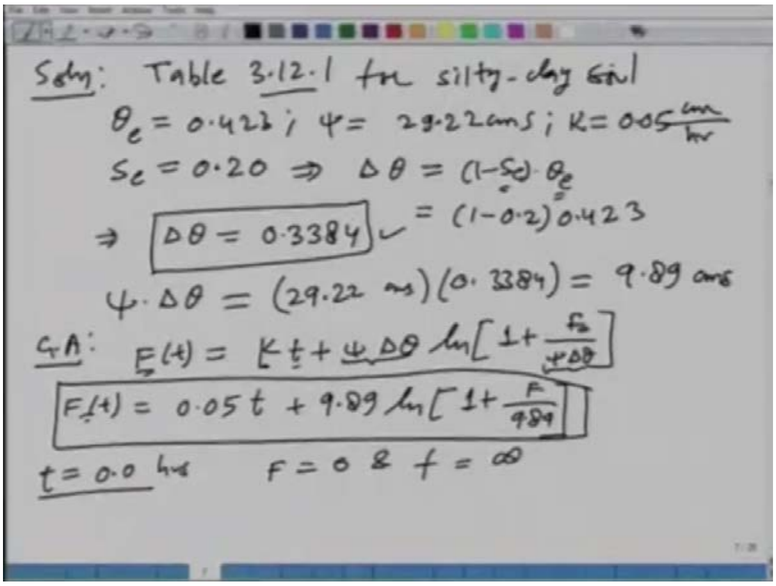

So, what we do is first of all we need to find out the data, what are the green ampt parameters? So, to be able to use any model in an equation, we need to estimate the values of the parameters of that equation. For the green ampt equation there is a table in the book this is 3.12.1. We can read out the parameters for silty clay type of soil. So, I will just give you this data directly so you can read it from this table in the Ven Ti Chows book. So, it will be effective porosity is 423, the suction had for this soil is 29.22 centimeters the hydraulic conductivity K is 0.05 centimeters per hour. What is the other data that is given to us is that the initial effective saturation S e as 20 percent. So, as a fraction it will be 0.2. So that we can calculate the initial conditions delta theta like we did in the last problem. So, it will be 1 minus S e times theta e. This is given theta is for that soil you have just written down. So, it will be equal to 1 minus 0.2 times 0.423 that will give us delta theta is equal to 0 0.3384.

So, this is we have determined the delta theta. So, once we have done that we can find out what is sin delta theta. Remember in the green ampt equation psi delta theta equation is a common expression in the natural log inside. So, we can find out what would be psi del theta psi is given to us as 29.22 centimeters multiplied by delta theta is a fraction 0.3384 we have just found out. So, that it will be 9.89 centimeters. Now, we write the green ampt equation, we have determined various parameters in the variables that are involved in this so F at anytime t by green ampt equation is given as K t plus psi delta theta times natural log of 1 plus capital F over psi delta theta. In this equation as you see what the knowns are and what are the unknowns? Well capital F is what we are trying to find out.

So, this appears on the left hand side and also on the right hand side both on the left and right inside. K is the hydraulic conductivity which is known is the time we are going to put at every 0.1 hour so that is known psi delta theta, we have just found out above. So, this is also known so everything is known in this equation then except capital F. So, we put the values we generate certain expression and then we will carry out the memory trials. Let us see how we do that. So, we will have F of t is equal to K is 0.05 and t is the time. So, let me put t here plus you have 9.89 natural log of 1 plus capital F over 9.89. This is my equation or this is the general equation which you are going to use by how do we carry out the trial and l procedure? Well what we will do is we will start or any alternative procedure starts with an initial guess. So, initial value or some assumption is made.

So we will just assume some value of infiltration F and we will put that in the equation. So, we have the left hand side F what will do is we will put that arbitrary value of F on the right hand side and evaluate this whole expression. If the 2 comes out to be equal we are fine if not we make slight adjustments in the assumed value of F. So, let us see how we are going to work on that. Well before we move at t is equal to 0 hours gets started your small f is 0 initially. There is no infiltration and small f is infinity initially this lot of these things. So, this is the trivial case.

