Advanced Hydrology
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Lecture – 7

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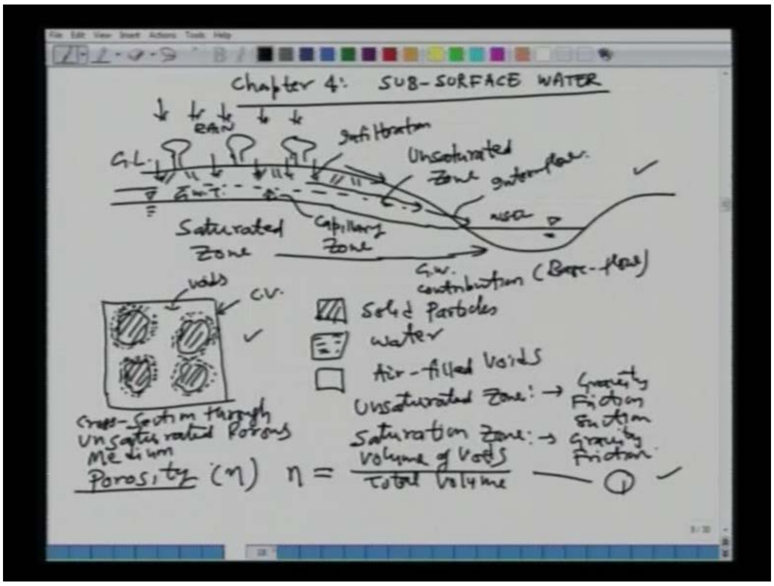

Good morning and welcome to this video course on Advanced Hydrology. Looking back in the last class we started the new chapter on sub-surface water, and I would like to go over these slides which we looked at in the last class, where you see initially we looked at the sub-surface water, the different physical processes, how they take place in the nature here. And then we looked at a cross-section through an unsaturated porous medium where we said how the air filled voids are changing as a function of time when we increase the water and then we define certain properties of soil that is porosity.

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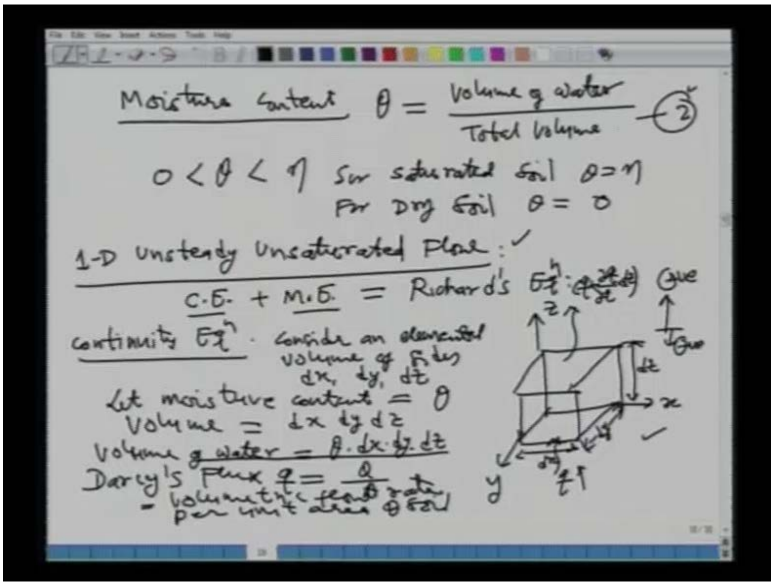

Then, we defined the moisture content like this equation and then we said that the moisture content in the soil is normally between 0 and eta, and eta is the porosity. Then we derived the basic governing differential equation for the one-dimensional unsteady unsaturated flow, which is called the Richard’s equation. It is a combination of the continuity equation and the momentum equation, where we looked at the control volume approach as shown here and then we applied the continuity and momentum equation. I would not go through this whole process again.

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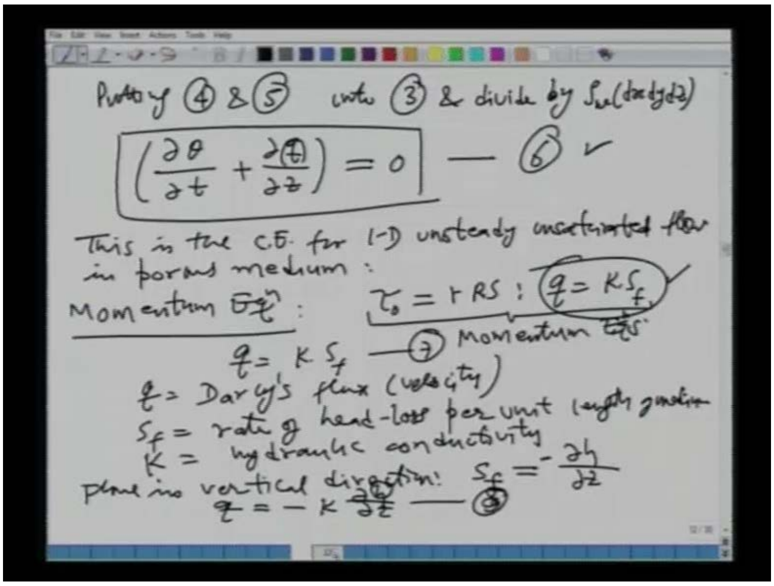

Continuing further, this is the continuity equation, alright. Equation number 6 we derived and then we took the momentum equation in the form of this Darcy's law, alright.

