Advanced Hydrology
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Lecture - 8

Good morning, and welcome to the lecture number 8 of this video course on Advanced Hydrology. In the last class, we looked at the derivation of the Green Ampt equation, which is the infiltration equation. And what I would like to do today is just go over it quickly, so that we understand the basic steps that the that are involved in the derivation.

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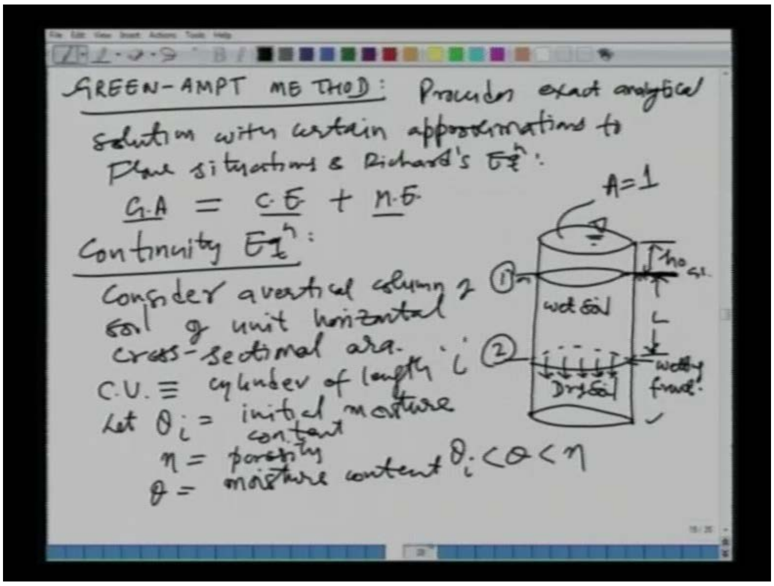

So, we come to the tabled here, we are looking at the Green Ampt method, and what we said is it provides an exact analytical solution with certain approximations to flow situations, and Richard’s equation. The Green Ampt equation is a combination of continuity and momentum equation, and how we derived it was that we first looked at the simplified soil moisture profile. And then we took a cylindrical volumetric element, and then we applied the continuity and momentum equation to that.

So, if we look here you see that, this is the cylindrical element, the length of this wetting front is L and it is cross sectional area of the cylinder is A is equal to 1, as the rain is falling water is infiltrating into the ground, and the depth of the wetting front is increasing. Then we define various parameters that are involved, I would not like to go over them again.

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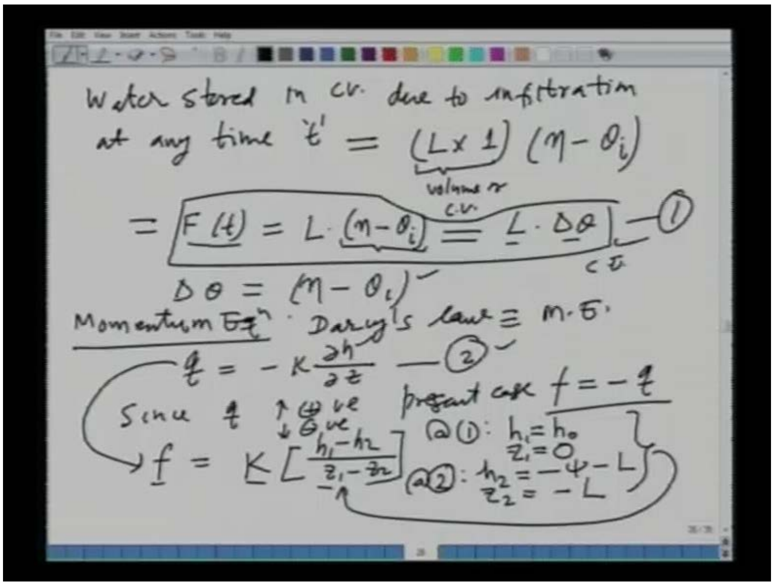

And then we applied the continuity equation in which we said that the cumulative infiltration at any given time is nothing but L times delta theta. Where delta theta is this quantity eta minus theta i, and we had seen earlier in the first few classes that the momentum equation for this kind of situation is nothing but the Darcy's law. So, we started with the Darcy's law which is equation number 2, and then we utilized the concept of various forces that are acting in this particular case, which were section forces and the gravity forces.

So, corresponding to that this energy h we said is the sum of two things, and then we derived the expression for small f using this, considering two cross sectional areas of 1 and 2. 1 is at the ground which is the datum, and 2 is at the depth of the wetting front.

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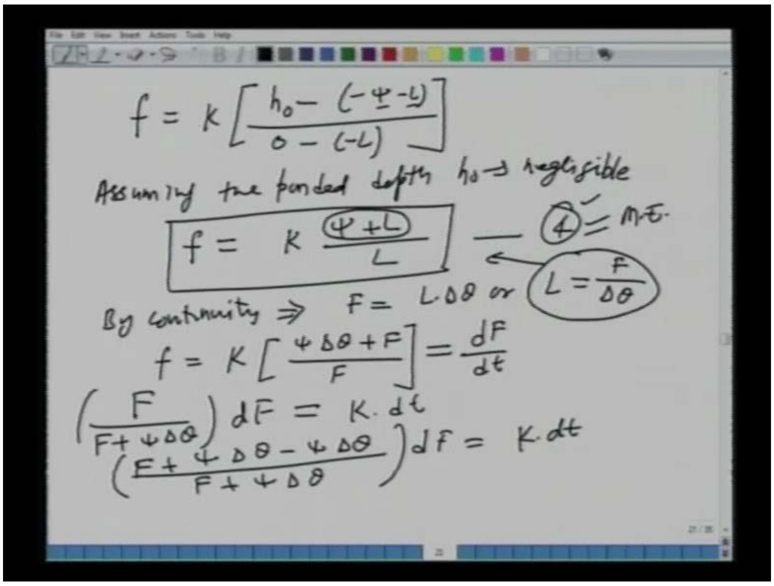

Then we continued further, the head h we sad is the sum of these two things as I said earlier, and once we derived this equation, this is called the momentum equation, equation number 4 which we found out. So, now what we did was we combined this momentum equation with the continuity equation, which is represented by this essentially.