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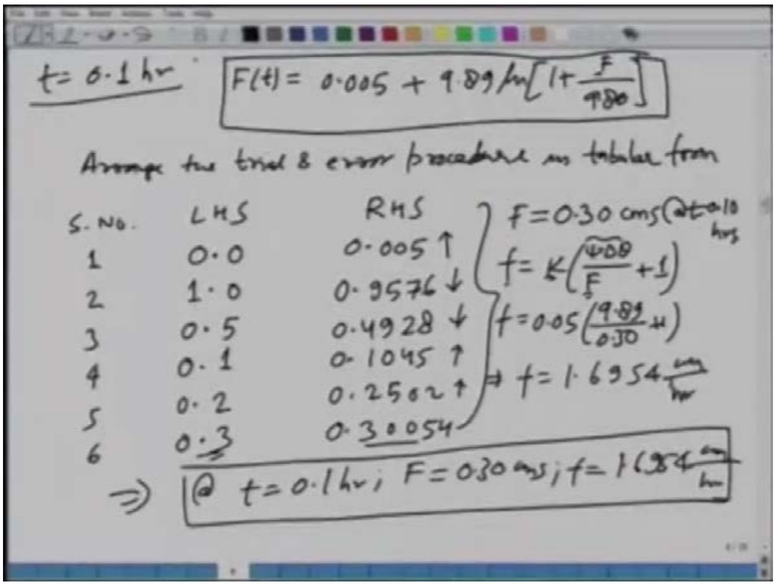

And then we look at the next case at t is equal to 0.1 hour once we put the value of t as 0.1 hour our equation will become F is equal to 0.05 times 0.1. So it will be 0.2 inside plus 9.89 is the psi delta theta to a log of 1 plus capital F over 98 phi. So, this is the equation you are going to use a balance out it is always a good practice to arrange all your trial and error alteration in a tabular form if you start writing the things then it will become very combustion and very long right. So, what we are going to do is arrange the trial and error procedure, this will be very helpful for all of you during your examinations and quizzes etcetera in tabular form that is to say you will have the serial number which will start from let us say 1 2 3 4 5 6 and so on. This depends upon how many trials you need then you have the left hand side then you are going to calculate the right hand side right. And then depending upon the relative magnitudes of the left hand side and right hand side you will judiciously keep changing your guess for F.

So let us get started with initial value as 0.0 to the start with right you could use any other values also. Once we put capital F is equal to 0 on the right hand side for this guy here right and calculates everything the right hand side will be 0.005. So right hand side is higher than the left hand side. So, what we need to do is we need to change the value of our initial values and it should be easy to see that the infiltration will have some value, although we started with 0.0 we could start with 0.1 or any other value. So, what we need to do is I will use certain notation which will be helpful. So, that it means to increase the value of F this arrow are up with which is pointing upwards which means that you need to increase the value. So, let us say that next time I use the value is 1.0 centimeters. If you do that this is your left hand side 1.0. And you put the value of F on the right hand side the value that will come out is 0.9576.

So, now you see that right hand side is smaller earlier the right hand side was higher than the left hand side. Now, we notice that the right hand side is smaller than the left hand side. So, what do we do we need to decrease the value you see that there is a pattern. So, this is our knowledge we will use in which direction we need to go. Now, I will put an arrow which is pointing downwards. So, whatever the value of F we had assumed was not right. So, we decreased it. So, let me go to 0.5 and then put this value of 0.5 on the right hand side. And then see what you get it will be 0.4928. Now, can you tell me what I need to do? The right hand side is slightly smaller than the left hand side and if that is the case what we had done in the first trial.

So if you do that we need to increase the value or decrease the value and let us say you have 0.1. So, if your right hand side is smaller then you need to decrease it. So, with 0.1 put the value as they do this quickly you will have 1045. So, you need to increase it then you have say 0.2 it will be 0.2502. And then you need to increase it further should be easy to see and then finally, if you keep it as 0 0.3 then you will have the right hand side as 0.0320054. So, you see that we can keep on doing these alterations and then we can achieve certain level of accuracy that is desired. So, here you see that our guess was 0 0.3 and which is the left hand side and the answer which has come up on the right hand side is 30054. So, we say that you know this is sufficiently are accurate.