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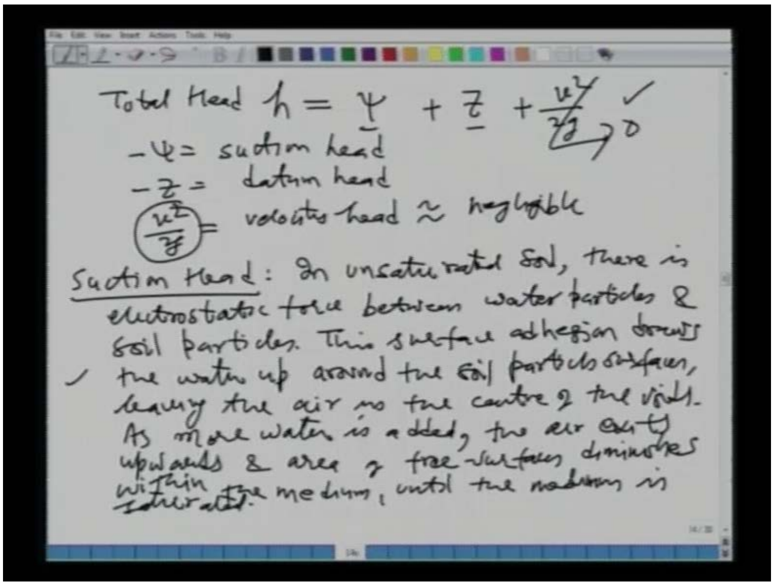

Then, we split the head or the energy which causes the water to infiltrate into two components. We neglected the kinetic energy and we saw it consists of suction head and the datum head. With this we look at the concept of the suction head here.

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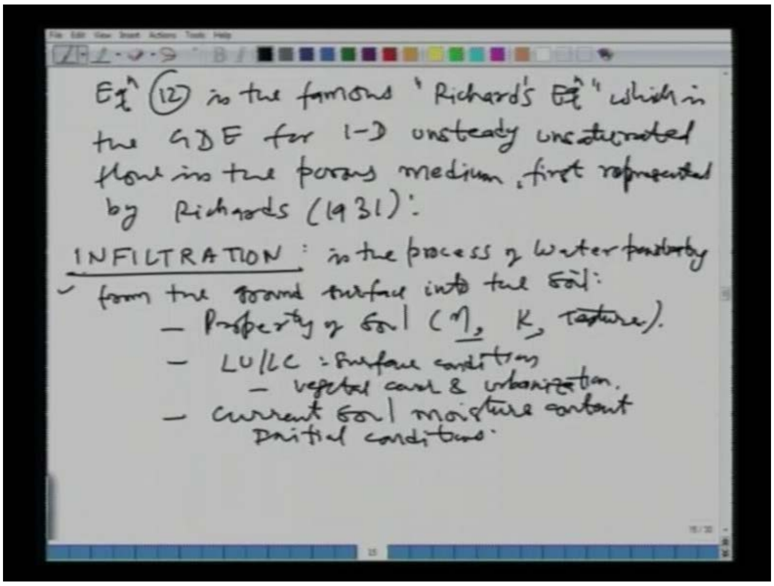

Then, we moved on to derive or combine the continuity and momentum equation to finally write the equation number 12 which is our Richard's equation, ok. Then, we moved on to the concept of infiltration. We defined what infiltration is. Then, we said what are the factors on which the infiltration depends and we said that it is the properties of the soil, it is the length use and length over properties of the catchment and also the antecedent moisture conditions or the initial conditions in the catchment.

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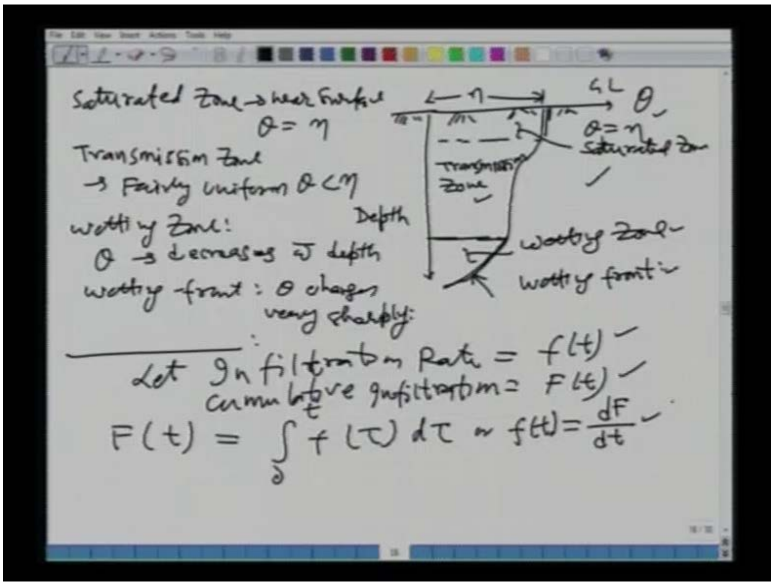