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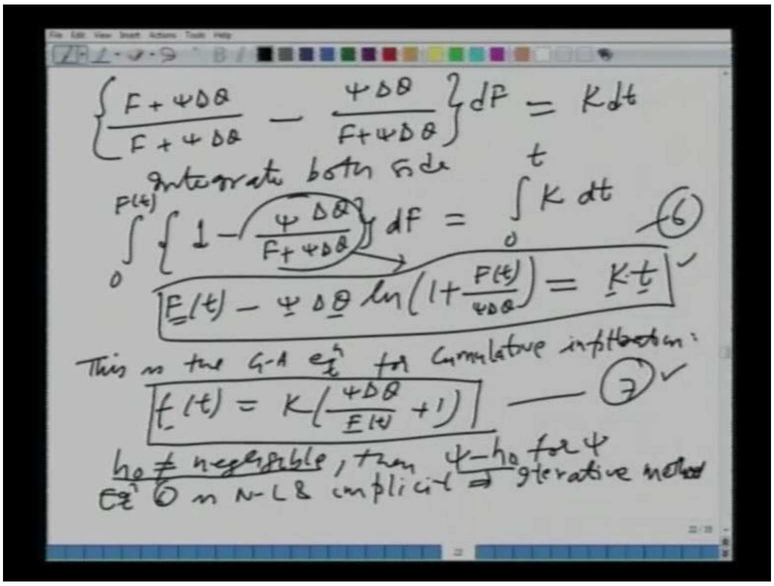

And we put it all that and then separate the variables perform the integration, and the final equation which we get comes out in this form or rather in this form, equation number 6. So, equation number 6, we said is gives you the cumulative infiltration as a function of psi, delta, theta and delta theta involves the soil properties that is porosity, and the initial moisture content, and the hydraulic conductivity and as a function of time. So, you see that, this equation 6 is an implicit equation and it is non-linear in nature, so we need some kind of iterative technique to solve it. We would like to take an example may be in the next class, on how to find the cumulative infiltration given the Green Ampt parameters.

One of the advantages of the Green Ampt equation is that, it gives a relationship between small f and capital F that is infiltration rate and the cumulative infiltration. So, capital F here is the cumulative infiltration, and small f is the infiltration rate, this we had derived already. Other thing we said is, one of the assumption was that the pounding depth h naught, we took was negligible, in deriving this equations 6 and 7.

However, if that is not the case, then all we need to do is instead of psi, we just take psi minus h naught; and we replace these psi by psi minus h naught and all these equations, and we will be fine with that.

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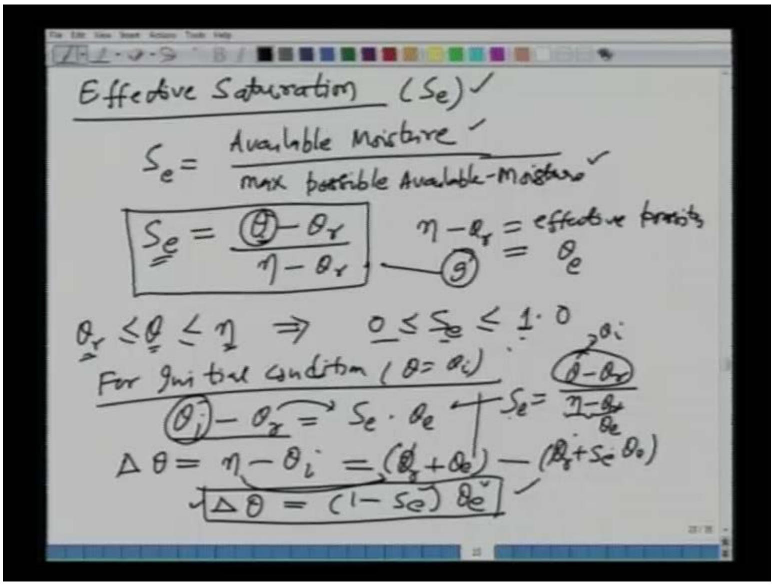

And then what we did is, we defined another parameter called the effective saturation, which is denoted as S e, and this S e was defined as the available moisture, the ratio of the available moisture divided by the maximum possible available moisture. And then we found out and expression for S e for the initial conditions in a soil, for soil moisture theta the minimum and the maximum limits are theta r which is the residual moisture content, and the eta which is the maximum soil moisture content equal to the porosity.

Corresponding to those two limits, the S e would vary between 0 and 1, so this is a convenient way of representing the percentage of saturation or effective saturation in a soil, when we are analyzing the infiltration. And then we wrote down an equation, in terms of delta theta the effective saturation and the effective porosity, because theta r is something which which cannot be replaced just by gravity forces. So, for all practical purposes, if we have the value of theta r, this is the equation which normally should be used, because it accounts for theta r.