So we can stop otherwise we can keep on carrying out these alteration. So, if we did that then you say your final answer capital F is equal to 0.30 centimeters at 3 is equal to 0.10 hours. So, what we have done is actually we have found out the value of accumulative infiltration at 3 is equal to 0.1 hour. Now, what we need to do is we need to determine the value of small f what is that there is relation as per the green ampt equation which is given by this. So, capital F we have just found out that psi delta theta we had already found out K is known. So, we can find out what is small f? So, at 0.1 hours we will have your F equal to 0.05 times 9.89 divided by 0.30 plus 1. Once you solve it you have F approximately 1.6954 centimeter per hour.

So therefore, at t is equal to 0.1 hour we have capital F as 0.30 centimeters and small F as 1.6954 centimeters per hour. So, these are the values at t is equal to 0.1 hour. You can see that we can continue in this fashion and find out the capital F and small F at other different times. This should be very easy to see that you can solve this problem using excel or you can write your own small computer program in which you can carry out these alterations or you can use some other alternative methods and Newton rapthon's method are the very important and very quick way of determining the roots of an non linear equation. So, here we can find out the value of capital F using the Newton rapthon's method if you would like the one more example I would like to just take or discuss.

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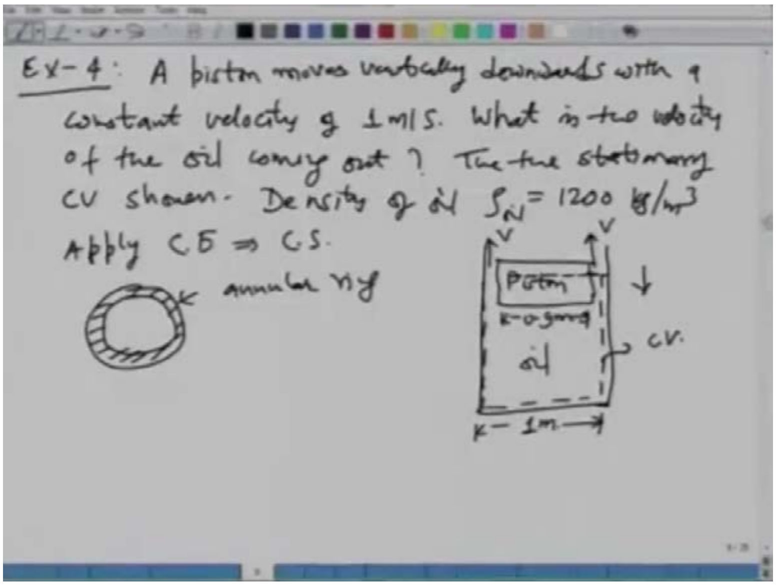

I may not have time actually to complete it, but I would like to just give it to you. So that you can look at, how we are going to or how you can solve it. So, I will just give you problem statements for this and then you see if you are able to solve it, it is about the application of the Renaults problems theorem.

So, you have a case of a piston which moves vertically downwards with a constant velocity of 1 meter per second. I will give you the figure in a second what you have to do is find out what is the velocity of the oil coming out? Take the stationary CV that is shown. I will give you the figure in a second and the density of oil row oil is given to you as 1200 kg per meter cube. So, this is the problem. So, if you look at symmetric of this problem, you have a cylindrical vessel in which there is a piston that is moving vertically downwards. This is our piston. The dimensions that are given to you are that this is diameter is 1 meter of the cylinder. And the piston is let me write it here inside it is 0.9 meters, the diameter of the piston is 0.9; the diameter of the cylinder is 1 meter and there is oil in this. And then this is moving down piston is moving down what it is causing is that there is oil that will come out with the velocity of V.

So, what we do is you take the control volume right next to the cylinder right like this. So, this is your CV that is short lived. So, what you do is you apply the continuity equation in which you will have the control surface, control surface will be this ring; this angular area from where the water will be coming out. So, we are looking vertically into it. So, this is angular ring from where what this water velocity V the oil is coming out with certain velocity. I will not solve this problem as I said this is like a homework for you. You work on this problem, this is slightly complicated and if you have any questions you can write to me at the email address or you can contact me. So, you take this as a homework assignment and I would like to stop at this point. Basically what we have done today is looked at these 3 problems which we have solved. The forth one is something for you as a challenging point. In the next class, we will start a new chapter on atmospheric hydrology in which we will look at how the rain is formed and different forms of rainfall evaporation and all those things. So, I would like to stop at this point.