Then, we looked at the moisture profile as we go below the ground, all right. This is soil moisture profile during any rain falling line in which we classified the ground moisture conditions into three different classes. First is the saturated, then we have a transmission zone and then finally, we have the vacuum zone and that in front is something at which the moisture content decreases abruptly, and the vacuum front is marching downwards as a function of time during rainfall event.

Then, we defined what infiltration rate is and what accumulative infiltration is and then, we said that one is accumulative, the other or once we know one of them; other can be calculated with this. We would like to start with slightly different concept in the form of the equations which we study, whether it is Horton equation or the Green-Ampt equation what does it represent, all right. So, the first concept which is very important is one to understand the most infiltration equations.

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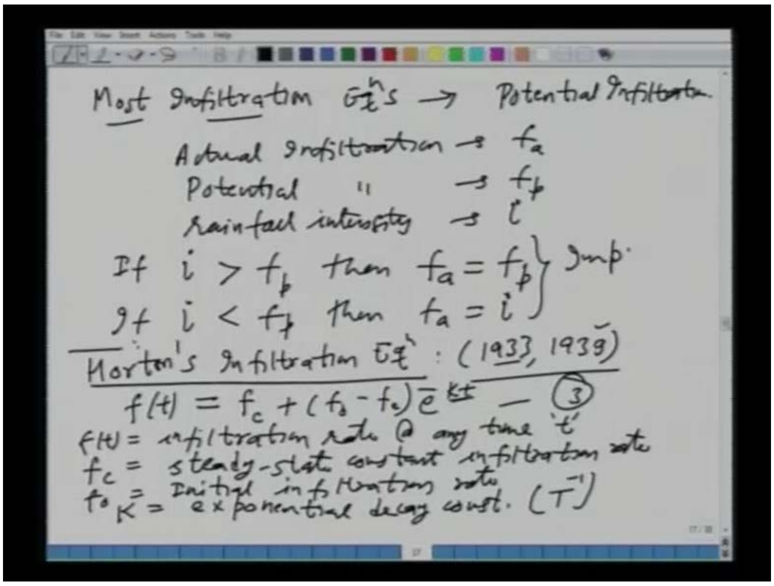

Do they give us the actual infiltration or the potential infiltration? I am sure you may have studied the Horton equation, all right. Now, does it give us the actual infiltration that is occurring, or it gives the potential? Some of you may know this, but all this infiltration equation, it gives us or they give us the potential infiltration that is under any given climatic and attachment conditions what is the maximum amount of infiltration that can occur at that given point of time. That is what any equation gives us.

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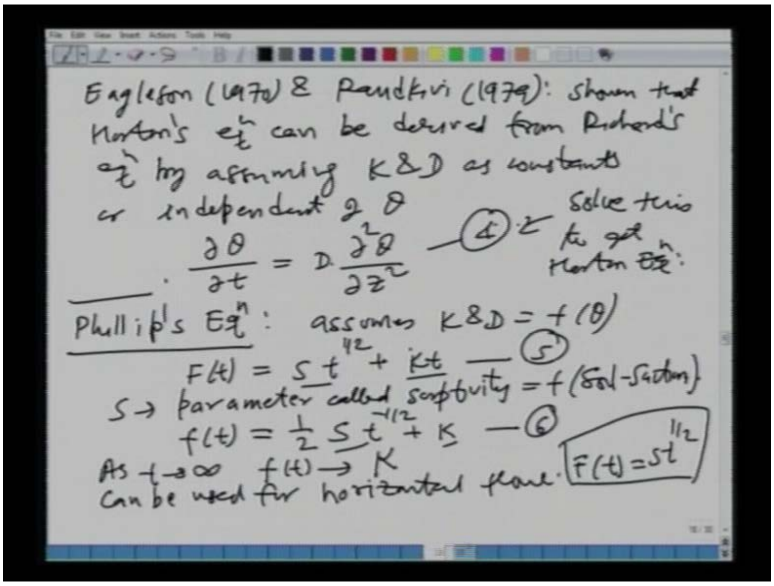