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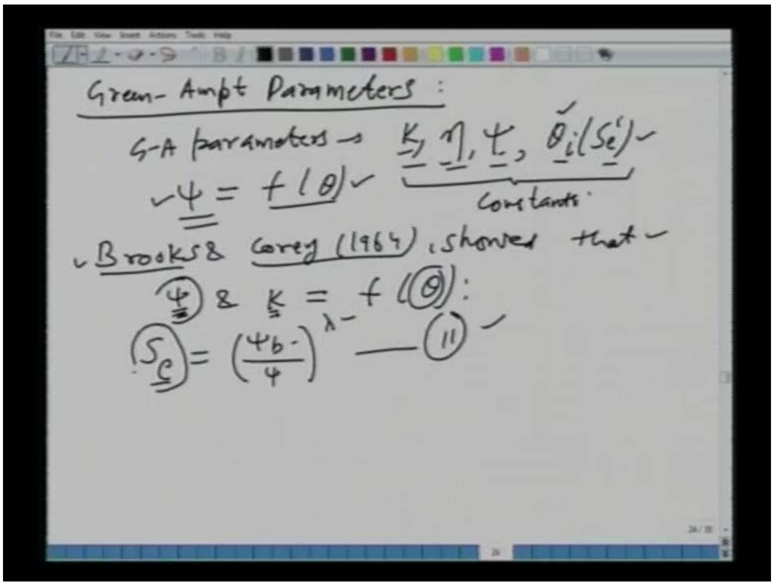

Then we moved on, and we started looking at the Green Ampt parameters, any mathematical model we have it involves various parameters which need to be calibrated, or the values of which need to be found out. Similarly, for the Green Ampt parameters we said there are three or four calibration parameters which are involved in a, first one is the hydraulic conductivity K, second one is the porosity of the soil, and psi is the suction head, all these are properties of the soil.

And then this theta i is nothing but the initial moisture conditions of the catchment, and all of these are normally assumed to be constant. So, whenever we apply Green Ampt equation to find the infiltration during a rainfall event, we assume these quantities or these calibrated parameters to be constant. Many people have conducted a lot of experiments to find out the values of these parameters, and in the book Chaos book you will find a table, which gives you the value of these values of these parameters, for different type of soils that is clay, sandy soil and and so on; for different types of soil these values are given which are constants.

However, many researchers have carried out some research or you know additional work, in which they have found out that some of these parameters are not constant during rainfall event. As the rain starts to fall, if the soil moisture is you know 0, the soil is completely dry, then the permeability or the suction head or relativity or the relative force which are acting will be different; and depending upon that the values of theseparameters will be different.

So, we said that this suction head, some people have found out that, this varies as a function of theta or as a function of time during a rainfall event. One of the relationships which we will look today is called the Brooks and Corey relationship, it was established in 1964, and they showed that both psi and k are function of theta. And the equation which they gave can be written like this, your effective saturation S e is equal to psi b over psi raise to the power lambda. And I am going to number this equation as equation number 11.

If you look at this equation, I would like to draw you attention to this equation 11, we are trying to find out a relation between psi and theta, that is to say the suction head as a function of soil moisture content. And this equation is in terms of S e, some psi b, some lambda and psi, so where is theta involved in this as we all know theta is involved in this effective saturation.

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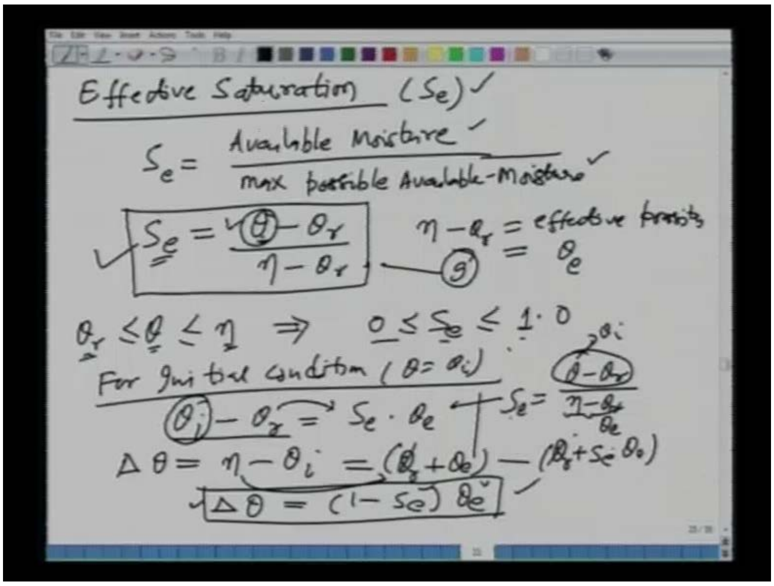

If you remember we had just defined, what is the effective saturation, this effective saturation is nothing but it involves this theta minus theta r over eta minus theta r.

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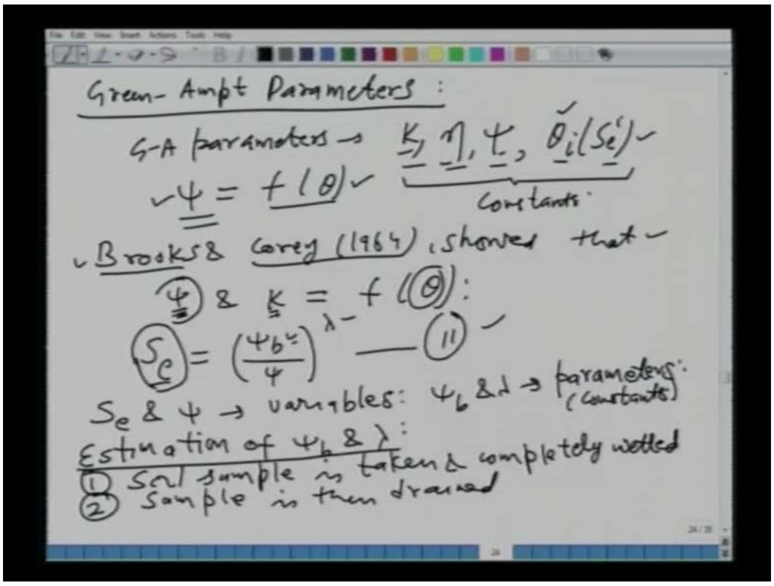

So, this is the equation and any equation which we have, we need to calibrate or we need to find out the parameters. So, in this equation what are the parameters, and what are the variables as it is easy to see, that S e and psi are your variables, and which relate basically your section with the theta, and your psi b and lambda are the parameters of this Brooks and Corey relationship, which are constants.