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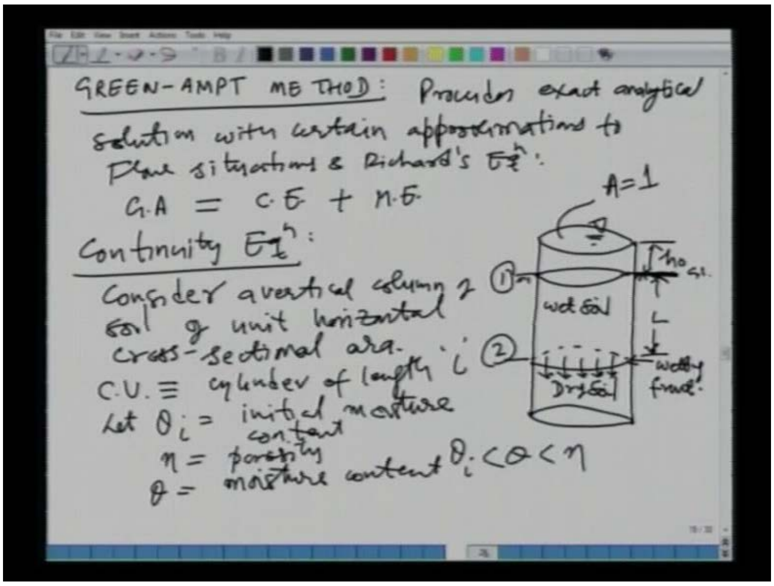

Then, the question that arises is how we find the actual which is called the Green-Ampt method? Green-Ampt method or Green-Ampt equation is as I said is very useful very important. It is widely acceptable and widely used. In fact, in many of the software is used to calculate infiltration. Why? It is because it has conceptual bases. So, it provides exact analytic solution with certain simplifications or approximation to the floor situation and your Richard's equation.

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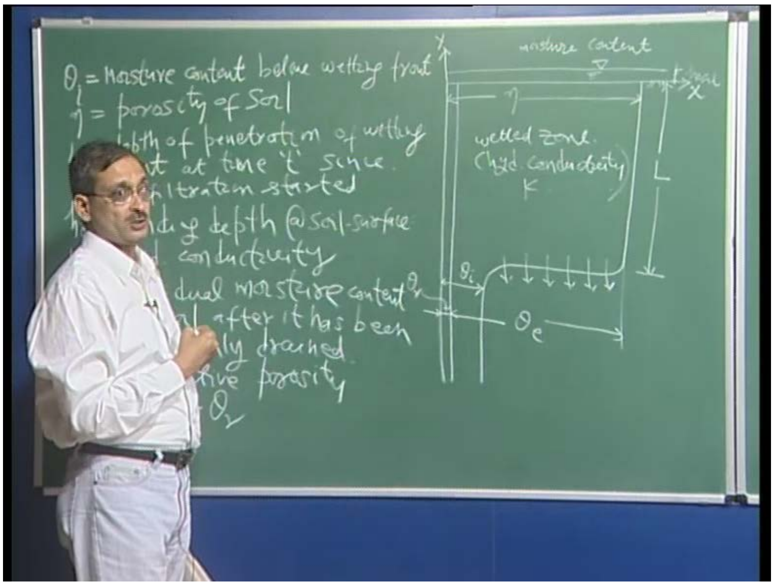

So, what we are going to do next is, I am going to look at the schematic of the soil measure profile which is used in the Green-Ampt equation to derive the final equations which can be used for calculating the infiltration. So, I am going to use the board. So, you come along with me and then, we are going to look at this soil moisture profile which is a simplified one that is h naught. It is the depth of the water on the ground, all right. This we are looking at in the ground level, all right. This is the soil moisture profile and this is the initial or residual moisture content. We are going to define everything here. This is called the wetted zone of hydraulic conductivity K.

Let us say this is my Y direction or the vertical direction, and this is your X direction on which we are plotting the moisture content. On the X direction, you have the moisture content. What is happening is this wetting front is continuously moving down into the ground. This is what is called Q per r or the residual moisture content, this is called theta e and I am going to define all those things in a second, and this is the maximum moisture content in the soil. This we have said already is equal to the porosity of the soil or theta. What is this? This is your theta i. So, initially this was the initial moisture content in the soil, but due to the rainfall, we have some ponding that here h naught and because of that we have saturated conditions close to the ground, all right and this is a snap shot at any given time t, all right. So, what is happening is this time, this is the depth of the vacuum front and this vacuum front will be marching down as a function of time. So, let us say at this any point of time, this length is L. L is the length or the depth of the wetting front at any given point of time.

So, let me define all these variables first. So, you have theta i is the moisture content below, the wetting front or the initial condition decision theta is the porosity of the soil or the maximum moisture content in the soil, L is the depth of penetration of the wetting front in or at time t. Since, the infiltration has started, this is important. Since, infiltration started that is to say when the infiltration starts, the length is 0 and in time t, the length is L. So, all the moisture content in this you know snap shot will represent the infiltrated water. We will come to that little later. H naught let us say is the ponding depth or the depth of the water which is ponded due to rainfall at the soil surface or on the ground. Let us assume that K is the hydraulic conductivity. We have said that K is the hydraulic conductivity of the soil and theta r is the residual moisture content of the soil after it has been thoroughly drained.