Now, how do we find these values, before we can establish a relationship between psi and theta, we need to be able to find this both psi b and lambda which are the constants in this equation. So, what they have done is they have given a procedure, a step by step procedure of finding out the values of these two parameters. So, what we will do is, we will look at this procedure of estimation of psi b and lambda which are Brooks and Corey parameters.

So, I will give it to you in a step by step procedure, what is done is a soil sample is taken from the field that is (Refer Slide Time: 11:14). And it is completely wetted that is completely drenched with water, and then what we do is the sample is now drained. Drained means, water is allowed to drain or pass through the sample or infiltrate through the sample; and then what we do is we take various measurement, while the water is getting infiltrated through this soil sample.

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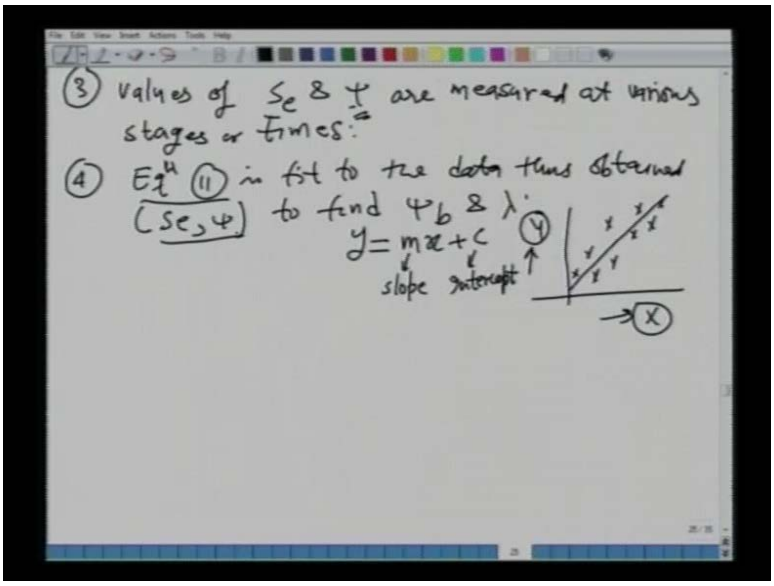

So, in the 3rd step, the values of S e and psi, the suction head are measured at various times or stages, how do we do that, S e as we know is nothing but the soil moisture content. So, we can measure the soil moisture content at any given time, in the field in the sample, how do we measure psi which is in the suction head well, we will not go into the details. But, we have very sophisticated instruments, which are called the (()) meters which can measure the suction head or the power of the soil to suck the water at different stages.

So, assuming that we can measure the soil moisture and the suction head at different times, we take the sample completely vatic, we allow it to drain the allow the water to drain, and then we measure the quantities of your S e and psi. So, what we have is the data in terms of the soil moisture content and the suction head, and then what we do is the equation 11 which we had just seen is fit to this data set. The data which we have just obtained which is in terms of S e and psi, to find the parameters psi b and lambda, how do we do that, or how do we find out the parameters of a two dimensional problem.

Where you have x and y, you know 2 only 2 variables involved, should be easy to say, if you have data or 2 variables x and y, so x is the independent variable and y depends upon the x. So, what we do is, let us say we have taken certain observations of x and y, let us say they look like or you just plot them. If you want to establish a relationship between these observed data, we look at the pattern, if it appears linear then we try to fit a line, or or a linear curve if it appears a non-linear curve, then we try to fit a non-linear curve.

So, let us say that if it is a, it appears like a straight line, then you just fit a straight line through them, graphically it is easy to understand, but mathematically also there are equations which we can use to find out the parameters of this line, or this model. What are those parameters it is nothing but the equation of the line is y is equal to m x plus c as you all know, and the parameters are m and c, what is m, m is the slope of the line and c is the intercept.

So, once we have found the value of m and c, we have determined the equation of the line, or we can find out the value of y given the value of x. In this particular case we are trying to fit equation 11 to data S e and psi.

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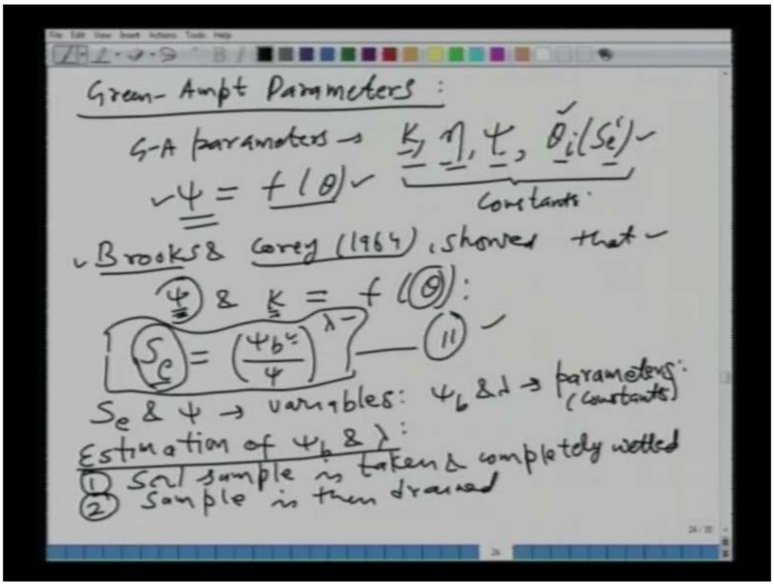

So, if we go back to your equation number 11 that is what it looks like, so how do you fit a straight line to this, this is a non-linear relationship or this is a non-linear equation. And as you all know, we can convert a non-linear problem into a linear problem.

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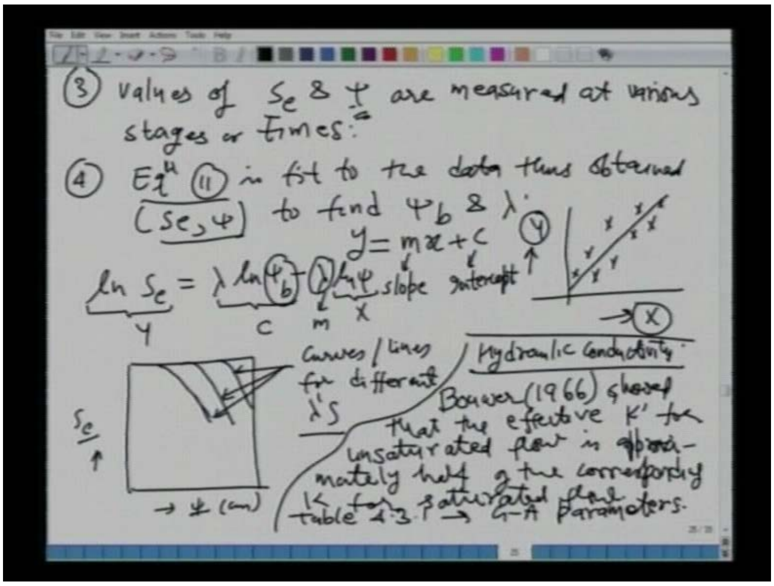

So, how do we do that, we can take log on either side and reduce this equation like this, log of S e is equal to lambda times natural log of psi b minus (No audio from 16:31 to 16:38), this whole thing let us say is your capital Y. And this whole quantity is your C and psi is your x, let us say let us say this is your x and so then this becomes your m. So, we have reduce this equation in the form of y is equal to m x plus c, we have the data in terms of S e and psi. So, what we can do is, we can either plot a or find the value of slope and intercept using the formulas which we have, and I do not want to go into the details of those.

So, that way we will be able to determine what the parameters which are, what are the parameters, one of them is the lambda and other will be the psi, we should be able to find it out, once we know the value of m and c. If we try to look at this relationship, it looks like this psi in centimeters as a function of soil moisture content, or in the form of effective saturation S e, I am not going toput the numbers here, you can you know see it in the book and it looks like this. It will appear as a straight line, and these different curves you see correspond to different value of lambdas or this curves or lines for different lambdas.

So, once we have established this relationship, we can find out what is psi as a function of S e or theta, once we have that we can use this relationship in the Green Ampt equation as a function of theta. During a rainfall event as the time elapses, the soil moisture content changesin the soil, and we can use the modified values of S e and psi. So, this was about the psi, the other parameter of the Green Ampt equation is the permeability or the hydraulic conductivity.

We will not go too much into the details, and except that, like that many researchers have worked on trying to find out, how the hydraulic conductivity changes during a rainfall event in the unsaturated zone. And one of them is Bouwer in 1966, showed that the effective hydraulic conductivity or effective k for unsaturated flow for unsaturated flow is approximately half of the corresponding k for saturated flow.

So, as far as this course is concerned, we are just going to say that the hydraulic conductivity in the unsaturated zone is approximately half of the saturated hydraulic conductivity that is what the research has shown. And the relationship of this hydraulic conductivity as a function of time or as a function of theta is beyond the scope of this course, although there are many you know studies that are available.

Table 4.3.1 in your chaos book gives you the values of various Green Ampt parameters, for different types of soils that is you know sand, and clay, and gravel and so on. So, this is about the Green Ampt equation, it is derivation and various types of parameters.

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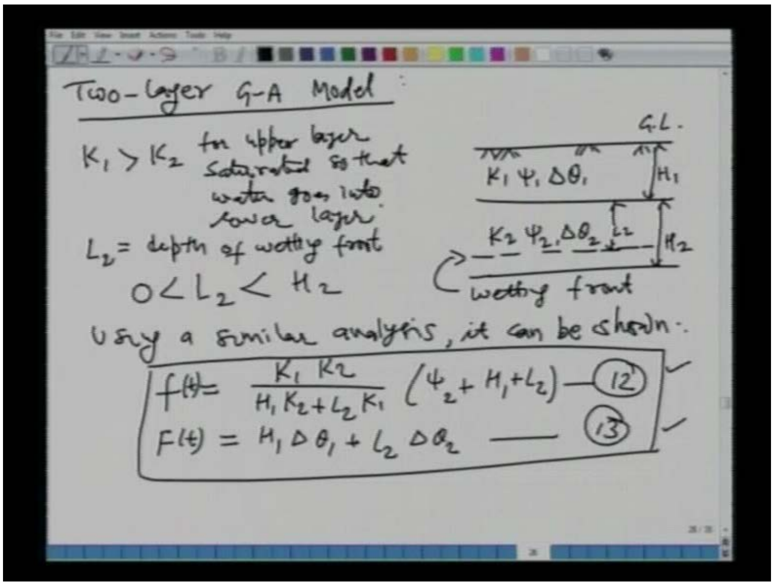

The next thing which we are going to look at is, the application of the Green Ampt equation when you have soil which consists of more than one type of soil, or soil stratum consist of more than one layer. This is called the two layer Green Ampt model two layer Green Ampt model. So, if we look at the field situation, this is your ground level, then you have two different types of soils, the first one close to the ground is let us say, consists of some soil which has hydraulic conductivity K 1, suction head psi 1, initial moisture content represented by delta theta 1.

And it is depth or height is let us say H 1, below that we have another soil which is K 2 is the hydraulic conductivity which has a different soil suction had, and initial condition there may be different, and it is height is let us say H 2. Now, we will look at this two layer Green Ampt model under certain assumptions, and one of them is that K 1 is greater than K 2 that is to say for the upper layer is saturated, so that water goes into the lower layer.