Yeah, I am sure you understand what do we mean by this concept of this residual content in which if we take a sample which is moist, and we start draining it, all right. If we just leave it, what will happen is most of the water will run out, but some water will be held back which is called the residual moisture content, all right and different soils will have different type of residual moisture content. So, it is a property of the soil we define what is theta e. Actually, theta is the effective porosity. What is effective porosity is the porosity which is available for water to get used the space, all right. So, this is nothing, but the total porosity minus the space which is not available for water to fill in the soil, all right. So, the effective porosity will be equal to the porosity and we subtract the moisture content. So, now, what we are going to do is, we will take a control volume within this depth of you know infiltrated water and we will use the control volume approach, and try to derive what will be the expression for the infiltrated water, ok.

So, we will move to our soft board here. So, we will derive the Green-Ampt equation as a combination of continuity equation plus the momentum equation, and what we have done is we have seen the simplified version of the soil moisture profile. So, using that with those definitions, we will write the continuity and momentum equation and we will combine these two and derive the final form of the Green-Ampt equation. So, what we do is, say we take, first we derive the continuity equation. So, for that what we do is we consider a vertical column of soil of unit horizontal cross sectional area. So, within that frame work which we have just seen, let us take that we have a cylinder. So, this is your depth h naught and this, obviously as you see is the ground level here. We have the depth of the wetting front if this is your L.

So, in the figure on the board, we are taking this cylindrical column within the length L and this is marching downwards. So, everything above the wetting front is the wet soil, this is your ground and this is your dry soil and what is this is your wetting front. We have taken a cross-sectional area of the cylinder as one square unit per meters or per square kilometers or whatever. So, with this frame work in mind, we will try to write the continuity equation. So, what we do then is, we consider our control volume as equivalent to the cylinder which we have shown on the right of length L. Let theta i is the initial moisture content in it, and we have defined eta as the porosity and theta is the moisture content at any time, and we know that theta will vary between initial moisture content. In this case which is theta i and eta.

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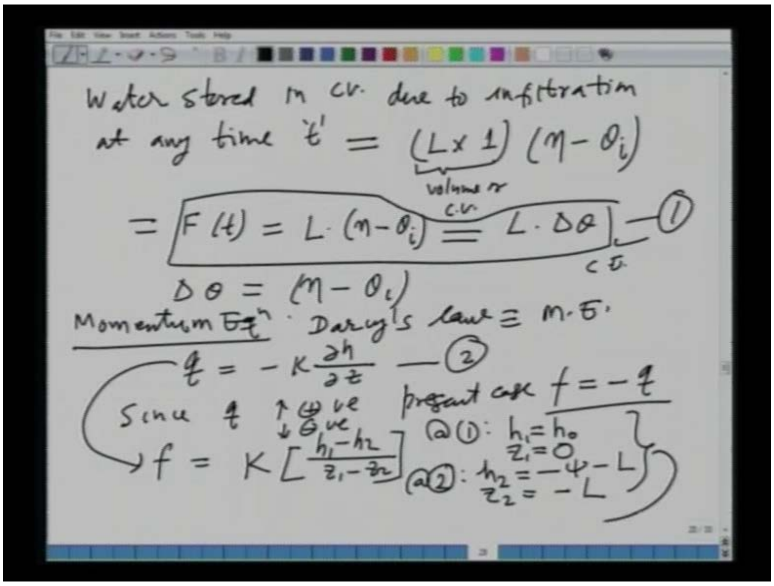

So, with this schematic in mind let us try to write down what will be the amount of water which is stored in the CV or in your cylinder. Due to infiltration at any time t, can we think about it? What will be the amount of water stored in that cylinder which we have just drawn at any given time t? Remember we have defined L as the length of the wetting front and wetting front represents, what it represents is the infiltrated water at any given time, all right. So, the amount of water stored as you can see will be equal to what will be equal to the length itself, and multiplied by the area square units, all right. So, this is the volume of the CV, this is the volume of a cylinder or the CV L times the cross-sectional area. That is the total volume how much water is there in this cylinder. What is the moisture content? It is the maximum one which is theta minus theta i. Theta i was initially already there.

So, the initial moisture conditions were there theta I, all right. They have gone up to eta or the maximum one due to infiltration, all right. So, the amount of infiltration then is equal to theta minus theta i times the volume. So, if that is clear, then we can write what will this be equal to this is your nothing, but your accumulating infiltration at any given time and that will be L times eta minus theta i and this is represented as L times delta theta. So, this is my continuity equation. I have used just the first preference where delta theta has been defined as eta minus theta i. It is a quantity which is representing the infiltration. It is the amount of soil moisture content in excess of the initial moisture content that is what basically delta theta is. So, this equation one represents your continuity equation.

Now, let us move on and try to write what will be the momentum equation for this and then, you can combine the two as we have seen earlier in the case of channel flow in the last couple of classes. Also, we have looked at the ground water flow situation. The Darcy's law represents nothing, but the momentum equation. So, in this case, you have q is equal to minus K del h over del z, that is your Darcy's law. Q is equal to q s f or K is the hydraulic conductivity and del h del x is negative. So, here put a negative sign here.

Now, since your q is positive upwards and it is negative downwards, all right, so in the present case, the water is going down or the rate of infiltration, then will be equal to minus q, the Darcy's law. With this we can write your f, that is your this will be equal minus will be taken care of and it will be equal to del h del z, and let me write it this one as h 1 minus h 2 over z 1 minus z 2. That is the first derivative, all right, del h del z.

Now, what we will do is when we go back to my previous line, this is your cross-section 1 and this is your cross-section 2. So, what we will do is we will write the del h del z term between these two cross-sections 1 and 2. So, what will be at 1? For example, what is h 1? H 1 is the head or energy which is causing the flow is h naught on the ground. At 1 we have h naught of the depth of water which is energy. What is z 1? X is that one is 0. That is our origin at the ground. We have datum. Let us say datum is at the ground z 1 will be equal to 0 now at the cross-section 2. What is h 2? This is going to be very important to understand. So, let me go back.

So, at this two, this is your direction, all right. So, this is your datum here. Now, what is the energy which is causing the water to infiltrate between these lengths L? So, there is infiltration is due to two forces. As we know, one is the suction head; another is the gravity head, all right. So, water is getting sucked into the soil due to the suction forces and let us say, the suction had a psi and it is because of the datum had which has length of L, all right and our notations are such that our positive direction is upwards. So, your h 2 then is equal to minus psi minus L. I would like to request you to try to understand this very carefully. H 2 is the energy which is causing the water to flow at cross-section 2. What will be z 2? Z 2 is nothing, but only the datum head at cross-section 2 which is minus L. So, what we do is we are going to plug these values of h 1, h 2, z 1, z 2 that into this equation, all of this, ok.

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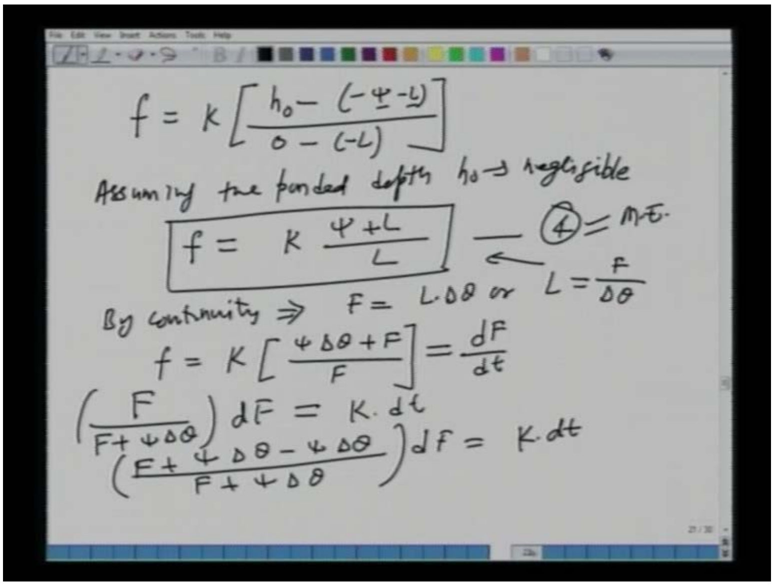

What do we get is your infiltration f is equal to k times h 1 which is h 0 minus h 2 which is minus psi minus L whole thing divided by z 1 minus z 2, all right. Z 1 minus z 2 will be nothing, but 0 minus minus L that is z 1 minus z 2. Now, what we do is we assume or assuming the pounded depth h naught to be negligible. As you know as engineers, we always try to simplify our life as compared to this psi and L which may be of the order of a few meters, all right. Then, h naught is a very small pounded depth on the ground which is you know less than an inch or may be a few millimeters you can neglect. So, if we neglect this into this equation, the final form what we get will be equal to k times psi plus L over L.

So, this is your equation number 4 which relates the rate of infiltration as per the Green-Ampt simplifications with the hydraulic conductivity, the suction head and the length of the wetting front. Now, what we do is by continuity this, this equation is what it is nothing, but your momentum equation. Now, what we do is we combine this momentum equation with the continuity equation by the continuity. What we have is remember it was f is equal to L times delta theta, right or we say L is equal to F over delta theta and h naught we have taken as 0. So, we put this into equation number 4. So, what we are going to get then is where f is equal to k times psi delta theta plus f over capital I should be you see that. Now, what we do is we say that this f is nothing, but dF over dt. Now, what we do is a separation of variables. So, we combine F comes on the left hand side here. So, it will be this, this dF. The f terms we take on the other side. This is equal to k dt. Now, what we do is slight algebraic manipulations. We take it on the other side and dF will be equal to K dt. So, all we have done is we have added psi del and subtracted psi del theta.