And also we define the certain length L 2 which is from this interface, L 2 this dash line and what is L 2, L 2 is the depth of the wetting front. Depth of wetting front means what the upper layer is completely saturated that is to say, the wetting front has completely penetrated the first layer, and now it has gone into the second layer. And L 2 is the depth of that wetting front from the interface from these two types of soils. So, in other words we say that the limits of this L 2 are such that, it is less than H 2 but greater than 0, so that it is in the second layer; and this let us say, let me define this as the wetting front.

Now, with this conceptual or with this, we can do is we can write down the continuity equation, momentum equation and combine both of them, we can carry out a very similar analysis which we did for the single. We are not going to do that, we will write this equation directly the final resulting equation. What we will say is that using our similar analysis, or method which we had described for a single layer. It can be shown that your small f which is the rate of infiltration is going to be equal to k 1, k 2 over H 1 K 2 plus L 2 K 1 times psi 2 plus H 1 plus L 2, this is your equation number 12.

And once we have found small f, we can find out capital F and remember that all of these are function of time, we are trying to find the rate of infiltration and the cumulative infiltration as a function of time. This is equal to H 1 delta theta 1 plus L 2 of your delta theta 2, and this is your equation number 13, so let me put these in the box both of these equation, which represent the Green Ampt equation for a two layer model.

So, if we have a field condition in which there are more than onedifferent type of soil, then still we can use the Green Ampt equations or Green Green Ampt method to find out the infiltration, and we can we can use this equation 12 and 13. You can easily see that, it should be very easy to extend this concept, if we have more than two different types of soils or three layers or four layers or you know may be more than that.

Similar analysis can be carried out, and we can find out the equation or we can extend this analysis to three layer or four layer. What do we do if there is only one layer, then we just need to consider what we had just derivedfor a single layer.

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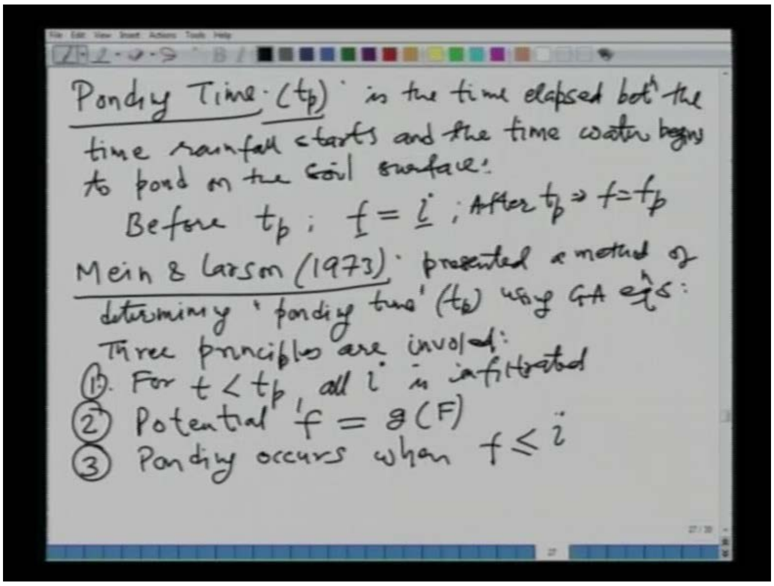

Now, we will move on further, and look at a very important concept, when we talk about the Green Ampt equation, and it is the concept of what is called ponding time. I think I said earlier in one of the classes that, any infiltration equation you take, whether it is Horton’s equation or it is Green Ampt equation, they all give you potential infiltration. What does that mean, that means that there is sufficient amount of water available or there is a ponding depth on the ground, water is ponded on the ground to certain depth and lot of water is available for infiltration. Under those conditions whatever we obtained using these equations is equal to your actual infiltration.

So, one of the basic assumptions of this Green Ampt equation is that, there is ponding available, knowing this now when does the water start to pond, initially let us say the catchment is dry, rain starts to fall the time up to which, the water will start ponding will be different for different types of catchments. Or it will be different in the same catchment for different types of soil, for different types of climatic conditions, rain fall patterns and so on. So, it is very important to determine or find what is this ponding time, so that we can use this equation after the ponding has occurred.

So, let us first define what the ponding time is, it is denoted as t p and the ponding time is the time elapsed ponding time is the time elapsed between the time rainfall starts or begins, and the time water begins to pond on the soil. So, what does it mean let us say, the rain starts at 8 AM or 8 o’clock in the morning, and the water starts to pond let us say at 8.20 AM, so you have 20 minute durations an in which the water completely penetrates or infiltrates into the soil, it will take the all of the water. And then water will start ponding on the soil surface on or on your garden or in the lawns, so that is called the ponding time and it is important to estimate the value of this ponding time.

So, what we will do is, we will try to estimate this ponding time using certain procedure, but before we we go for the derivation of this ponding time, it is important to understand that, before the ponding. What is f, f is nothing but your I whatever rain is falling that is infiltrating into the ground, therefore this i is equal to f, and after t p or after the ponding has occurred, your f is equal to what, v maximum possible or the potential infiltration.

So, calculation of this ponding time is a extremely complexes process, there are various researchers who have worked on this, and try to give the expressions. What we will do in this course is, we will look at one method which is proposed by Mein and Larson and it uses the Green Ampt equation. The advantage of this is it uses the Green Ampt equation, which is a exact analytical solution of simplified version of the schematic.

So, let us look at the Mein and Larson's procedure, which was given in 1973 what they did is they presented a method of determining the ponding time, which is denoted as t p, using the Green Ampt equation. How they did it they used three different important concepts, some of them we have already looked at, so they used three principles in finding out what will be the ponding time. One of them we have actually already written, but let me do that again for the time less than the ponding time all rain fall gets or is infiltrated, it should be very easy to understand.