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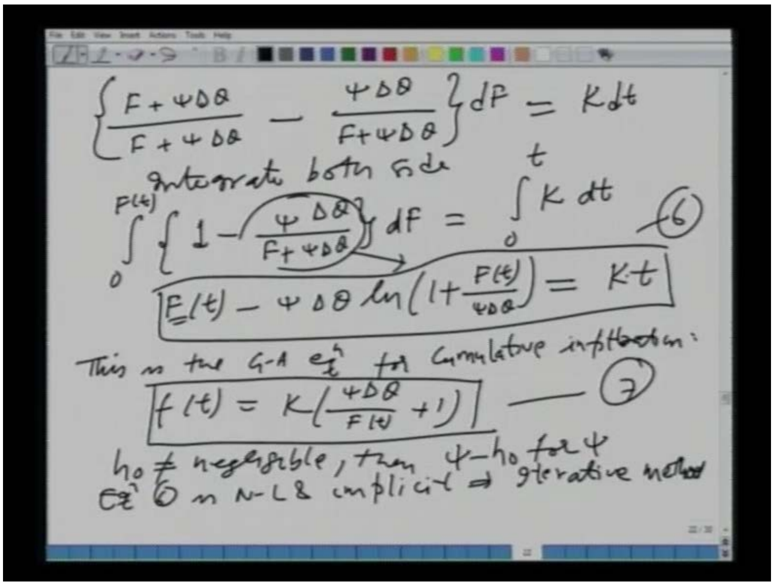

Moving on further, you will have minus psi delta theta and dF. We are just simplifying this equation a little bit is equal to K dt should be easy to see that. Now, what we do is we integrate on both sides. One with respect to F and other is with respect to time. If we did that, we will have 0 t between 0 and t, all right. So, on the left hand side, the limits will be 0 to F and what we have is 1 minus psi delta theta over this dF is equal to limit would be between 0 and t of your K dt. After the integration and putting the limits, you can do that (( )) f t minus psi delta theta and this is the natural log of 1 over X will be equal to 1 plus F of t over psi delta theta. We integrate this quantity, all right. That is what you will get and on the right hand side, K is constant. So, it will be K t minus K times 0. So, it will be K dt. So, this is your equation number 6 and this is your Green-Ampt equation, the Green-Ampt equation for accumulative infiltration.

Now, once this F is found, we can always find the F by the equation which we have already written. So, it will be K times of your psi delta theta over F of t plus 1. This equation we have already written. So, there is a relation between the infiltration rate F and accumulative infiltration F and we can find out is this equation number 6, and let me name this as number 7. In case this h naught which we have neglected, we cannot neglect it to the significant. All we need to do is let me say if h naught is not negligible, then what we do is we use psi minus h naught for psi in this equation. So, that is the only change going to take place is you can verify that.

Now, this equation number 6 which you have just seen or derived, there is F. It appears on both the sides and it is non-linear in nature. So, it is a non-linear implicit equation which is not easy to solve, all right. Equation 6 is non-linear and implicit, all right. So, some alternative procedure has to be used to solve it. So, this alternative procedure can either be a manual trial and error or the Newton (( )) or the successive approximation. Any kind of alternative method needs to be used with this like to move on and define another important concept sometimes in terms of the certain parameters of this Green-Ampt equation.

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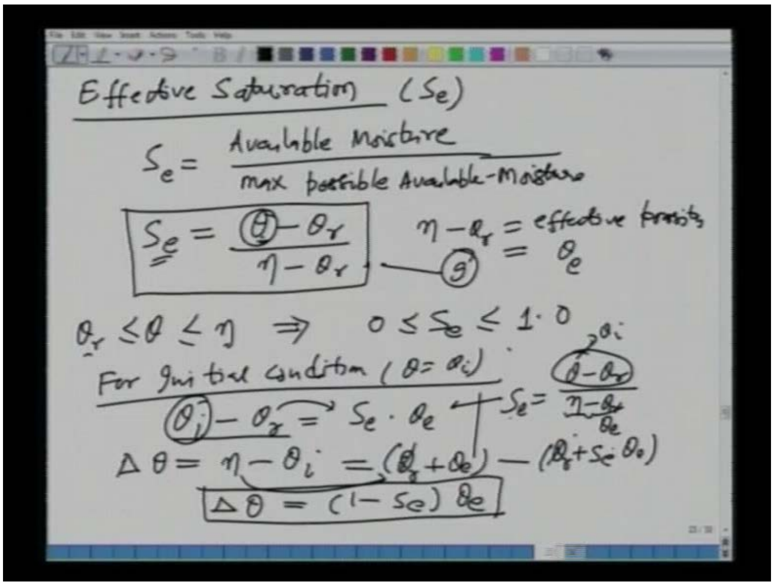

We call it effective saturation and this effective saturation is defined or denoted rather Se. This is defined as the ratio of the available moisture divided by the maximum possible available moisture in the soil. So, what is this then? What is the available moisture in the soil? At any given point of time, theta is the moisture content at any point of time and available is we subtract the theta r and maximum possible is nothing, but then maximum is neta minus theta r, but eta minus theta r we have defined as the effective porosity, right. It is let us say, theta e. So, when your theta varies between theta r and eta, what will be the limits of your Se? You look at the definition of the effective saturation Se. When theta is equal to theta r, then that is the lower limit of moisture content. Then, if you put theta is equal to theta r here, you can see that Se will be equal to what 0. Similarly, when theta is equal to eta, all right, that is this one then your Se will be equal to 1.0. It should be very easy to see, ok.

So, we have this important equation which relates the effective saturation with the moisture content at a given point of time. Now, what we do is we will write this effective saturation for initial condition, that is when theta is equal to theta i. So, what we have is initially theta i minus theta r will be equal to Se times theta e. You see that how is this equation coming from the definition of Se. It is theta i, sorry theta minus theta r over eta minus theta r. In this we put theta is equal to theta i and this whole thing is effective porosity and from here that is where I have come, all right, that is theta i minus theta r. That is the numerator here will be equal to Se times theta e and then, name this, number this equation as equation number 9.

So, now, what will be the delta theta which is one of the parameters of the Green-Ampt equation? So, this will be equal to eta minus that is what we have defined initial this is equal to theta r minus theta e which is eta minus. What is theta i? From this equation, theta i is equal to if you can take this on the other side, that is what we will have theta r plus Se times theta e. So, all I am doing is I am putting theta i from this equation here and eta from the basic definition. It is the sum of effective porosity in the residual moisture content. From this your theta r will get cancelled out and finally, what we will have is your delta theta is equal to 1 minus Se times theta e. This is an important relation because delta theta is the initial moisture conditions for any type of soil we need to know that for calculating the infiltration, ok.

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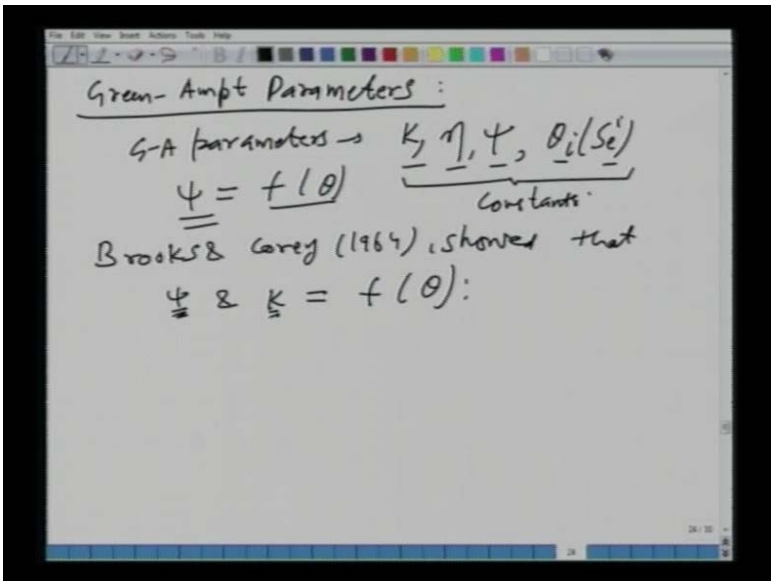

Now, the last thing today we are going to look at is on the Green-Ampt parameters, we have seen the derivation of the Green-Ampt equation and it involves certain parameters that we need to know before we can calculate infiltration for applying any model. As you know we need to do the model calibration. Similarly, we need to find out what the infiltration parameter is or the Green-Ampt parameters for using this. As you can see the GA parameters are those hydraulic conductivity is what we need to know for that soil, the porosity is what we need to know, also the suction hand psi for that particular type of soil and also the initial conditions. So, for a particular type of storm or a particular situation theta i or in other words, your Se for initial conditions is what we need to know.

Normally, this psi is function of the moisture content. Most of these parameters are considered them as constant. Let me write them as we consider them as constants, that is to say their values are not changing as a function of time during the rain falling event. What is the constant? It is which does not change with time as far as these parameters are concerned. However, we know that the suction head and the hydraulic conductivity, these parameters can change as a function of time. So, this psi is equal to function of theta and what many people have done is, they have tried to work on this relationship and one of them is Brooks and Corey. In 1964, they showed that these both psi and K can be derived as a function of the soil moisture content, and they gave an equation which will be used for calculating their values as a function of time or as a function of theta, all right. So, psi as well as K.

I am running out of time today. So, what we will do is, we will come back and look at these Green-Ampt parameters, and the equation which is given by Brooks and Corey how we can represent these two parameters as a function of theta.