And see we have a mention this many times; number 2 is that the potential f has some relationship with capital F that is to say, you need an equation or expression which relates the infiltration rate with the cumulative infiltration, and we have that for the green Ampt equation. So, and we are find there, and then the 3rd step is that ponding occurs when your f is less than or equal to i, what does that mean ponding will occur when therate of rainfall becomes higher than the infiltration rate. Then the when the rain is falling at a higher rate, then what the soil can absorb that is when the water will start to pond.

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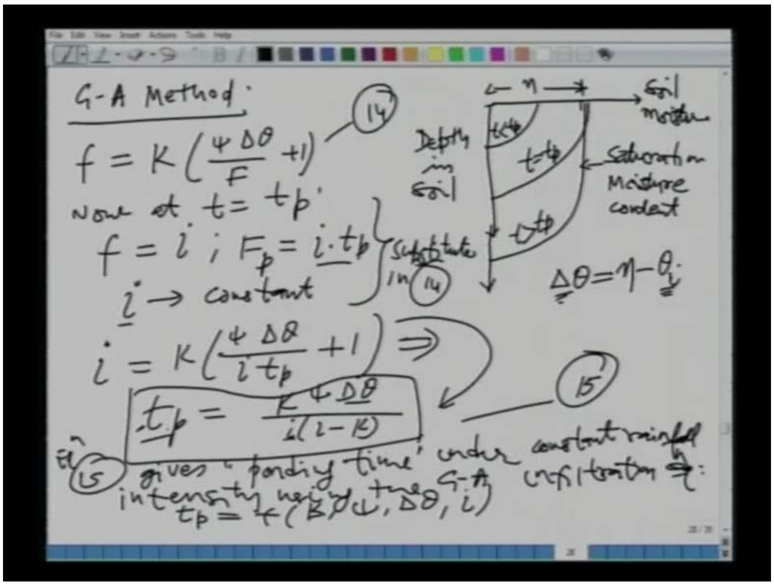

So, let us look at this concept and before I move further, I would like draw a sketch which gives the moisture content as a function of the depth. So, this is your depth in the soil vertically downward direction and this is your soil moisture content. So, if I draw three curves during the rainfall event at any three different times, and if I tell you that these three curves correspond to three different times.

So, on the x axis you have the soil moisture and on the y axis in the downward direction it is the depth, as you go down into the ground, the moisture content will be different. And we have taken three snap shots at three different times, and if I ask you one is before the ponding occurs, one is exactly at the ponding, and the third one is at the time which is after the ponding. So, which curve will correspond to which one, so if we look here, there are a three curves, the moisture content this less than or this first curve this one, the moisture content is less than the maximum which is let us say eta is the maximum.

So obviously on this curve the time is les than t p, because right at the ponding you have saturated conditions close to the ground, so just when the ponding has occurred that is t is equal to t p. Then on the ground you have saturated conditions, and as you go down the soil moisture reduces. And the third curve which you see here, (Refer Slide Time: 37:45) this is for the time greater than t p, after the ponding has occurred what happens, the wetting front starts to move down or the saturated zone starts to increase by certain length. So, this one I can write is the saturation, moisture content which is equal to eta.

So, coming back to this derivation of your ponding time, we are going to do this using the Green Ampt method or the Green Ampt equation of infiltration. According to the Green Ampt method we have seen, the relationship between infiltration rate, and the cumulative infiltration is that small f is equal to k times of your psi delta theta divided f plus 1, and let me number this equation as equation number 14. Now, what we are going to do is, we will write this equation just at t is equal to t p, so at t is equal to t p, if we write this equation, what is equal to small f, what is small f equal to.

I think we have justwritten it in the last sheet, just at the time to ponding your, I have just before that what is f, f is nothing but the rate at which rain is falling right. So, we can say that at t is equal to t p your f is equal to nothing but the rainfall intensity, and what is your F p going to be, if we know this that the, just at that time is equal to time to ponding, your f is equal to y then what will be the cumulative infiltration at t is equal to t p.

As you can see it should be nothing but your i which is the rate at which rain is falling times t p right, intensity is centimeters per hour and let us say t p is in hours, so the cumulative infiltration will be nothing but i times t p. Now, one thing we should understand here is that, during this whole analysis we are assuming that the rate at which rain is falling or i is constant. During that time interval for an hour, 2 hours or whatever, the i is constant, if i is not constant then this whole analysis will not be valid, so under this constant rainfall intensity assumption, we are deriving this.

So, now what we do is, we just substitute these things in equation number 14, if we did that then left hand side is f which is, we said is i is equal to k times psi delta theta divided by capital F, which is at t is equal to t p is F p and which we have said is i times t p plus 1. So, this is the simple equation, and we can find out what will be the explicit expression for t p, so your t p will come out to be from this equation as k psi delta theta divided by i times i minus k.

I am not going to derive that; it should be very easy for you all see this, to come from this equation to this equation. You can take this as an exercise and verify this, and let me say that this is my equation number 15. So, this equation 15 gives, that is your equation 15 gives the ponding time under constant rain fall intensity using the Green Ampt infiltration equation, looking at equation number 15 it should be very easy for you to see that, your t p is function of your hydraulic conductivity k, psi is the suction head, delta theta is the initial conditions and i. If the hydraulic conductivity is more, if you are comparing let us say two different catchments, one has lower hydraulic conductivity than the other, which one do you think will have more ponding time, or which catchment will take more time to pond as compare to the other one.

Obviously higher is the value of k, higher will be the ponding time, because water will infiltrate quickly into the catchment, and t p is directly proportional to k. Similarly, higher is psior the suction head higher will be the ponding time, because psi is appearing in numerator, what about delta theta, if the initial conditions are such that, the catchment is already saturated. If the catchment is saturated means what let us say, your what is delta theta you want to look at, it is eta minus theta i, if theta i is higher delta theta will be lower, for higher theta i delta theta will be lower, so that your t p will be lower.

So, because of the negative sign it is an inverse relationship, higher theta i lower t p, lower theta i higher t p that should be easy to see that. What about the rainfall intensity, if you consider a situation in a catchment when rain is falling at a very high rate as compare to other case or other time, when the rain is falling at a smaller rate. In which case you will have higher ponding time, a obviously one which has slower rain fall higher the rate of rain fall, quickly the water will pond and time to ponding will be small. So, it is easy to see that the equation which we have derived intuitively next sense.

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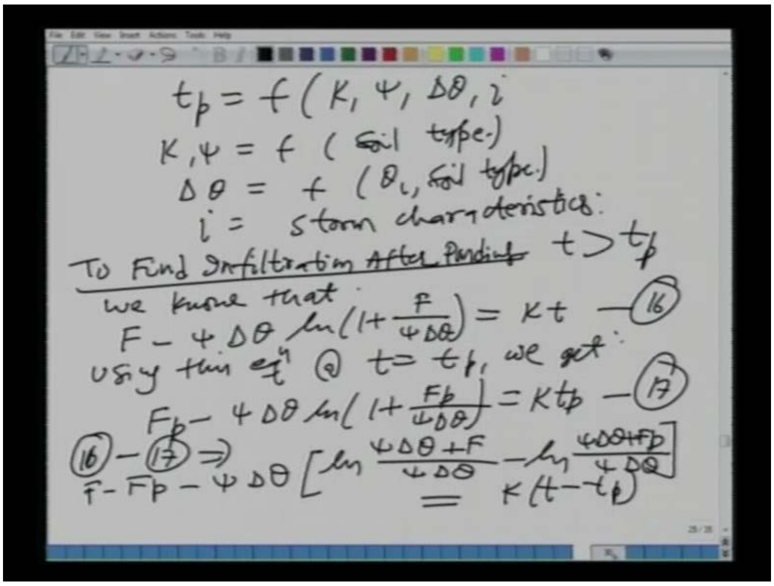

So, let us move on further from here, and say that you already said that in the previous page that, your time to ponding is function of many parameters which is k, psi, delta theta and i all these four things. Wherein your k and psi are the function of soil type, so the time to ponding depends upon the type of soil in the catchment, what is delta theta, delta theta is the function of your theta I, and also the soil type. So, it will depend upon the initial conditions, and i of course, is the storm characteristics, so you see that the time ponding will depend upon all these things that is soil type, the initial conditions and also the storm characteristics.

Now, the next thing which we are going to look at, is how do we find infiltration after ponding that is to find infiltration after ponding, that is for the time and time ignore then t p well. We know that from the Green Ampt equation, this is true, this relationship, let me number this as 16 this is nothing but our Green Ampt equation for single layer. Now, what we do is we can write this equation at t is equal to t p that is to say, using this equation at t is equal to t p what do we get, at t is equal to t p your F is F p minus psi delta theta natural log of 1 plus F p over psi delta thetais equal to k times what, t is t p.

So, if I say that this is number 17 and then I subtract 17 from 16 that is 16 minus 17 will give me what, will give me F minus F p minus psi delta theta, and in the brackets you have natural log of psi delta theta plus F over psi delta theta one part. And if you subtract the other one you will have natural log of psi delta theta plus F p over psi delta theta, so that is this is on the left hand side, and on the right hand side you will have k timest minus t p.

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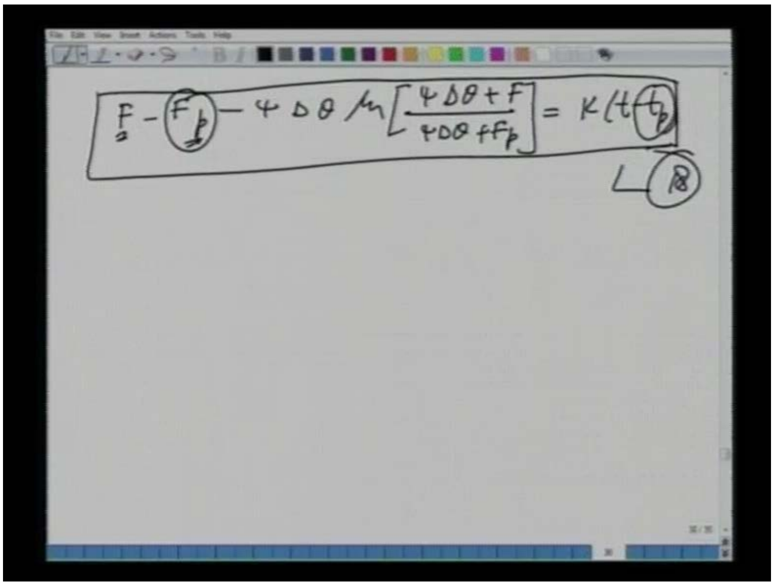

You can simplify this further to obtain F minus F p minus psi delta theta of your natural log of psi delta theta plus F over psi delta theta plus F p is equal to k times t minus t p. So, this is your final form of Green Ampt, equation which can be used after ponding, let me number this as 18, so equation 18 gives you theinfiltration F at any time after ponding. It involves F p, F p is nothing but i times t p, once we have determine what is time to ponding t p, using this equation which you have just seen, we can find out what is F p, we know what is t p.

Everything else in this equation is then know except capital F, and we can find out capital F using certain numerical iterative technique, once we have done that we can find out small f, so this is the procedure for finding the or using the Green Ampt equation rather. I think I would like to stop at this point of time, because I am at a good stopping point, and in the next class what we are going to do is we will look at a few examples of infiltration or the use of the infiltration equations.

So, we will look at the example of the Green Ampt or may be the Horton or some practical problems, in which we will use these infiltration equation to solve certain problems.

Thank you.